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PROOFS OF THE NORMALIZATION OF THE FUNCTION OF THE THICKNESS CLASSES ONE-DIMENSIONAL NORMAL DISTRIBUTIONS

ABSTRACT. One-dimensional two parameters the normal distribution assorts basic probability distributions in the statistics. in last years into being many generalized versions, taking into account parameters of the asymmetry and the kurtosis. They there create the class of normal distributions *m*-parameter, properly with parameters, $m = 1$ – positions (shifts), $m = 2$ – positions and variations (scale), $m = 3$ – positions, variations and skewness and $m = 4$ – positions, variations, skewnesses and kurtosis.

On the job we give 7 chosen one-dimensional probability distributions from the class of normal distributions. For them one mentioned functions of the thickness and one averred normalizations to show, that the integral after area of the determinates of these functions is equal one.

Key words: normal distribution, normalization of density function.

I. INTRODUCTION

The density functions probability distributions of one-dimensional random variables can be expressed: (a) with one parameter – positions, (b) with two parameters – positions and variabilities, (c) with three – positions, variabilities and the asymmetry or (d) with four – positions, variabilities, asymmetries and the kurtosis. There refers this particularly of the class from the normal distribution. In classical way it is expressed by two parameters: expected value (or) and the standard deviation (the parameter of the variability). It's modifications with asymmetry and kurtosis. Lead to formulate the new types of distributions with more complex density functions. The integrals of this functions on suitable integrals should fulfill the normalization condition (equal to 1). For the proof of the indicated condition one uses the integral calculus and numeric methods.

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In this paper we consider several distributions from the normal distribution. These distributions were described by given density function. Giving for these parameters example-values, we tabulated density functions we presented their graphs. We also proved the condition of normalization for density function presented in this paper.

II. NOTATIONS AND BASIC FORMULAS

Let X be a continues random variable with density function $f(x;\boldsymbol{\theta})$, where $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_m)$ is m-dimensional vector of the parameters of distribution. This vector is a real vector, i.e. $\Theta \subset R^m$.

Until now we will use some formulas:

(w1) $\Gamma(p) = \int_0^\infty x^{p-1} e^{-x} dx$ – gamma function,

(w2) $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$,

(w3) $\Gamma(a) = \exp\left\{ \ln\sqrt{2\pi} + (a - \frac{1}{2})\ln a - a + \frac{1}{12a} - \frac{1}{360a^3} \right\}$ – (the stirling formulate see Fichtenholz 1995, t.2, p. 680, Jones 2001),

(w4) $e^{-\frac{1}{2}t^2} = \sum_{k=0}^{\infty} (-1)^k \frac{t^{2k}}{2^k k!}$.

For computing the integrals we used approximations by composite quadrature using the following formulae (CQ) (see Mizerski, 1999, p. 138)

$$\int_a^b f(x)dx \approx h\left\{ \frac{1}{2}(f(a) + f(b)) + \sum_{k=1}^{N-1} f(a + h \cdot k) \right\},$$

were h is the step of tabulating of interval (a, b) and $N = \left[\frac{b-a}{h}\right] + 1$ is a number of steps.

III. TWO-PARAMETERS AND STANDARIZED NORMAL DISTRIBUTION

The density function of normal distribution is of the from

$$f(x;\mu,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\cdot\left(\frac{x-\mu}{\sigma}\right)^2\right\},$$

with $\boldsymbol{\theta}=(\mu,\sigma)\in\Theta=R\times R_+\subset R^2$, mean μ and standard deviation σ. We say that $X\sim N(\mu,\sigma)$. The standardized random variable $Z=\dfrac{X-\mu}{\sigma}$ give us the standardized normal distribution $Z\sim N(0,1)$ with density function $\varphi(z)=\dfrac{1}{\sqrt{2\pi}}e^{-z^2/2}$ and distribution function $\Phi(z)=\int\limits_{-\infty}^{z}\varphi(t)dt$. The values of both functions we can obtain using procedure ROZKLAD NORMALNY in EXCEL.

For the $N(0,1)$ distribution we give several properties:

(rn1) $\Phi(0)=\frac{1}{2}$, $\Phi(-\infty)=0$, $\Phi(\infty)=1$ – chosen values of c.d.f. $\Phi(x)$.

(rn2) $\int\limits_{x}^{\infty}\varphi(t)dt=1-\Phi(x)$,

(rn3) $\Phi(x)=\frac{1}{2}+\Phi_0(x)$, where $\Phi_0(x)=\int\limits_{0}^{x}\varphi(t)dt$ is the distribution function, describe on the interval $(0,\infty)$,

(rn4) $\Phi_0(x)=\frac{1}{\sqrt{2\pi}}\sum\limits_{k=0}^{\infty}\frac{(-1)^k}{2^k k!}\cdot\frac{x^{2k+1}}{2k+1}$, extentions of c.d.f. $\Phi(x)$

(rn5) $\int\limits_{-\infty}^{\infty}x^{2k+1}\varphi(x)dx=0$, for k=0, 1, 2, ... , (see w4)

(rn6) $\Phi'(z)=\varphi(z)$ –

At calculations which we made for imitable integrals, often one takes place e^x changes of variables $x\to Z,\ dx=\sigma dz$.

IV. SOME DISTRIBUTIONS CONNECTING WITH NORMAL DISTRIBUTIONS

For simplicity of suitable notation of given distribution we will present it in the following scheme:

a) the author or authors to other works,

b) the density functions,

c) the specification of variable parameters at which distribution runs value of parameters at which distribution runs into the normal distribution or into other well-known distribution,

d) The graphs of the density function for given values of parameters,

e) The proof of normalization in analytic wad or application of (CQ),

f) Representation of the density functions with the regard to parameters.

by

4.1. The exponential power distribution (EP)

a) Subbotin (1923), DiCiccio, Monti (2004),

b) $f_{EP}(x;\mu,\sigma,\alpha)=\frac{1}{c\cdot\sigma}\exp\left\{-\frac{1}{\alpha}\left|\frac{x-\mu}{\sigma}\right|^{\alpha}\right\}$,

$$x\in R,\ \boldsymbol{\theta}=(\mu,\sigma,\alpha)\in\Theta=(-\infty,\infty)\times(0,\infty)\times(1,\infty),\quad c=2\cdot\alpha^{\frac{1}{\alpha}-1}\Gamma\left(\frac{1}{\alpha}\right),$$

c) $\alpha=2\ \Rightarrow N(\mu,\sigma)$,

d) (w3) we made three graphs for different values of parameters of variability and shape (fig. 1), counting the following values (tab. 1)

Table 1.

Graphs	1	2	3
mi	0.5	0.5	0.5
sigma	0.75	1	1.5
alfa	1.25	1.5	2.5
1/alfa	0.8	0.6667	0.4000
gamma	1.1390	1.3211	2.1529
c	2.1785	2.3081	2.4848

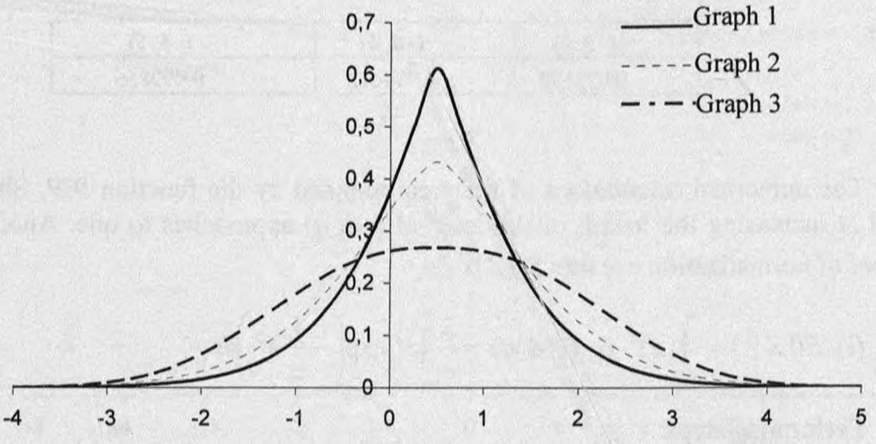

Fig. 1. Graphs of density function of the exponential power distribution

Fig 1 shows that at any values of parameters α and δ, the curves of density functions stay symmetrical, but they flats with growing values of α. The proof of the normalization after the done twice exchange variables and using w1 have three steps.

e) i) $\Rightarrow$ ZZ: $x \to z, \ \Rightarrow \ I_\alpha = \frac{1}{\sigma \cdot c} \int\limits_{-\infty}^{\infty} \exp\left\{ -\frac{1}{\alpha} \left| \frac{x-\mu}{\sigma} \right|^\alpha \right\} dx = \frac{2}{c} \int\limits_{0}^{\infty} \exp\left\{ -\frac{1}{\alpha} z^\alpha \right\} dz,$

ii) ZZ: $z \to v, \ \Rightarrow \ v = \frac{1}{\alpha} z^\alpha, \ z = \alpha^{\frac{1}{\alpha}} v^{\frac{1}{\alpha}}, \ dz = \alpha^{\frac{1}{\alpha}-1} v^{\frac{1}{\alpha}-1} dv,$

iii) (w1): $I_\alpha = \frac{2}{c} \alpha^{\frac{1}{\alpha}-1} \int\limits_{0}^{\infty} v^{\frac{1}{\alpha}-1} e^{-v} dv = \frac{2}{c} \alpha^{\frac{1}{\alpha}-1} \Gamma\left(\frac{1}{\alpha}\right) = 1.$

Table 2. Given constans are the following

μ	σ	α	$1/\alpha$	gamma	c	$1/c$	a	h
0	1	1,25	0,8	1,1627	2,2239	0,4497	–5	0,025
–5	–4,975	–4,95	...	–0,025	0	0,025	0,05	...
0,001136	0,001178762	0,001224	...	0,446102	0,449663	0,446102	0,441238	...

For these constants we calculations using CQ for intervals ($-a$, a). For example for $a = 5$ we have obtain the integral equal to 0,999937.

(−3, 3)	(−4, 4)	(−5, 5)
0.973692	0.994692	0.999856

The numerical calculations of the area bounded by the function fEP. Show that at increasing the length of the interval (−*a*, *a*) approaches to one. Another proof of normalization use the $E(|Z|)^k$:

(i) $E(|Z|^k) = \int_{-\infty}^{\infty} |z|^k f_{EP}(z;\alpha)dz = \frac{2}{c}\int_0^{\infty} z^k \exp\left(-\frac{1}{\alpha}z^{\alpha}\right)dz,$

Performing steps:

(ii) ZZ: $t = \frac{1}{\alpha}z^{\alpha}, \quad z = \alpha^{\frac{1}{\alpha}} \cdot t^{\frac{1}{\alpha}}, \quad dz = \alpha^{\frac{1}{\alpha}-1} \cdot t^{\frac{1}{\alpha}-1} dt,$

(iii) $E(|Z|^k) = \frac{2 \cdot \alpha^{\frac{1}{\alpha}-1}}{2 \cdot \alpha^{\frac{1}{\alpha}-1} \cdot \Gamma\left(\frac{1}{\alpha}\right)} \cdot \alpha^{\frac{k}{\alpha}} \cdot \Gamma\left(\frac{k}{\alpha}+\frac{1}{2}\right) = \alpha^{\frac{k}{\alpha}} \cdot \frac{\Gamma\left(\frac{k+1}{\alpha}\right)}{\Gamma\left(\frac{1}{\alpha}\right)}.$

The condition of normalization holds for $k = 0$.

4.3. The skew-normal distribution (SN)

a) Azzalini (1985), DiCiccio, Monti (2004), Jadamus-Hacura (2006),

b) $f_{SN}(x;\mu,\sigma,\lambda) = \frac{2}{\sigma}\Phi\left[\lambda\left(\frac{x-\mu}{\sigma}\right)\right]\varphi\left(\frac{x-\mu}{\sigma}\right),, \quad x \in R,$

$Q = (\mu,\sigma,\lambda) \in \Theta = (-\infty,\infty)\times(0,\infty)\times \times(-\infty,\infty),$

$\varphi(z), \Phi(z),$ ....

c) $\lambda = 0 \Rightarrow N(\mu,\sigma),$

d) Fig. 2 present the graph of density function for parameters $\mu = 0,\ \sigma = 1,$ and λ'+1.5; 1; 1.5; 2.

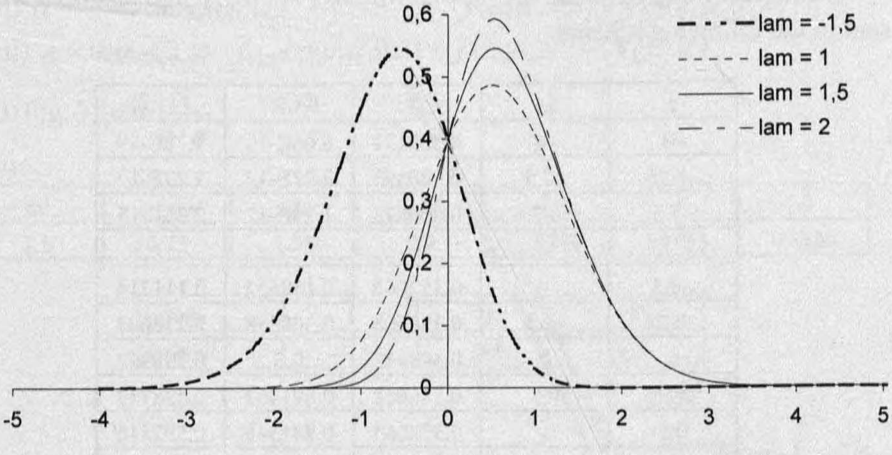

Fig. 2. Graphs of density function of skew-normal distribution

e) The proof of normalization lead to exchange to x and using the properties rn3, rn4 and rn5.

(i) ZZ, $x \to z$: $f_{SN}(z;\lambda) = 2\Phi(\lambda z)\varphi(z)$, $z, \lambda \in R$,

(ii) (rn3): $I_\lambda = \int\limits_{-\infty}^{\infty} f_{SN}(z;\lambda)dz = 2\int\limits_{-\infty}^{\infty}\varphi(z)\Phi(\lambda z)dz = 2\int\limits_{-\infty}^{\infty}\varphi(z)\left\{\frac{1}{2} + \Phi_0(\lambda z)\right\}dz$

$= 1 + 2\int\limits_{-\infty}^{\infty}\varphi(z)\cdot\Phi_0(\lambda z)\,dz,$

(iii) (rn4): $\Phi_0(\lambda z) = \frac{1}{\sqrt{2\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k}{2^k k!}\cdot\frac{\lambda^{2k+1}}{2k+1}\cdot z^{2k+1}$,

(iv) (rn5): $I_\lambda = 1 + \frac{2}{\sqrt{2\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k}{2^k k!}\cdot\frac{\lambda^{2k+1}}{2k+1}\cdot\int\limits_{-\infty}^{\infty}\varphi(z)z^{2k+1}dz = 1$.

The proof of normalization for special case, when $\mu = 0, \sigma = 1, \lambda = 1$ or the case $\lambda = 1$:

(i) $\int\limits_{-\infty}^{\infty} f_{SN}(z;1)dz = 2\int\limits_{-\infty}^{\infty}\varphi(z)\Phi(z)dz = I_1$,

(ii) (rn1), ZZ: $z \to v$: $v = \Phi(z)$, $dv = \varphi(z)dz$, $z \in (-\infty,\infty)$, $v \in (0,1)$,

(iii) $I_1 = 2\int\limits_0^1 v dv = 1$.

Numerical integration are for $\lambda = 2$ with a = -4, b = 6 and h = 0,25 we present in the following scheme:

z	$2z$	$\varphi(z)$	$\Phi(2z)$	$f(z;2)$
–4	–8	0.000134	6.66E-16	1.78E-19
–3.75	–7.5	0.000353	3.22E-14	2.27E-17
–3.5	–7	0.000873	1.29E-12	2.25E-15
...	...	...	...	...
–0.5	–1	0.352065	0.158655	0.111714
–0.25	–0.5	0.386668	0.308538	0.238603
0	0	0.398942	0.5	0.398942
0.25	0.5	0.386668	0.691462	0.534733
0.5	1	0.352065	0.841345	0.592416
...	...	...	...	...
5.5	11	1.08E-07	1	2.15E-07
5.75	11.5	2.64E-08	1	5.28E-08
6	12	6.08E-09	1	1.22E-08

The method CQ evas using when $\mu = 0, \sigma = 1, \lambda = 2$ and the limits of integration a = –4, $b = 6$. In calculation procedure ROZKŁAD.NORMALS was used)in the case h = 0,25)

$$f(x;\mu,\sigma,\lambda_1,\lambda_2) = \frac{2}{\sigma}\varphi\left(\frac{x-\mu}{\sigma}\right)\Phi\left(\frac{\lambda_1(x-\mu)}{\sqrt{\sigma^2+\lambda_2(x-\mu)^2}}\right)$$

The fragment of these calculation as given in table 3
The obtained integral was equal to 0,999996.

4.4. The skew-exponential power distribution (SEP)

a) Azzalini (1986), DiCiccio, Monti (2004),

b) $f_{SEP}(x;\mu,\sigma,\lambda,\alpha) = 2\Phi(w) f_{EP}(x;\mu,\sigma,\alpha)$,

$x,\mu,\lambda \in R$, $\sigma \in R_+$, $\alpha \in (1,\infty)$, $w = \lambda \cdot sign(z) \cdot | z = \frac{x-\mu}{\sigma} |^{\frac{\alpha}{2}} \cdot \sqrt{\frac{2}{\alpha}}$,

$f_{SEP}(z;\lambda,\alpha) = [1+2\Phi_0(z)] \cdot f_{EP}(z;\alpha)$,

c) (rn1): $\lambda = 0 \Rightarrow w = 0$, $f_{SEP}(x;\mu,\sigma,0,\alpha) = f_{EP}(x;\mu,\sigma,\alpha)$,

ii) $\alpha = 2 \Rightarrow w = sign(z) \cdot \lambda \cdot |z|$, $f_{SEP}(x;\mu,\sigma,\lambda,2) = f_{SN}(x;\mu,\sigma,\lambda)$,

iii) $\lambda = 0, \alpha = 2 \Rightarrow f_{SEP}(x;\mu,\sigma,0,2) = f(x;\mu,\sigma) - N(\mu,\sigma)$,

d) Fig. 3, $d = \dfrac{1}{c \cdot \sigma}$,

e)

mi	sigma	alfa	1/alfa	gamma	c	d
1.5	0.75	1.25	0.8	1.1390	2.1785	0.6120

graph	1	2	3
Lambda	–0.5	0.5	1.5

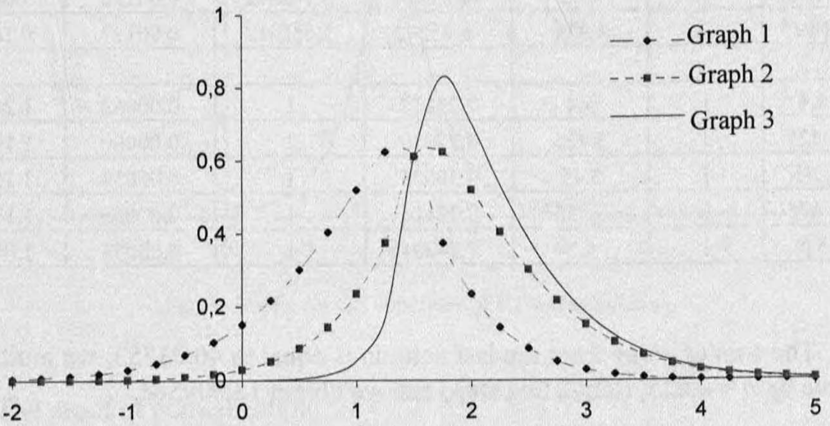

Fig. 3. Graphs of density functions of skew-normal distributions

(e) condition of normalization results directs from the following formula:

$$E(Z^{2m}) = \alpha^{2m/\alpha} \frac{\Gamma\left(\dfrac{2m+1}{\alpha}\right)}{\Gamma\left(\dfrac{1}{\alpha}\right)}, \quad m = 0, 1....$$ (DiCiccio, Monti, 2004, p. 439)

for $m = 0$).

The proof of normalization by numerical integration CQ for given values of parameters

mi	sigma	alfa	1/alfa	gamma	c	constans	lambda	x0	h
2	2.5	1.25	0.8	1.1627	2.2239	0.1799	2	–1.5	0.025

i) ZZ : $x \rightarrow z$: $f_{SEP}(z;\lambda,\alpha) = 2\Phi(w) f_{EP}(z;\alpha)$, where w like in (b).

ii) Calculating the integral $I_{SEP} = 2\int_{-\infty}^{\infty}\Phi(w) f_{EP}(z;\lambda,\alpha)dz$, with suitable parameters:

The part of these calculations is the following:

z	sign	expected value	w	Hi(w)	fEP	fSEP
–5	–1	5	–6.91745	2.31E-12	0.00114	5.25E-15
–4.975	–1	4.975	–6.89582	2.69E-12	0.00118	6.35E-15
–4.95	–1	4.95	–6.87414	3.14E-12	0.00122	7.68E-15
–4.925	–1	4.925	–6.85242	3.65E-12	0.00127	9.28E-15
...	...	...	...	...	...	...
5.4	1	5.4	7.25832	1	0.00062	1.24E-03
5.425	1	5.425	7.27930	1	0.00060	1.19E-03
5.45	1	5.45	7.30025	1	0.00058	1.15E-03
5.475	1	5.475	7.32116	1	0.00055	1.11E-03
5.5	1	5.5	7.34204	1	0.00053	1.07E-03

The sum of value from the last column is equal to 40,03753, we multiple his value by $h = 0{,}025$, (tabulating step) and we obtain 1,0009384.

4.5. The double truncated normal distribution (OTN)

a) Damilano, Puig (2004),

b) $f_{DTN}(x;\mu,\sigma,\theta) = \frac{c(\theta)}{2\sigma}\exp\left[-\theta\left|\frac{x-\mu}{\sigma}\right| - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$,

$x,\mu,\theta \in R,\ \sigma \in R_+$,

where $c(\theta) = \frac{\varphi(\theta)}{1-\Phi(\theta)}$ is the reverse of Mill's quotient

c) $\theta = 0 \Rightarrow c(0) = \frac{\varphi(0)}{1-\Phi(0)} = \frac{2}{\sqrt{2\pi}}$, $N(\mu,\sigma)$,

d) $\theta = 0{,}5,\ 2,\ 3$, Fig. 4

mi	sigma	theta	fi	Fi	c	constants
2	1.5	0.5	0.35207	0.69146	1.14108	0.38036
2	1.5	1	0.05399	0.97725	2.37322	0.79107
2	1.5	3	0.00443	0.99865	3.28293	1.09431

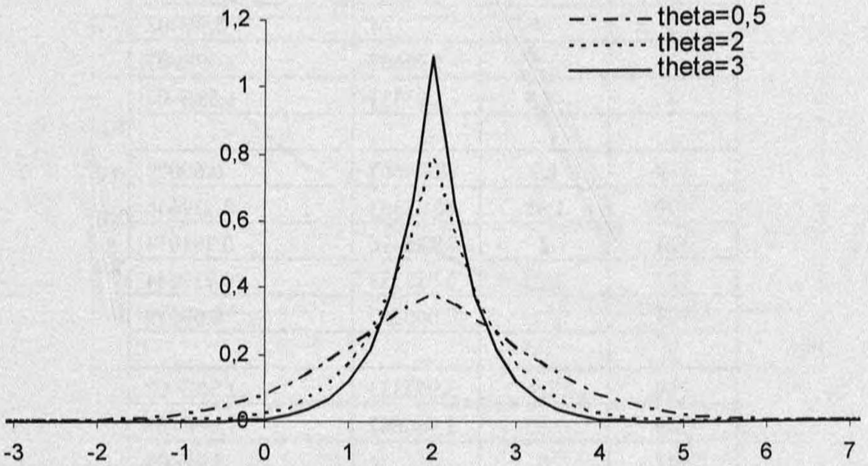

Fig. 4 Graphs density functions of PTN distributions

e) The proof of normalization:

(i) ZZ: $z = \dfrac{x-\mu}{\sigma}$, $dz = \dfrac{1}{\sigma}dx$,

(ii) $f_{DTN}(z;\theta) = \dfrac{c(\theta)}{2}\exp(-Q(z,\theta))$, $Q(z;\theta) = \theta\,|\,z\,| + \dfrac{1}{2}z^2$

(iii) $I_\theta = \dfrac{c(\theta)}{2}\displaystyle\int_{-\infty}^{\infty}\exp(-Q(z,\theta))dz$,

(iv) $Q(z,\theta) = \dfrac{1}{2}(z^2 + 2\theta\,|\,z\,|) = \dfrac{1}{2}[(z^2 + 2\theta\,|\,z\,| + \theta^2) - \theta^2] = \dfrac{1}{2}(|\,z\,| + \theta)^2 - \dfrac{1}{2}\theta^2$,

(vi) $I_\theta = A_\theta \displaystyle\int_{-\infty}^{\infty}\exp\left[-\dfrac{1}{2}(|\,z\,|+\theta)^2\right]dz = 2A_\theta\int_{0}^{\infty}\exp\left[-\dfrac{1}{2}(z+\theta)^2\right]dz$,

$$A_\theta = \frac{c(\theta)}{2}\exp\left(\frac{\theta^2}{2}\right),$$

(v) ZZ: $u = z + \theta$, $z = u - \theta$, $dz = du$, $(0,\infty) \to (\theta,\infty)$,

(vi) $I_\theta = 2A_\theta \cdot \sqrt{2\pi} \cdot (1-\Phi(\theta)) = \dfrac{\varphi(\theta)}{1-\Phi(\theta)} \cdot \exp\left(\dfrac{\theta^2}{2}\right) \cdot \sqrt{2\pi}(1-\Phi(\theta)) = 1$.

The proof of normalization by CQ:

Lp	*x*	*z*	*f(x)*
1	–4	–4	8,9E–08
2	–3.95	–3.96667	1.09E–07
3	–3.9	–3.93333	1.33E–07
...	...	...	...
119	1.9	–0.06667	0.69079
120	1.95	–0.03333	0.739644
121	2	5.92E–16	0.791074
122	2.05	0.033333	0.739644
123	2.1	0.066667	0.69079
...	...	...	...
239	7.9	3.933333	1.33E–07
240	7.95	3.966667	1.09E–07
241	8	4	8.9E–08

The sum of values of $f(x)$ is equal to 20.008839. We multiple this sum by 0.05 (tabulating step) and we obtain the value of integral $I_\theta = 1.00044197$.

4.6. The singly truncated normal distribution (STN)

a) Del Castillo, Puig (1999),

b) $f_{STN}(x;\mu,\sigma,\nu,\psi) = \dfrac{1}{\sigma \cdot NC(\nu,\psi)} \exp\left\{-\nu\left(\dfrac{x-\mu}{\sigma}\right) - \psi\left(\dfrac{x-\mu}{\sigma}\right)^2\right\}$,

$x,\mu,\nu \in R,\ \sigma,\psi \in R_+,\quad NC(\nu,\psi) = \sqrt{\dfrac{\pi}{\psi}} \cdot \exp\left(\dfrac{\nu^2}{4\psi}\right)\left(1-\Phi\left(\dfrac{\nu}{\sqrt{2\psi}}\right)\right)$,

c) $\nu = 0, \psi = 0{,}5 \Rightarrow NC(0;0{,}5) = \sqrt{2\pi} \cdot (1-\Phi(0)) = \sqrt{\dfrac{\pi}{2}}$,

$f_{STN}(x;\mu,\sigma,0,0{,}5) = \sqrt{\dfrac{2}{\pi}} \cdot \exp\left(-\dfrac{1}{2} \cdot \left(\dfrac{x-\mu}{\sigma}\right)^2\right) = f_{HN}(x;\mu,\sigma)$, (HN – half-normal),

d) $a = \dfrac{\nu^2}{4\psi}$, $b = \sqrt{a}$, Fig. 5.

e)

mi	sigma	ni	psi	a	b	dystr.	NC	stała
0	1	0.5	0.75	0.08333	0.28868	0.61358	0.85959	1.16335

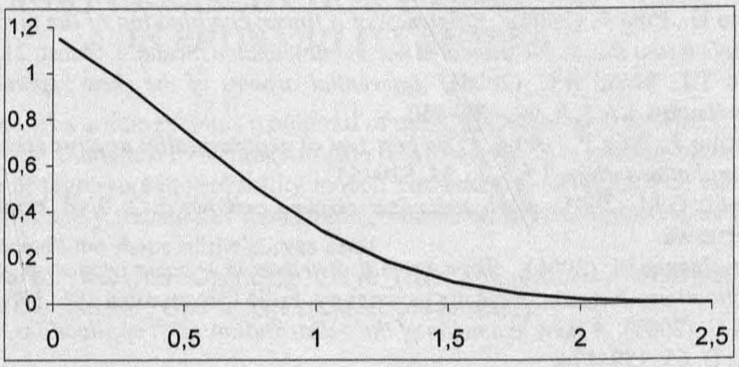

Fig 5. Graphs of density function of STN distribution

e) The proof of the condition of normalization:

(i) ZZ: $f_{STN}(x;\nu,\psi) = \dfrac{1}{NC(\nu,\psi)}\exp(-Q(z;\nu,\psi))$, $Q(z;\nu,\psi) = \nu \cdot z + \psi \cdot z^2$,

(ii) $Q(z;\nu,\psi) = \psi\left[z^2 + 2z\cdot\dfrac{\nu}{2\psi} + \left(\dfrac{\nu}{2\psi}\right)^2 - \left(\dfrac{\nu}{2\psi}\right)^2\right] = \psi\left(z + \dfrac{\nu}{2\psi}\right)^2 - \dfrac{\nu^2}{4\psi}$,

(iii) $I_{\nu,\psi} = A_{\nu,\psi}\int\limits_0^\infty \exp\left[-\dfrac{1}{2}\cdot 2\psi\left(z + \dfrac{\nu}{2\psi}\right)^2\right]dz$, $A_{\nu,\psi} = \dfrac{\exp\left(\dfrac{\nu^2}{4\psi}\right)}{NC(\nu,\psi)}$,

(iv) ZZ: $u = \sqrt{2\psi}\left(z + \dfrac{\nu}{2\psi}\right) \Rightarrow z = \dfrac{1}{\sqrt{2\psi}}u - \dfrac{\nu}{2\psi}$, $dz = \dfrac{1}{\sqrt{2\psi}}du$,

$z \in (0,\infty)$, $u \in \left(\dfrac{\nu}{\sqrt{2\psi}}, \infty\right)$,

(v) $I_{\nu,\psi} = \dfrac{A_{\nu,\psi}}{\sqrt{2\psi}}\cdot\int\limits_{\frac{\nu}{\sqrt{2\psi}}}^\infty \exp\left(-\dfrac{1}{2}u^2\right)du = \dfrac{A_{\nu,\psi}}{\sqrt{2\psi}}\cdot\sqrt{2\pi}\left(1 - \Phi\left(\dfrac{\nu}{\sqrt{2\psi}}\right)\right) = 1.$

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DOWODY UNORMOWANIA FUNKCJI GĘSTOŚCI KLASY JEDNOWYMIAROWYCH ROZKŁADÓW NORMALNYCH

Jednowymiarowy dwuparametrowy rozkład normalny należy do podstawowych rozkładów prawdopodobieństwa w statystyce. W ostatnich latach powstało wiele jego uogólnionych wersji, uwzględniających parametry asymetrii i kurtozy. Tworzą one klasę rozkładów normalnych m-parametrowych, odpowiednio z parametrami: $m = 1$ położenia (przesunięcia), $m = 2$ - położenia i zmienności (skali), $m = 3$ - położenia, zmienności i skośności oraz $m = 4$ - położenia, zmienności, skośności i spłaszczenia.

W pracy podajemy 7 wybranych jednowymiarowych rozkładów prawdopodobieństwa z klasy rozkładów normalnych. Dla nich wymieniono funkcje gęstości oraz przedstawiono dowody unormowania pokazując, iż całka po obszarze określoności tych funkcji jest równa jeden.