ACTA UNIVERSITATIS LODZIENSIS
FOLIA OECONOMICA 228, 2009

*Czesław Domański**

# HUGO DIONIZY STEINHAUS (1887–1972)


Hugo Steinhaus was born on 14 January 1887 in Jasło in south –eastern Poland.

His father Bogusław was the director of the credit co-operative and his mother Ewelina Lipschitz-Widajewicz ran house.

In 1905 Steinhaus graduated from public grammar school in Jasło and later that year he enrolled at Lvov University where he studied philosophy and mathematics. In 1906 he moved to Geting University where he listened to lectures of Hilbert and Klein – the two eminent scientists who exerted enormous influence on development of mathematics. At that time Steinhaus developed friendly contacts with Poles studying there, namely: Banachiewicz, Sierpiński, the Dziewulski brothers, Łomnicki, Chwistek, Stożek, Janiszewski and Mazurkiewicz. In Geting in 1911 he earned his doctor's degree on the basis of the doctoral dissertation *Neue Anvendungen des Dirichlet'schen Prinzip* which was written under the scientific supervision of David Hilbert. He then went to University of Munich and to Paris in order to continue his studies.

In the years 1911–1912 Steinhaus lived in Jasło and Cracow. At the outbreak of World War I he joined the army and took part in a few battles of the First Artillery Regiment of Polish Legions. Since 1916 he stayed in Lvov where he worked first for The Centre for Country's Reconstruction and then as an assistant at Lvov University. In 1917 he obtained the postdoctoral degree on the basis of the dissertation entitled *On some properties of Fourier series.* One year later he published *Additive und stetige Funktionaloperationen* (Mathematische Zeitschrift",5/1919) considered to be the first Polish work on functional operations. In the years 1918–1920 he held the post of office worker in the Bureau of

* Professor, Chair of Statistical Methods, University of Lodz.

Natural Gas in Niegłowice near Jasło. In 1920 he returned to Lvov where he was appointed for the post of associate professor of Lvov University and in 1923 he became full professor.

The year 1923 marks also the publishing in "Fundamenta Mathematicae" his first work in the field of theory of probability "Les probabilities denombrales et leur rapport a la theorie de la measure" Many years later in his 'Autobiography" (1973) the author pointed at the fact that "It is one of the first steps towards mathematization of probability calculus by reducing the notion to the theory of measure. And also the first work where number series of the form $\Sigma\pm a_n$ exist when $a_n$ terms are given, and signs "+" and "–" are drawn.

In 1925 Steinhaus published the article entitled "Definitions for games and chase theory" in "Academic thought" – the gazette of Lvov students. According to the author "this is the first work which deals with the problem of chase in the approach adopted by the modern game theory. The fundamental concept of the game theory i.e. "minimax" concept had been invented by Emil Borel, which I was not aware of then. It took some time for his works to be recognized, too ".

Ryll-Nardzewski wrote in 1972: "This is but a small work not really mathematical in its nature; it is rather a collection of remarks, yet remarks which were extremely innovative at that time and which have become the foundations of the modern game theory. Firstly, it introduced the notion of strategy (named "the mode of game") .The second important element was the so called games normalization, and finally, the notion of payment, indispensable in every game, and the principle of minimax strategy selection".

K. Urbanik (1973) makes an attempt to axiomatize probability calculus in the scientific output of Hugo Steihaus. He writes: "In 1923 Volume 4 of "Fundamenta Mathematicae "is published. It contains two works: *Nouveaux fondements du calcul des probabilites* by Jan Łomnicki and *Les probabilites denombrables et leur rapport a la theorie de la mesure* by Hugo Steinhaus. The two works are the milestones on the long way towards giving probability calculus a solid mathematical foundation. Łomnicki, making the use of measure – set notions, provides precise descriptions of such notions as: elementary events, random events and probability as a measure based on random events .They are interpreted on the basis of examples available at the time. However, the notion of independence – the most fundamental notion of probability theory, which sets it apart from the theory of measure, is not discussed in his work. What it lacks is a precise definition of random variable or expected value. This will be done much later by Andrzej Kołmogorow who formed the axiomatic foundations of the probability theory. In his work Hugo Steinhaus showed in a natural way how one case of non-classical probability system i.e. infinite series of toss-ups can be formalized by assigning toss-up results to series. Without reducing the level of generalization they can be considered as zero-one series of real numbers of (0,1) interval, or rather their two expansions. In the course of this operation random events are transformed to measurable sets, in Lebesgue sense, and probability is transformed into Lebesgue measure.

In this way the infinite game of "heads or tails" is fully mathematized. It allows to translate the probabilistic problems into the language of the theory of measure and the theory of real functions i.e. the language of well-developed, mathematical theories. For instance, the Steinhaus –Wiener problem of determining probability whether the series of real numbers

$$\sum_{k=1}^{\infty} \pm c_k$$

with signs chosen independently with the same probability of 1/2 is convergent, can be reduced, according to Steinhaus, to finding Lebesgue measure of the set of $t$ numbers for which the series

$$\sum_{k=1}^{\infty} c_k r_k(t)$$

is convergent,and where $r$ are Rademacher functions :

$r_k(i) = \mathrm{sign}(\sin 2^k \pi t)$

The main result consists in the fact that the series $\sum_{k=1}^{\infty} c_k r_k(t)$ is convergent or divergent almost everywhere depending on whether $\sum_{k=1}^{\infty} c_k^2$ is convergent or divergent. Rademacher functions are an example of a sequence of independent functions i.e. the Steinhaus formalization of a sequence of independent random variables. Steinhaus fully appreciated the role that independence played in his research. The result of that research conducted together with his disciples, mainly with Marek Kac, was a series of works published in "Studia Mathematicae" under a common title *Sur les functions independantes* Part 1 written by Marek Kac was published in Volume 6 of "Studia Mathematicae" in the year 1936, and part 10, the final one,whose author was Hugo Steinhaus, came out in print in Volume 13 in 1953.

Steinhaus provides a description of the period discussed here and he writes: "The decade of 1930–1940 is characterized by the following topics: independent functions, the problem of length measuring, cooperation with doctors, *Mathe-*

*matical Kaleidoscope,* square rate . Independent functions are a step ahead towards mathematization of probability calculus. When I presented this definition to my former student Marek Koc he managed to find an analytical criterion of functions' independence . It was the first of the series of works called *Sur les functions independantes* (On independent functions – what is meant by independence is really a stochastic independence i.e. the one which corresponds to independence of random variables in probability calculus").

Making an attempt to evaluate Hugo Steinhaus' output in the field of applications of mathematics professor Łukaszewicz, wrote that:

"It is by no means an easy task because over a half of 200 of his publications is related to applications of mathematics".

In 1936 a work entitled "On the so called line of ethnic equality " came out where Steinhaus pointed out that the term "borderline for minor population" used by Jan Czekanowski was ambiguous. It also assumed that the two minorities lying on the two sides of the borderline and separated from the fatherland should be absolutely equal. Steinhaus proposed to find such borderlines for minorities which would be not only equal, but also possibly small.

Another example of practical application of mathematics quoted by Łukaszewicz is the problem of electricity rates. Steinhaus proposes a few versions of the new method of settlements between the supplier and the end user of electric power. One of them is the so called square rate which consists in reclaiming the due amount proportionately to the square root of integer of the squared amount of power received. Not only did put forward the proposal but he also cooperated with a group of engineers to construct a new type of electric meter. Łukaszewicz (1973) observed that Steinhaus paid a lot of attention to conflicts which may arise between the supplier and the customer in the situation of receiving goods. For many years Steinhaus cooperated with The National Normalization Committee and took part in establishing standards of statistical quality control (see: Oderfeld,Steinhaus, Wiśniewski 1951, Steinhaus 1953). The problems related to basic ideas of statistical inference were in the scope of his interest till the end of his life.

The universal character of applications of mathematics can be best seen in Steinhaus' works on conflicts which occur in court during fatherhood proceedings (see: Steinhaus 1954, 1958). Łukaszewicz (1956) stresses that Steinhaus was able to propose a method of evaluation of fraction of erroneous sentences in cases where serological analysis was conducted but judges ruled without taking into account the probability of fatherhood. This seems to be the only case in the history of law when the consistence of court sentences with the material truth was verified with the use of statistical methods.

The verification did not deal with individual sentences, so each judge could claim that his sentences were infallible. Yet on the basis of 844 cases one can see that the fraction of sentences contrary to the material truth ranged from 12 to 22%.

The work of Hugo Steinhaus ( 1958) is still considered by many lawyers to be the most interesting publication in the field of civil law in Poland after World War II.

Steinhaus is also highly praised for his achievements in the field of geography and his name is the most often quoted Polish name in the literature of the subject. He is appreciated for his works on geographical coefficients (Steinhaus 1936, 1947), his innovative ideas in the area of measuring the length of flat curves and generalization of the notion of length, and particularly for proposing a longimeter ( Steinhaus 1931).

The output of Hugo Steinhaus in applications of mathematics is so rich that it is impossible to discuss all its aspects but it is worth saying that his great merit lies not only in his individual work but also in establishing two schools of his name. They promulgated the Master's ideas and developed ideas of their own.

Since 1926 Steinhaus was the member of the Scientific Society in Lvov and in 1939 he became the full member of the Polish Statistical Society (see: Domański 1984). At the annual general meetings of The Polish Statistical Society held on 15 June 1947 and 29 May 1949 he was chosen the member of the Board of the Society.

In 1926 together with Stefan Banach he co-founded a periodical "Studia Mathematica", and since 1931 he was the member of the editing committee of a series called "Mathematical Monographs ".

When the Red Army seized Lvov in 1939 he was appointed for the post of professor of the Chair of Higher Analysis at the State Ukrainian University (formerly Jan Kazimierz University ) and a lecturer of the Academy of Sciences in Kiev. On 4 July 1941, when the German Army entered Lvov, Steinhaus managed to leave the city thanks to professor Bulanda – the former president of Lvov University and – and Mrs Morska-Klastrowa. He found shelter in Osieczyn near Lvov in the house of Witold Otto – an ex-worker of Bursar's Office of the University. Since 26 November 1941 till 26 August 1945 he assumed the identity of a Grzegorz Krochmalny, an inhabitant of Przemyśl, whose birth certificate was found by the poet Tadeusz Hollender.

On July 13, 1942 Steinhaus moved to Bezdechowo near Gorlice where he got engaged in a large – scale project of clandestine courses. After the war, since autumn of 1945 – Steinhaus took an active part in organizing the works of Wroclaw University where he later became the dean of the Faculty of Mathematics Physics and Chemistry. The Faculty was incorporated into the structures of both the University and the Technical University of Wrocław, and the Department of Natural and Economic Applications of the State Mathematical Institute. Hugo

Steinhaus also was the head of the Chair of Mathematical Applications, the first president of the Wroclaw division of the Polish Mathematical Society and the secretary general of the Wroclaw Scientific Society.

In 1945 he became the corresponding member, and in 1952 the fellow member of Polish Academy of Knowledge, and when Polish Academy of Sciences was established he became its full member. In the years 1948–1962 he headed the department of applications of mathematics operating within the structure of the State Institute of Mathematics.

Since 1947 – the year when the periodical "Colloquium Mathematicum" was founded – he was a member of its editing board . During the decade of 1953–1963 he was the editor in chief of another periodical entitled "Applications of Mathematics"

In his "Autobiography"(1973 ) Hugo Steinhaus presents the basic areas of his research in the following way: "the main directions of my scientific work carried out in Wroclaw include: A) Bayes law: B) Fatherhood investigation; C) Random numbers; D) Application of typology for geometry E) Mathematization of probability calculus and random processes; F) Didactics G) Others ".

The list of scientific publications of Hugo Steinhaus is impressive and it encompasses 247 items . It seems worthwhile to mention here the following books: "What is mathematics and what it is not" (1923), "Mathematical Kaleidoscope" (1938)(1958 3-rd edition) – translated into 10 languages; "One hundred problems"(1958), "Heads or tails" (1961); "Mathematics – an agent between the spirit and the matter"(2000).

In the field of theory of probability Steinhaus worked towards constructing axiomatic probability theory based on the theory of measure. He is thought to be the pioneer of the game theory. His works became an inspiration for development of theory of stochastic processes. Moreover, he initiated work on fundamental notions and problems of mathematical statistics in Poland . Methods of classification and ordering devised by Czekanowski inspired Steinhaus to design another method which he named the Wroclaw taxonomy. He was the author of tables of gold and iron numbers and shuffled numbers which are commonly used in the representative method. Finally, he proposed a method of parameter estimation in binomial distribution known as alpha Steinhaus estimator. Professor Kuratowski quotes professor Orlicz's opinion on Steinhaus: "We have very few scholars who, like Steinhaus, managed to co-found two different scientific schools". Let us remind here their names: the school of basic research, mainly functional analysis, and the school of applications of mathematics.

In the final part of his "Autobiography" Hugo Steinhaus wrote:

"My scientific output would be much smaller without the help of my disciples and co-workers: Stefan Banach, Marek Kac, H. Auerbach, J. Schauder, W. Sierpiński, Zbigniew Łomnicki, Al. Rajchman, Z. Janiszewski, K. Jantzen, Stanisław Ułam, A. Zygmund, B. Knaster, K. Kuratowski, Cz. Ryll-Nardzewski. E. Marczewski, J. Perkal, J. Łukaszewicz, St.Trybuła, Kazimierz Urbanik, Jan Oderfeld, Aleksander Deitzius, H. Fast, A. Goetz, St. Hartman, A.Zięba, St. Drobot, D. Blackwell, Otto Toeplitz, Edmund Landau, L. Hirszfeld, H. Kowarzyk, Jan Mycielski and others".

## REFERENCES


Domański Cz. (1984), *Hugo Steinhaus w „Sylwetki Statystyków Polskich*, WUS, PTS Łódź, s. 67–68.

Kuratowski K.( 1973), *Pół wieku matematyki polskiej 1920–1970*, Biblioteka Wiedzy Współczesnej Omega, Warszawa.

Łukaszewicz J. (1956), *O dochodzeniu ojcostwa*, Zastosowania Matematyki 2, s. 349–378.

Łukaszewicz J. (1973), Rola Hugona Steinhausa w rozwoju zastosowań matematyki, Wiadomości Matematyczne t. XVIII s. 51–63.

Maraszewski E. (1967), Hugo Steinhaus, Nauka Polska, t. XV, s. 82–93.

Oderfeld H., Steinhaus H., Wiśniewski K. (1951), *Statystyczna kontrola jakości (odbiór towarów według oceny alternatywnej)*, Polskie Normy PN/N-03001, PKN, Warszawa.

Steinhaus H. (1936), *O tak zwanej linii równowagi etnicznej*, Czasopismo geograficzne 8 s. 297–298.

Steinhaus H.(1973), *Autobiografia*, Wiadomości Matematyczne 1973. t. XVIII 4.

Steinhaus H. (1931), Longimetr, Czasopismo geograficzne 3, s. 1–4.

Steinhaus H. (1936), *O charakterystyce skupień osiedli*, Czasopismo geograficzne 8, s. 288–297.

Steinhaus H. (1954), *O dochodzeniu ojcostwa*, Zastosowania Matematyki 1, s. 67–82.

Steinhaus H. (1958), *Dochodzenie ojcostwa i alimentów (Uwagi de lega ferenda)*, Ruch prawniczy i ekonomiczny 20 s. 1–16.

Steinhaus H. (2000), *Między duchem a materią pośredniczy matematyka,* Wydawnictwo Naukowe PWN.

Urbanik K. (1973), Idee Hugona Steinhausa w teorii prawdopodobieństwa, Wiadomości Matematyczne t. XVIII, s. 39–50.

*Wielka Encyklopedia Powszechna*, Państwowe Wydawnictwo Naukowe, Warszawa 1962–1970, t. 1, s. 13.

*Wielka Ilustrowana Encyklopedia Powszechna*, Wydawnictwo Gutenberga, Kraków, t. 1, s. 20.

Caricatures by Leon Jeśmanowicz.