A C T A   U N I V E R S I T A T I S   L O D Z I E N S I S
FOLIA OECONOMICA 292, 2013

Alicja Olejnik[*]

ASSESSING THE SPACE-TIME STRUCTURE WITH A MULTIDIMENSIONAL PERSPECTIVE

1. INTRODUCTION

The correct assessment of the dependence structure among cross-sectional observations is one of the most important problems in spatial econometrics. Ina standard approach researcher is obligated to specify a set of spatial neighbours for each location which is used for assessing of the spatial stochastic process where the covariance structure is modelled indirectly by the relation of membership in the set of neighbours (Anselin and Bera 1998, pp. 237–289).

For a classic spatial autoregressive SAR model for cross-sectional observations with normal disturbances: $\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \mathbf{u}, \quad \mathbf{u} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$, $\mathbf{y}(N\times 1)$ is a spatially lagged endogenous variable and $\mathbf{X}(N\times K)$ matrix of observations on K exogenous variables, where matrix **W** is a given *a priori* spatial weight matrix representing the neighbourhood structure (w_{ij}= 1 - if the locations i and j are neighbours and zero otherwise).The specification of matrix **W** is a matter of considerable arbitrariness and should be done with a great attention. Typically, in regional science, the spatial weight matrices, are based on distance relationship and contiguity (Anselin 1988). However, let us notice that in some empirical studies these matrices may be insufficient (among others, cf. Deng 2008, pp. 26–51).

As an example, Dacey in (1968) defined a non-symmetric spatial weight matrix that combined the relative area of spatial units with a binary contiguity factor and relative length of border between spatial units. Similarly, Cliff and Ord (1981) suggested a combination of distance measure and the boundary measure. On the other hand Bodson and Peeters (1975) introduced a general accessibility weight matrix that combined the influence of several channels of communication between regions with the relative importance of the means of communication (such as roads, railway lines etc.) and the distance between spatial units. Alternatively, Besner (2002) presented a model with the spatial weight matrix constructed on the basis of similarity measures in socio-economic variables. Recently, Getis and Aldstadt (2004) suggested a model where the spatial weights are constructed using the Getis-OrdGi local statistic (cf. Ord and Getis 1995, pp. 286–306).

[*] PhD, University of Lodz.

Most of the above solutions for spatial weight matrices are asymmetric and they attempt to differentiate the strength of spatial dependence. However, they all incorporate unknown parameters in the weights. As noticed by Anselin (1988) this can create estimation and interpretation problems as well as lead to the inference of spurious relationships. Given the application difficulties of the parameterised weight matrix **W** an alternative approach has been demonstrated in Deng (2008). The author proposed a general model in which parameters associated with the spatially lagged dependent variable term are the arguments of a non-linear function which constitutes a general weight matrix.

According to the field of research different concepts of distances or neighbourhood are considered, since different factors result in spatial dependences or directly affect it. For example, in some cases the researcher must take into account isolated units without any direct geographical connection with other regions. Sometimes it may seem to be appropriate to diversify the strength of the influence of the neighbours. For instance, the spatial dependence of commercial activities between a municipality and its surrounding rural area appear to be asymmetric. As a result the spatial structure introduced in the model sometimes should represent not only geographical relations but also socio-economic interactions such as commuting and trade flow or even ethnic linkages and many others. As the spatial weight matrix is, in fact, meant to approximate the true spatial relations (regardless of its nature) it seems that it is justified to let for even more flexibility in the representation of the spatial structure.

The objective of this paper is to present some remarks on a procedure for the analysis of multidimensional spatial dependence structure in spatial, time and space-time context. In this study we discuss the idea of *multidimensional spatial weight matrix* and *multidimensional spatial coefficient matrix* which give the opportunity for better description of the complete pattern of the dependence structure (cf. Olejnik 2012a).The model with additional time dimension allows for more detailed analysis of the spatial relations in terms of stability of space, time and even investigation of time lag in the spatial structure. The remainder of this paper is structured as follows. Section 2 introduces the general idea of the multidimensional spatial weight matrix with a strategy for incorporation of multidimensional matrix into a spatial econometric model. Finally, Section 4 provides a summary and some concluding remarks.

2. MULTIDIMENSIONAL SPATIAL WEIGHT MATRIX

Spatial autoregressive model contain spatial parameter ρ multiplied by the spatial weight matrix **W**. Then it is impossible for various degrees of different types of spatial influence to be estimated in the model. The general concept is to let the spatial weight matrix be a matrix with more than two dimensions and, as a result, better describe the spatial interactions of the cross-sectional units. The natural consequence of this is that the spatial autoregressive coefficient ρ becomes at least a vector of parameters. Apparently, such idea is a straightforward generalization of standard concept and enables more complex analysis of the spatial relations.

We assume that the elements of the multidimensional spatial weight matrix are derived from information on the spatial arrangement of the observations in its general sense. All elements of the multidimensional spatial weight matrix must be positive and the matrix itself must be observable and exogenous. The last assumption is especially significant for the specification of the socio-economic interactions, where a great attention must be paid to avoid the endogeneity problem.

Employing multidimensional weight matrix allows for considering in one model several sources of dependency between spatial units at the same time. In this paper, for simplicity of presentation, we restrict our examples to the case of three dimensional weight matrix (cf. Olejnik 2012a, 2012b). Then, in the case of the three-dimensional spatial weight matrix each 'slice' of the matrix can be a separate, independent weight matrix itself so that the multidimensional spatial weight matrix can incorporate several measureable spatial relationships. For example, one slice can describe purely geographical interactions, other can concern economic, social, political and trade or ethnic relations. The additional dimension in the three dimensional weight matrix will be called shortly the *economic dimension*.

In some cases, establishing the proper structure of the spatial interactions may require considering explicitly a three dimensional space. For example, the third dimension could be the size of the enterprises or even the level of the development of the spatial region. We expect that large neighbouring enterprises interact differently from the small ones. Similarly, spatial interactions between neighbouring high-developed regions are different from the low-developed ones.

Let us propose the following notation (cf. Olejnik 2012a, 2012b, Suchecki 2012):

$$\mathbf{y} = \boldsymbol{\rho}\mathbf{D}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \mathbf{u} \quad \mathbf{u} \sim \mathrm{N}\left(\mathbf{0}, \sigma^2\mathbf{I}\right), \tag{1}$$

where the spatial structure of the process is represented by the general expression $\boldsymbol{\rho}\mathbf{D}\mathbf{y}$. The matrix $\mathbf{D}$ denotes a multidimensional spatial weight matrix of dimension $N \times P1 \times P2 \times \ldots \times PR \times N$, while $\boldsymbol{\rho}$ is a multidimensional spatial coefficient matrix. The form of $\mathbf{D}$ and $\boldsymbol{\rho}$ depends on the subject of interest of the empirical study. In the expression $\boldsymbol{\rho}\mathbf{D}\mathbf{y}$ we use a multidimensional multiplication which is determined by the dimensions of $\mathbf{D}$ and $\boldsymbol{\rho}$ (cf. Olejnik 2012a, 2012b).Henceforth, the model incorporating a multidimensional spatial weight matrix $\mathbf{D}$ of the form (1) has been called the *Multidimensional Spatial Autoregressive Model* (MSAR) (cf. Olejnik 2012a).

In some applications of the New Economic Geography theory it may appear to be useful to employ a three dimensional spatial weight matrix which distinguishes the strength of influence between core and peripheral regions (Fujita 1999). Let us consider the case in which we have *N* different spatial units. Thus, from a set of all *N* regions we can choose a subset of regions being local centres with high level of technology, employment and with high potential for innovation and growth (e.g. metropolitan regions). Therefore it seems to be reasonable to assume that core regions have not only greater impact on their neighbours but also interact within greater radius than the peripheral ones as they affect more distant regions e.g. due to extensive trade or specialized labour market. In the following example the three-dimensional spatial weight matrix will take the form of two layers (flat matrices) revealing the pure distant effect and core regions effect. Therefore, the first layer of the matrix represents the geographical proximity whereas the second one considers only core regions and their wider range of neighbours. Let us define $S(j)$ as a set of neighbours of region j i.e. a set of nearest neighbours affected by region j or regions located within a certain radius r from the centre of region j. Furthermore let C be a set of core regions selected from all N regions and SC be a set of neighbours of core regions (i.e. regions located within a greater radius: $r_c>r$, or larger number of nearest neighbours). Then, the matrix **D** is of dimensions $N\times 2\times N$ with elements defined by

$$D_{i1j} = \begin{cases} 1 & \text{for } i \in S(j) \\ 0 & \text{otherwise} \end{cases}$$
$$D_{i2j} = \begin{cases} 1 & \text{for } \left(i \in SC(j) \wedge j \in C\right) \\ 0 & \text{otherwise} \end{cases} . \qquad (2)$$

In this simple example we associate one parameter with a layer. The parameter ρ_1 connected with the first layer reflects the spatial dependence of pure geographical effect. From the definition of matrix **D** (equation **Błąd! Nie można odnaleźć źródła odwołania.**) it can be seen that the parameter ρ_2 associated with the second layer express the level of conditional spatial effect, namely how regions 'surrounded' by core regions benefit from their location. However, it should be emphasized that the precise interpretation of this parameter is not straightforward. Indeed, the change: by α in neighbouring, but not core regions, by β in neighbouring core regions and by γ in neighbouring core, but distant regions (distant with radius greater than r and less than r_c), *ceteris paribus*, change the dependent variable y by:

$$\rho_1\left(\alpha\sum_{j:i\in(S(j)\setminus SC(j))} D_{i1j} + \beta\sum_{j:i\in(SC(j)\cap S(j))} D_{i1j}\right) +$$
$$+\rho_2\left(\beta\sum_{j:i\in(SC(j)\cap S(j))} D_{i2j} + \gamma\sum_{j:i\in(SC(j)\setminus S(j))} D_{i2j}\right).$$

It is noteworthy that this kind of matrix may be interesting not only from the viewpoint of empirical studies. Let us notice that **D** does not satisfy the, somewhat demanding 'row-wise orthogonality condition':

$$(D_{ikj} \neq 0) \Rightarrow (\forall_{l \neq k} D_{ilj} = 0)\,.$$

(cf. Anselin and Smirnov 1996; Anselin and Bera 1998). Nevertheless it is still a proper spatial weight matrix which can be introduced into an econometric model.

The extension of the MSAR model with the additional time dimension allows the researcher to look at the *spatial coefficient* in a different way; now it is a vector of parameters rather than a single number. At the same time with time dimension added the ability to estimate additional parameters is enhanced.

The motivation for multidimensional line of thinking has been the need to solve some issues concerning regional development in the EU. To conceptualize the idea we focus on a number of problems considering NUTSII regions (cf. Olejnik 2012b).In a classic approach we assume geographically neighbouring regions do interact with each other and these interactions are the same within and outside countries. However it does not necessary reflect the true spatial structure. Similarly the assumption of the same power of spatial interaction within and outside the Euro Zone might be too restrictive. Correspondingly the same dubious issues arise for old or new EU countries, inside and outside Schengen Area or other historically or culturally close or distant regions. Therefore, it seems to be justified to test whether all of the regions of interest influence their neighbours in the same way. Let us notice that the time dimension gives new 'spatial' possibilities to account for spatial dependence in terms of more possible spatial parameters to estimate. As a consequence employing the time dimension raise the question whether the power of interaction remains the same over time. Finally is there any time lag in the spatial interaction? Therefore in this section we propose a methodological solution to deal with the problems mentioned above by presenting the *Multidimensional Spatial Panel Model*- MPSAR.

Following Paelinck (2012) the dependence should be adopted to the specific region since every region has its own level of economic activity. In particular, as the transport flows are different in urban and rural regions the spillover effect may vary from region to region. Let us assume that the spatial coefficient diversified across some groups of regions and is constant within the groups. Let $P=\{\rho_k\}$ denote the set of all possible spatial coefficients. Therefore, with N spatial observations the maximal cardinality of the set is also N. In classic spatial setting the estimation of such econometric model is not feasible. In space-time case however theoretically one can consider estimation of the parameters over the time dimension obtaining different spatial coefficients for each or almost each location (assuming that T is sufficiently large). If the number of time observations is relatively small one can still consider a coefficient for a few spatial locations. Henceforth we will call the former as the *local spatial effect* ρ_i and the latter: *group specific coefficient* ρ_k^{gr}.

Therefore, in the first case the *local spatial effect* ρ_i measures the power of spatial interaction for a specific location *i* with its neighbours $N(i)$ over time. In particular it indicates the strength of influence of region *i* on its neighbours or how the neighbours affect region *i*. Alternatively, we could exclude some locations and focus (within one slice of the multidimensional weight matrix) on only a few regions of interest. It should be emphasised that in our model the set of neighbours $N(i)$ is allowed to change over time as the spatial weight matrix can change over time (eg. different $\mathbf{W}^t$ for old and new Schengen Area). In the same manner, the power of interaction with its neighbours may change over time.

The *group spatial coefficient* $t\rho_k^{gr}$ averages spatial effect within a group of interest. For example, considering the EU regions, the subject of interest might be spatial spillover effect separately among new-member and old-member countries. The group spatial coefficient for the first cluster accounts for spatial interaction specific for new member countries where the second one explains spatial dependence within the group of old member countries. Alternatively we could consider clusters for countries with similar convergence objectives. In this case the group spatial coefficient embodies the spatial spillover effect among regions of countries with similar convergence objectives. Another example of the use of would be the group spatial coefficient is a differentiation of spatial effect for strong and catching-up economies or any other in the case where we expect significantly dissimilar spatial dependences between groups but similar within the group. Analogically to the local spatial effect the group spatial effect may very over time $-\rho_k^{gr,t}$. It is noteworthy that theoretically each group can be a singleton with only one region making the *local spatial effect* a special case of the *group specific coefficient*. On the other hand the *dynamic spatial effect* δ^d_{it} reflects the power of spatial interaction for a specific location *i* in moment *t* with its neighbours $N(i)$ at the moment $t+1$, $t+2$,…. Hence, it can embody an economic spatial interaction which is spread with a certain delay (e.g. economic growth). Let us notice that in some cases the *dynamic spatial coefficient* may appear even more appropriate than the classic one.

In *Multidimensional Spatial Autoregressive Model* – MSAR the spatial structure is represented by **W** or **D** matrix. The typical weight matrix **W** considers only neighbourhood relations and omits the power of interactions. The multidimensional spatial matrix **D** is built a priori to incorporate varied strength of interactions between neighbours. In effect, in MSAR model the information about the spatial structure must be put into the model. On the other hand, in *Multidimensional Spatial Panel Model* (MPSAR) the information on the strength and significance of the spatial local interactions is given by the model.
In MPSAR model the *multidimensional spatio-temporal structure* is represented by Υ term:

$$\Gamma_{ij(TN\times TN)} = \mathbf{\Psi}_{ij}^{1}\mathbf{D}_{ij}^{1} + \mathbf{\Psi}_{ij}^{2}\mathbf{D}_{ij}^{2} + \ldots + \mathbf{\Psi}_{ij}^{\eta}\mathbf{D}_{ij}^{\eta}$$

where $\mathbf{\Psi}$ refers to *multidimensional spatial coefficient matrix* and **D** – *multidimensional spatial weight matrix*, *i,j* – refers to locations, $\eta<E$ indicates economic dimension.

In **P** matrix addition the spatial coefficients are allowed to vary for both: different regions, group of regions, time and economic dimension. Therefore the matrix of spatial coefficients in general combines local, group and dynamic effect as well as economic factors. Thus, the dimension of triangular matrix $\mathbf{\Psi}$ is ($TN\times TN\times E$). Hence for fixed η the main diagonal of matrix $\mathbf{\Psi}$ is given by: $[\mathbf{\Phi}^{\eta,1},\mathbf{\Phi}^{\eta,2},\ldots,\mathbf{\Phi}^{\eta,T-1},\mathbf{\Phi}^{\eta,T}]$ and $[\mathbf{\Delta}_{t-j}^{\eta,t}]$ elements below the main diagonal, where: $\mathbf{\Phi}^{\eta,t} = \boldsymbol{\iota}_T \otimes [\rho_1^{\eta,t} \ldots \rho_N^{\eta,t}]$, $\mathbf{\Delta}_d^{\eta,t} = \boldsymbol{\iota}_T \otimes [\delta_1^{\eta,t,d} \ldots \delta_N^{\eta,t,d}]$ with $\boldsymbol{\iota}_T$ as vector of ones.

The associated *multidimensional spatial weight matrix* **D** (*TN*x*TN*) is also a triangular matrix with the main diagonal: $[\mathbf{W}^{\eta,1},\mathbf{W}^{\eta,2},\mathbf{W}^{\eta,T-1},\mathbf{W}^{\eta,T}]$ and $[\mathbf{\Omega}_{t-1}^{\eta,t}]$ elements below the main diagonal. Matrices**W**-s referring to a different spatial structure in a separate moments in time and **Ω**-s as matrices of spatio-dynamic interactions. In particular $\mathbf{\Omega}_d^{\eta,t}$ represent dynamic spatial dependence of region *m* with *d*-th delay. Thus, for an individual observation:

$$y_m^{\eta} = \mathbf{\Delta}_d^{\eta,t}\mathbf{\Omega}_d^{\eta,t} y_{m-d} .$$

Therefore, the multidimensional panel spatial autoregressive model takes the following form:

$$\mathbf{y}_{(it)} = \mathbf{\Gamma}\mathbf{y}_{(it)} + \mathbf{X}_{(it)}\boldsymbol{\gamma} + \boldsymbol{\varepsilon} \quad \boldsymbol{\varepsilon} \sim \mathrm{N}(\mathbf{0},\sigma^2\mathbf{I}) ,$$

with **y** as dependent variable ($TN\times 1$), *i*=1,…, *N*, t=1, …, *T*, **X**– ($TN\times K$) – matrix of explanatory variables and $\mathbf{\Gamma}$ – *spatio-temporal structure* ($TN\times TN$).

3. CONCLUSION

This study is fundamentally based on the concept of the importance of correct assessment of the *multidimensional space-time structure*. It has been argued that taking into account the time dimension makes it possible to fully describe the pattern of spatial dependence structure. In our paper we have explained the advantage of the MPSAR model in which the information on the strength and significance of the spatial local interactions is given by the model. It has been stressed that failure to recognise these multidimensional effects may lead to incorrect inference and therefore to biased conclusions. In multidimensional spatio-temporal analysis different forms of spatial interactions across different spatial objects are allowed for to successfully model the complex structure of economic processes.

Concluding, further work needs to be done to develop a proper technical tool and to make use of our findings in the designed experiment. However, we hope that our contribution will draw the attention of researchers to the interesting topic of *multidimensional spatio-temporal structure* and will encourage consideration and employment of those ideas in empirical studies in regional science.

REFERENCES

Anselin L. (1988), *Spatial Econometrics: Methods and Models*, Kluwer, Dordrecht.

AnselinL., Bera A.K. (1998), *Spatial dependence in linear regression models with an introduction to spatial econometrics*,(in:) Ullah A., Giles D. (eds.), Handbook of Applied Economic Statistics, Marcel Dekker, New York.

Anselin L., Smirnov O. (1996), *Efficient algorithms for constructing proper higher order spatial lag operators*, Journal of Regional Science, Wiley, 36.

Besner C. (2002), *A Spatial Autoregressive Specification with a Comparable Sales Weighting Scheme*, Journal of Real Estate Research, American Real Estate Society, New York, 24

Bodson P., Peeters D. (1975), *Estimations of the Coefficients of a Linear Regression in the Presence of Spatial Autocorrelation: An Application to a Belgian Labour-Demand Function*,‘Environment and Planning’ A 7 (4)

Cliff A., Ord, J.K. (1981), *Spatial Processes: Models and Applications*, London: Pion.

Dacey M.F. (1968), *A Review of Measures of Contiguity for Two and K-Color Maps*,(in): Englewood Cliffs SpatialAnalysis: A Reader in Statistical Geography, Prentice-Hall.

Deng M. (2008), *An anisotropic Model For Spatial Processes*, Geographical Analysis, Wiley 40(1).

EUROSTAT (2002), *European regional Statistics. Reference guide*, European Communities, Luxembourg.

Fingleton B. (2006),*The new economic geography versus urban economics: an evaluation usinglocal wage rates in Great Britain*, Oxford Economic Papers, 58.

Fujita M., Krugman P., Venables A. (1999), *The Spatial Economy: Cities, Regions, and International Trade*, MIT Press, Cambridge MA.

Getis A., Aldstadt J.(2004), *Constructing the Spatial Weights Matrix Using A Local Statistic*,‘Geographical Analysis’,Wiley, 36

Kelejian H.H., Prucha I.R. (1998), *A Generalized Spatial Two-Stage Least Squares Procedure for Estimating a Spatial Autoregressive Model with Autoregressive Disturbances*, ‘Journal of Real Estate Finance and Economics’, Springer, 17

Mankiw N., Romer D. Weil D. (1992), *A contribution to the Empirics of Growth*, Quarterly Journal of Economics, 107.

Olejnik A. (2008), *Using the spatial autoregressively distributed lag model in assessing the regional convergence of per-capita income in the EU25*, Papers in Regional Science, Wiley, 87/3.

Olejnik A. (2012a), *Multidimensional spatial process of productivity growth in EU 22*, working paper.

Olejnik A. (2012b), Spatial *autoregressive model – a multidimensional perspective with an example study of the spatial income process in the EU 25*, working paper.

Ord J.K., Getis A. (1995), *Local Spatial Autocorrelation Statistics: Distributional Issues and an Application*,‘Geographical Analysis’ 27.

Alicja Olejnik

ASSESSING THE SPACE-TIME STRUCTURE WITH A MULTIDIMENSIONAL PERSPECTIVE

This study presents some remarks on procedure for space-time process investigation by the use of multidimensional panel spatial autoregressive model. It is shown that information on the strength and significance of the spatial interactions is given by the model. Motivation for the use of multidimensional dependence structure as well as some empirical examples are provided. It is argued that such approach could allow for more accurate description of the spatial dependence, whose true form often has a spatio-temporal character. It is emphasised that failure to recognise these multidimensional effects may lead to incorrect inference and therefore to biased conclusions.

ZASTOSOWANIE PODEJŚCIA WIELOWYMIAROWEGO DO OCENY STRUKTURY PRZESTRZENNO-CZASOWEJ ZJAWISK EKONOMICZNYCH

Przedmiotem referatu jest ocena procesu przestrzenno-czasowego z zastosowaniem wielowymiarowej macierzy wag przestrzennych. W szczególności zakłada się, że podejście wielowymiarowe pozwala na lepszy opis struktury zależności przestrzennych. Praca ma na celu pokazać nowo opracowaną metodologię dotyczącą wielowymiarowego autoregresyjnego modelu przestrzennego WAMP z uwzględnieniem wymiaru czasowego. Zatem całość rozważań stanowi nowy element ekonometrii przestrzennej, a poprzez włączenie dodatkowej informacji na temat badanego zjawiska umożliwia wnikliwszą jego analizę.