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THE PROPOSITION OF MEASURE OF PENSION FUNDS' EFFECTIVENESS [1]

Abstract. The aim of this paper is to propose a new measure of pension funds' effectiveness. The presented measure is an alternative to the classical rate of return, but the additional assumption takes into consideration the results of whole group of funds. In the light of the Polish law the relation between the financial results of the fund and the results of the group is very important. The Polish legislations also requires the presented measure to take into account the interval of 36 months, but the last 12 months are considered to be the most significant period. The construction of the measure is based on the author's proposition of the price index (in this case – the index of values of units).

Key words: price indexes, Attraction of Dynamics of Funds, open Pension Funds.

I. INTRODUCTION

Open Pension Funds are institutions which should invest their clients' money in the most effective way. There are a lot of measures of the efficiency of these investments. The measures should be well constructed – it means that all changes of funds' assets, connected with any investment, should influence the given measure. It is very important to calculate the average rate of return of a group of pension funds. Firstly, having this result we can compare any fund with the group. The *good* fund should be more effective than, on average, the group. But, first of all, in the Polish law regulations (The Law on Organization and Operation of Pension Funds, Art. 173, Dziennik Ustaw Nr 139 poz. 934, Art 173; for the English translation see *Polish Pension*..., 1997) the definition of the average return of a group of funds determines a minimal rate for any fund (see Białek(2005), Gajek, Kałuszka (2000)). In this paper we propose a definition which – according to the Polish law – takes into account 36 months of fund's activity. The proposed measure is based on relation between the result of a given fund and the average result of the group. In other words – we are going to

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construct a measure which would show the dynamics of unit of the given fund in relation to the group. Firstly, we give a definition of the average unit price dynamics of the group of Open Pension Funds.

II AVERAGE UNIT PRICE DYNAMICS IN TIME INTERVAL

Consider a group of N Open Pension Funds ($N > 1$). We observe at discrete time moments $t = 0, 1, 2, ...$ the following variables:

$w_i(t)$ – value (price) of unit of i-th fund at time t,

$k_i(t)$ – number of unit of i-th fund at time t,

$A_i(t)$ – the net assets of i-th fund at time t,

Hence we have

$$A_i(t) = w_i(t) \cdot k_i(t), \quad i = 1, 2, 3, ..., n, \ t = 1, 2, ... \tag{1}$$

The assets shares of the commodities at time t are defined by

$$A_i^*(t) = \frac{A_i(t)}{\sum_{i=1}^{N} A_i(t)}. \tag{2}$$

Certainly we have

$$\sum_{i=1}^{N} A_i^*(t) = 1, \text{ for each } t = 1, 2... \tag{3}$$

The construction of index of average unit price dynamics is based on the paper by Białek (2006). The presented definition differs from the classical price indexes based on Laspeyres, Paasche, Fisher or Lexis formulas (see Diewert (1976), Dumagan (2002) etc.). Let $[T_1, T_2]$ be the time interval of monitoring the group of funds $1 \le T_1 < T_2 \le T$. Using the above significations the definition of the index is as follows:

$$I^w[T_1, T_2] = \sum_{i=1}^{n} [\frac{\sum_{u=T_1}^{T_2} w_i(u)k_i(u)}{\sum_{m=1}^{n}\sum_{u=T_1}^{T_2} w_m(u)k_m(u)} \times \sum_{u=T_1+1}^{T_2} \frac{1}{2} \cdot (\frac{w_i(u-1)k_i(u-1)}{\sum_{y=T_1+1}^{T_2} w_i(y-1)k_i(y-1)} +$$

$$+\frac{w_i(u)k_i(u)}{\sum_{y=T_1+1}^{T_2} w_i(y)k_i(y)})\cdot\frac{w_i(u)}{w_i(u-1)}]=$$

$$=\sum_{i=1}^{n}[\frac{\sum_{u=T_1}^{T_2} A_i(u)}{\sum_{m=1}^{n}\sum_{u=T_1}^{T_2} A_m(u)}\times\sum_{u=T_1+1}^{T_2}\frac{1}{2}\cdot(\frac{A_i(u-1)}{\sum_{y=T_1+1}^{T_2} A_i(u-1)}+\frac{A_i(u)}{\sum_{y=T_1+1}^{T_2} A_i(u)})\cdot\frac{w_i(u)}{w_i(u-1)}]. \quad (4)$$

Let us denote additionally

$$\alpha_i^u[T_1,T_2]\equiv\alpha_i^u=\frac{1}{2}(\frac{w_i(u-1)k_i(u-1)}{\sum_{y=T_1+1}^{T_2} w_i(y-1)k_i(y-1)}+\frac{w_i(u)k_i(u)}{\sum_{y=T_1+1}^{T_2} w_i(y)k_i(y)}), \quad (5)$$

for $i=1,2,...,n,\ \ u=T_1+1,T_1+1,...,T_2,$

$$\beta_i[T_1,T_2]\equiv\beta_i=\frac{\sum_{u=T_1}^{T_2} w_i(u)k_i(u)}{\sum_{k=1}^{N}\sum_{u=T_1}^{T_2} w_k(u)k_k(u)},\quad i=1,2,...,N. \quad (6)$$

Now the definition (4) is as follows

$$I^w[T_1,T_2]=\sum_{i=1}^{n}\beta_i\times\sum_{u=T_1+1}^{T_2}\alpha_i^u\times\frac{w_i(u)}{w_i(u-1)}, \quad (7)$$

where β_i informs the investor how profitable i-th fund is on a global scale, and α_i^u informs the investor how important u-th moment is in case of i-th fund.

III BASIC PROPERTIES OF THE AVERAGE UNIT PRICE DYNAMICS

Next, we formulate a list of properties of the average unit price dynamics defined by (4). Since the proofs are simple they will be omitted here. Before the presentation of the properties let us notice that in the case of only one n_0-th fund our definition is as follows

$$I_{n_0}^{p}[T_1,T_2]=\sum_{u=T_1+1}^{T_2}\alpha_{n_0}^{u}\frac{w_{n_0}(u)}{w_{n_0}(u-1)}, \tag{8}$$

where

$$\alpha_{n_0}^{u}=\frac{1}{2}\left(\frac{w_{n_0}(u-1)k_{n_0}(u-1)}{\sum_{y=T_1+1}^{T_2}w_{n_0}(y-1)k_{n_0}(y-1)}+\frac{w_{n_0}(u)k_{n_0}(u)}{\sum_{y=T_1+1}^{T_2}w_{n_0}(y)k_{n_0}(y)}\right). \tag{9}$$

Property 1. Certainly we have

$$\forall T_1\le t\le T_2 \;\; w_i(t)=c_i \Rightarrow I^{w}[T_1,T_2]=1. \tag{10}$$

This property has almost axiomatic character. It says that in case when the price of each unit is constant during the time interval $[T_1,T_2]$ then the index defined by (4) must absolutely inform us about that situation.

Property 2. Let us assume that all units of funds are infinitely divisible. If for some $m\in\{1,2,3,...,N\}$ holds

$$\max_{i\in\{1,2,...,N\}\setminus\{m\}} A_i^{*}(u)\le\theta\cdot A_m^{*}(u), \text{ for } u=T_1,...,T_2$$

then we have

$$\lim_{\theta\to 0} I^{w}[T_1,T_2]=I_m^{w}[T_1,T_2]. \tag{11}$$

This property says that the influence of unprofitable funds on the average unit price dynamics is asymptotically negligible.

Property 3. If all prices of units grew at about the same $m\%$ then the value of the average unit price dynamics would not change. Similarly, if all numbers of units grew at about the same $s\%$ then the index defined by (4) would have the same value before, and after the growth.

Property 4. With probability one we have

$$\min_m I_m^w[T_1,T_2] \le I^w[T_1,T_2] \le \max_m I_m^w[T_1,T_2]. \tag{12}$$

Property 4 means that the average unit price dynamics is not greater than the highest unit's price dynamics of a single fund, and not smaller than the smallest unit price dynamics of a single fund.

IV. THE CONSTRUCTION OF MEASURE PENSION FUNDS' EFFECTIVENESS

According to the Polish law let us assume $[T_1,T_2]=[1,36]$. Let us denote

$$I_{1i} = I_i^w[25,36];\ I_{2i} = I_i^w[13,24];\ I_{3i} = I_i^w[1,12];$$

$$I_1 = I^w[25,36];\ I_2 = I^w[13,24];\ I_3 = I^w[1,12]$$

and

$$\gamma_{ki} = \frac{I_{ki}}{I_k},\ \text{for}\ k=1,2,3. \tag{13}$$

So we divide the considered time interval into three one-year intervals.

The γ_{ki} informs the investor about the relation between the monthly dynamics of value of i-th fund's unit and the average, monthly dynamic of units of the group and it takes into consideration the k-th, one-year period. The influence of these three periods on the measure of pension funds' effectiveness should be different. It means that, the last year should be the most important. The new and old data cannot weight the same. The proposed measure of ***Attraction of Dynamics of Fund (ADF)*** is as follows:

$$ADF_i = \left(\sum_{k=1}^{3} \exp(-k\beta)\gamma_{ki} - 1\right)\cdot 100\% \tag{14}$$

where the weights for γ_{ki} we get from the following exponential equation:

$$\exp(-\beta)+\exp(-2\beta)+\exp(-3\beta)=1\,. \tag{15}$$

The numerical solution of (15) is $\beta = 0.609358$.

We can present the formula (14) in an equivalent way, it means:

$$ADF_i = (\sum_{k=1}^{3} \hat{\beta}_k \cdot \gamma_{ki} - 1)\cdot 100\%, \tag{16}$$

where

$$\hat{\beta}_1 = 0.5437,\ \ \hat{\beta}_2 = 0.29561,\ \ \hat{\beta}_3 = 0.160723\,,$$

and certainly we have

$$\sum_{k=1}^{3} \hat{\beta}_k = 1. \tag{17}$$

The interpretation of the formula (16) is very simple and natural. If $ADF_i > 0$ for some i-th fund then we have the situation, where the fund has a more attractive (faster) dynamics of unit price than the group of funds. Where $ADF_i < 0$ we have the situation where the average dynamics of prices of all units is faster than the dynamics of price of i-th fund's unit. In the case of $ADF_i = 0$ we have a common, average fund.

Example 1. We consider the period of January 30, 2002 – 30 December 2004 (three years) for Polish pension funds. In case of Poland we have $n = 15$. The results are given below:

Table 1. Rates of return and average assets of Open Pension Funds for time period: Jan 2002 – Dec 2004

Pension fund	Relative, average assets: $\frac{1}{2}(A_i^*(1)+A_i^*(36))$ [%]	Rate of return
1	2	3
AIG	8.70	39.41
Allianz	2.58	36.15
Bankowy	3.20	44.39

Table 1 (cont.)

1	2	3
Commercial Union	28.8	31.50
Credit Suisse	2.80	33.92
DOM	1.60	37.38
Ergo Hestia	1.80	36.34
Generalli	3.53	37.86
ING NN	22.18	39.39
Pekao	1.63	33.92
Pocztylion	2.12	32.4
Polsat	0.66	40.64
PZU Złota Jesień	14.33	38.83
Sampo	3.25	36.83
Skarbiec Emerytu-ra	2.77	34.98

Source: author's calculation based on data from www.money.pl.

The calculation of ADF leads to the following results:

Table 2. ADF measure for time period Jan 2002 – Dec 2004

Pension fund	ADF_i
AIG	0.053
Allianz	**-0.103**
Bankowy	0.088
Commercial Union	**-0.058**
Credit Suisse	0.006
DOM	0.049
Ergo Hestia	0.003
Generalli	0.065
ING NN	0.053
Pekao	**-0.006**
Pocztylion	**-0.054**
Polsat	0.118
PZU Złota Jesień	0.043
Sampo	**-0.010**
Skarbiec Emerytura	**-0.132**

Source: author's calculation based on data from www.money.pl.

The rankings of Open Pension Funds based on rates of returns and ADF measure are presented in Table 3 (similar positions are in bold):

Table 3. Ranking of Open Pension Funds for time period Jan 2002 – Dec 2004

Pension fund	Rate of return (%) and position in ranking	ADF_i and position in ranking
AIG	**39.41 (3)**	**0.053 (4)**
Allianz	36.15 (10)	–0.103 (14)
Bankowy	**44.39 (1)**	**0.088 (2)**
Commercial Union	31.50 (15)	**–0.058** (11)
Credit Suisse	33.93 (12)	0.006 (9)
DOM	**37.38 (7)**	**0.049 (6)**
Ergo Hestia	**36.34 (9)**	**0.003 (8)**
Generalli	37.86 (6)	0.065 (3)
ING NN	**39.39 (4)**	**0.052 (5)**
Pekao	33.92 (13)	**–0.006** (10)
Pocztylion	**32.4 (14)**	**–0.054 (12)**
Polsat	**40.64 (2)**	**0.118 (1)**
PZU Złota Jesień	**38.83 (5)**	**0.043 (7)**
Sampo	36.83 (8)	–0.010 (13)
Skarbiec Emerytura	34.98 (11)	–0.132 (15)

Source: author's calculation based on data from www.money.pl.

V. CONCLUSIONS

Both presented ranking lists of Open Pension Funds, based on rates of return and *ADF* measure seem to be similar (Spearman's rank correlation coefficient is equal to 0.8). But *ADF* measure has a more interesting interpretation. This measure can show even "rich" funds with very slow dynamics of unit price (for example Commercial Union). It is also interesting to notice that there are "poor" funds (with small value of relative, average assets) having very attractive dynamics of their unit prices (see Polsat and Bankowy). The ranking based on *ADF* measure seems to be a good alternative to existing methods of ranking of Pension Funds.

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PROPOZYCJA MIARY EFEKTYWNOŚCI FUNDUSZY EMERYTALNYCH

Celem pracy jest propozycja nowej miary efektywności funduszy emerytalnych. Prezentowana miara z założenia ma stanowić alternatywę dla klasycznej stopy zwrotu, przy czym dodatkowym postulatem jest tu wymóg odniesienia się do kondycji całej grupy funduszy. W świetle polskiego ustawodawstwa bardzo istotna jest bowiem relacja wyników funduszu w stosunku do przeciętnych rezultatów całej grupy. Zgodnie z ustawą, prezentowana miara rozważać będzie również 36-o miesięczny interwał czasowy, przy czym za najbardziej istotny traktować będzie miniony rok. Konstrukcja miary oparta jest na propozycji autora indeksu dynamiki cen