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STANISŁAW MARCIN ULAM (1909–1984)

The year 2009 was the centenary of the birth, and the 25th anniversary of the death, of the outstanding Polish mathematician Stanisław Marcin Ulam. I will try to present you with a brief portrait of this extraordinary scientist.



Stanisław Marcin Ulam was born on 13 April 1909 in Lwów (Lviv), and gained his education in that city. His father, Józef Ulam, was a lawyer, and his mother Anna (*née* Auerbach) was the daughter of an industrialist. It was a wealthy, Polonized Jewish family which had arrived from Venice three generations earlier. After the outbreak of the First World War the Ulams moved to Vienna. During the war Ulam's father was a staff officer in the Austrian army, which meant that his family was frequently on the move. For a time they lived in Ostrava, where Stanisław went to school. After that he had only private teachers – the family was travelling too much for him to attend school regularly. In 1918 the Ulams returned to Lwów, and in 1919, at the age of 10, Stanisław passed the entrance exam to the city's 7th *Gimnazjum* – a secondary school which pupils would attend for eight years. In 1927 he passed his school-leaving exams, and in autumn of that year began studying at the General Faculty of Lwów Polytechnic, where he studied in the mathematical group. At set theory lectures he met the young Professor Kazimierz Kuratowski, recently arrived from Warsaw, a former student of Sierpiński, Mazurkiewicz and Janiszewski. From the start Ulam participated more actively than his colleagues in the discussions led by Kuratowski. Kuratowski quickly came to regard him as one of the best students, and often talked with him after lectures. In this way, thanks to Kuratowski's encouragement, Ulam's mathematical career began. Between lectures he would

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usually sit in the room of one of the mathematics lecturers. It was in one of these rooms that he first met Stanisław Mazur, a young doctoral student at the university, who had come there to work with Orlicz, Nikliborc and Kaczmarz, who were a few years older than he. Through his conversations with Mazur, Ulam began to learn about the problems of functional analysis, which was being developed by Banach. At the start of the second semester of his studies, Kuratowski told Ulam about a certain problem in set theory relating to transformations of sets. Ulam succeeded in solving this problem, and his paper appeared in *Fundamenta Mathematicae*, a leading Polish mathematical journal edited by Kuratowski, in 1929. By the end of his first year of studies Ulam had written his second paper, again published in *Fundamenta Mathematicae*. In a very long paper which appeared one year later, Alfred Tarski obtained the same result. When Kuratowski pointed out to him that it followed from Ulam's theorem, Tarski mentioned the fact in a footnote. This event was described by Ulam (1976, p. 30) as follows: "In view of my youth, this seemed to me a little victory – an acknowledgment of my mathematical presence."

In 1932 he was asked to present a paper at an international mathematical congress in Zurich. After the congress ended he paid a short visit to Montreux with Kuratowski and Knaster, and then he returned to Poland to pass some remaining exams and write his master's thesis. Ulam (1976, p. 46) recalls: "I had an almost pathological aversion to examinations. For over two years I had neglected to take the examinations which were usually necessary to progress from one year to the next. My professors had been tolerant, knowing that I was writing original papers. Finally, I had to take them – all at once."

The last academic year of Ulam's regular four-year course was that of 1930/31, but he passed his last four course exams only in November and December of 1932. In order to obtain a degree it was necessary, as well as passing the required course exams, to present a thesis and to pass a written and oral final exam. Ulam's application to the Degree Examinations Committee at the General Faculty of the Polytechnic was accompanied by a list of 12 of his papers, nine of which had already appeared in print. Ulam's master's thesis was titled "On the Theory of Combinatorial Products". His supervisor, Professor Kazimierz Kuratowski, assessed it thus: "The work is a study of the 'product' operation, which has been little researched until now, but is playing an increasingly important role in mathematics. The author analyses this concept in relation to problems in set theory, group theory, topology, metric space geometry, combinatorics and measure theory in connection with probability. Since the author has demonstrated complete mastery of the subject and due knowledge of the relevant literature, that moreover the work contains a number of original results, and finally that the author presents many interesting problems in this work, I consider this thesis to be of the highest class." (In the original the

word *celująca*, meaning “of the highest class”, was underlined.) On the subject of his master’s thesis Ulam (1976, p. 46) recalls: “I wrote my Master’s thesis on a subject which I thought up myself. I worked for a week on the thesis, then wrote it up in one night, from about ten in the evening until four in the morning, on my father’s long sheets of legal papers. I still have the original manuscript. (It is unpublished to this day.) The paper contains general ideas on the operations of products of sets, and some of it outlines what is now called Category Theory. It also contains some individual results treating very abstractly the idea of a general theory of many variables in diverse parts of mathematics. All this was in the fall of 1932 upon my return from Zurich.”

Ulam passed his final oral exam on 15 December 1932, gaining the highest grade, in front of a committee consisting of Włodzimierz Stożek (chairing), Antoni Łomnicki and Kazimierz Kuratowski. Consequently the Council of the General Faculty of Lwów Polytechnic awarded him a master’s degree.

In 1933 Ulam defended his doctoral thesis, titled “On the Theory of Measure in General Set Theory”, which was published by Ossolineum of Lwów in the same year. In this work the author combined several of his earlier papers, theorems and generalizations in measure theory. The most important results in the thesis had already been published by Ulam in *Fundamenta Mathematicae* in 1930. This was the first doctorate awarded by the new General Faculty created at Lwów Polytechnic in 1927 – it was the only faculty in which master’s and doctor’s degrees were awarded (the others gave engineering qualifications).

The circumstances in which Ulam’s doctoral thesis came into being are recalled by Kazimierz Kuratowski (1981, p. 95): “Returning to the portrait of Ulam as a scientist, it may be worth recalling his doctoral thesis, which was a typical product of the atmosphere holding sway in the Lwów mathematical community. As I remember, it was sometime in 1928/29 during one of my numerous conversations with Banach at the Scottish Café that we took an interest in the ‘measure problem’ which had been posed by Hausdorff many years earlier, and then remained unsolved. During a conversation lasting several hours we tried different ways of attacking the problem, but without effect. I returned home around midnight. However I could not sleep until (to my great joy) I found a solution. The next day I met Banach and said ‘You know what, Stefek, I’ve solved Hausdorff’s problem.’ ‘So have I,’ Banach replied. What’s more, it turned out that Banach’s method of reasoning and mine were almost identical: it was essentially a continuation of our discussion at the café. Naturally we decided to publish our result in the form of a joint paper (in *Fundamenta*). In that paper we posed a certain problem which had not been solved (and which in fact we had not tried to solve). I told Ulam about it. After some time Ulam came to me with a ready solution. For me, naturally, this was a great satisfaction, and since I considered the result to be of great value, I encouraged Ulam to base his

doctoral thesis on it. As it turned out, Ulam's doctoral thesis aroused great interest in the academic world, remaining relevant even today, while for my part I had acquired 'my' first doctor." In the same memoirs, on page 93, Kuratowski writes: "Stanisław Ulam is the most outstanding of my students. I could speak of him as Steinhaus did of Banach: Ulam is my great scientific discovery."

Mathematics embraced the whole of Ulam's life. Stanisław Mazur, Kazimierz Kuratowski and Stefan Banach introduced him to the secrets of mathematical thinking and the process of discovery. Ulam recalls the long hours they spent together at the Scottish Café in Lwów, over a sheet of paper with a single symbol or function on it. They would pore over it and exchange thoughts and suggestions. For them it was a crystal ball, making it easier to concentrate on some problem. These meetings came at Banach's initiative. Ulam was the youngest of those present. While still a student, he and his friend Józef Schreier attained the honour of being admitted there among the mathematical geniuses. Professor Andrzej Alexiewicz has said that being invited to the Scottish Café was considered like being dubbed a knight. Banach regarded some of Ulam's approaches to mathematical problems and proofs as strange, but admitted that they led to correct results. This was a great compliment to Ulam – he considered Banach (rightly) as a real born genius with a subconscious ability to discover "hidden paths". Hugo Steinhaus (1961, p. 257) describes the atmosphere at the Scottish Café thus: "Banach, Mazur and Ulam made up the most important table at the Scottish Café in Lwów. There was even a session that lasted 17 hours – it resulted in a proof of a certain important theorem in Banach spaces, but no-one wrote it down, and today no-one is able to recreate it. Probably the table top covered in the chemical traces of the pencil was wiped clean after that session, as always, by the café's cleaning woman. Thus a great service was done by Banach's wife Łucja – now at rest in a cemetery in Wrocław – when she bought a thick notebook in hard covers and entrusted it to the barman of the Scottish Café. There problems were written down, on the first side of each sheet, so that any answers could at some time be entered on the blank sides next to the text of the questions. The original 'Scottish Book' was available to any mathematician at the café who demanded it; some problems were announced there with the promise of a prize for a solution – prizes ranged from a small black coffee to a live goose."

The notebook bought by Łucja Banach quickly became known as the Scottish Book, and throughout its several years of existence played a great part in the life of Lwów's mathematicians. The first problem was entered in the Book by Stefan Banach with the date 17 July 1935, and the last by Hugo Steinhaus on 31 May 1941. In total 198 problems were written in the book. The record-holder in terms of the number of problems entered was Stanisław Ulam, who wrote 40

himself and 22 jointly with others, including with Stefan Banach (5), Józef Schreier (6) and Stanisław Mazur (7).

After the war and after her husband's death, Łucja Banach brought the Book to Wrocław, where Professor Hugo Steinhaus copied it out by hand (exactly word for word) and in 1956 sent the copy to Stanisław Ulam at Los Alamos. Ulam translated it into English, and then made 300 copies (at his own cost) which he sent to friends and to various universities in different countries. The Book became famous, and many mathematicians asked Ulam for additional copies. There were so many such requests year after year that it was decided at Los Alamos that a new edition would be produced (with various corrections included), no longer at Ulam's own cost, which happened in 1977. In May 1979 at North Texas State University the "Scottish Book Conference" took place (attended by Ulam, Kac and Zygmund among others), after which, with updated information on solutions to the problems and on related problems, the Book (supplemented with several papers from the conference, reminiscences in particular) was published in 1981 by Birkhauser (edited by R. Daniel Mauldin).

Today the original Scottish Café Book is in the possession of Banach's family. There is a copy of the original in the library of the PAN (Polish Academy of Sciences) Institute of Mathematics in Warsaw.

In 1934, not having much chance of finding teaching work, he travelled (at his father's cost) to Vienna, Zurich, Paris and Cambridge, in order to attend and give mathematical lectures. Ulam recalled that these journeys were an attempt to make an impression in the world with his mathematical results – and not without reason. The situation in Europe had changed dramatically since Hitler's coming to power, and augured the worst, particularly for Jews. In Poland, through mild analogy, aggressive anti-Semitism appeared. Ulam's uncle, Michał Ulam, encouraged him to take up a career abroad. At the end of 1934 Ulam began corresponding with John von Neumann, a very young professor at the Institute for Advanced Studies in Princeton. Ulam wrote to him about several problems in measure theory. In reply, he invited Ulam to come to Princeton for a few months. In autumn 1935 Ulam met von Neumann in person in Warsaw. Von Neumann, returning from a topology conference in Moscow, stayed in Warsaw for several days and gave a lecture at the Warsaw branch of the Polish Mathematical Society. In December 1935 Ulam left the French port of Le Havre, aboard the English ship *Aquitania*, on his first voyage to New York. At Princeton Ulam got to know (among others) Bochner, Birkhoff and Weyl. However the greatest authority and model for most scientists was Albert Einstein. Ulam (1976, p. 72) recalls the following event: "A cousin of mine, Andrzej Ulam, a banker, came to New York on business about two months after my arrival, and I invited him to visit me in Princeton. It happened that during that week I was giving a talk in some seminar, and my name was listed on the

same page of the Institute's Bulletin as the announcement of Einstein's regular weekly seminar. This impressed him enormously; he mentioned it in a letter home, and my reputation among friends and family in Poland was made."

In view of the worsening situation in Europe and the increasing threat to Poland, and as a Jew, Ulam sought means by which he could remain in the United States. With the support of George David Birkhoff, Ulam obtained a nomination to the post of junior fellow at Harvard, for three years starting in autumn 1936. The conditions were extremely attractive: \$1500 a year plus board and lodging, as well as travel allowances. In those days it seemed a lavish offer. He began working with John Oxtoby, leading to a joint publication on statistical mechanics in 1941, which Ulam later regarded as one of the most important in his mathematical career. He lectured at Harvard and at Brown University in Providence, Rhode Island. Between 1936 and 1939, he always spent the three summer months in Poland with his family. In 1937, together with Banach, he played host to von Neumann in *Lwów*. Next year Ulam made a return visit to von Neumann in Budapest. In 1938 Ulam's mother died of cancer, and he himself obtained an American immigrant's visa from the US consulate in Warsaw. A few months later this would have been virtually impossible.

In August 1939, his father Józef and uncle Szymon took Ulam and his 17-year-old brother Adam to the passenger quay at Gdynia. The Ulam brothers sailed aboard the *Batory* to America. Adam, 13 years Stanisław's junior, whom the latter helped during many years at Brown University, later became a well-known historian, author of one of the first books about Stalin in the West, and director of the Centre for Russian Studies at Harvard University. They would never see their family again. Ulam's entire family, including his sister Stefania (but apart from two cousins), died during the war.

In 1939 Ulam's three-year contract with the Society of Fellows expired, and it could not be renewed because he was over the age limit. Thanks to Birkhoff, then the guru of American mathematics, he obtained permission to remain at Harvard for a year as a lecturer in the Mathematics Faculty. After that, in 1940, he gained the post of lecturer at Wisconsin State University in Madison. There he became friends with Cornelius Everett, with whom he wrote many joint papers on group theory and projection algebras.

In 1941 Ulam acquired American citizenship and tried to join the air force as a volunteer, but his application was rejected due to his poor vision. At Madison in 1942 he married the French student Françoise Aron, who he had met at Cambridge, Massachusetts. In 1944 their only daughter, Claire, was born. When Claire was asked by her friends why her father didn't play ball with her, she answered "My daddy just thinks, thinks and thinks! Nothing else, just thinks." Thinking was Ulam's principal activity. In late spring 1943 he wrote to von Neumann to ask about the possibility of doing work for the army. The whole

world was engrossed in war, and Ulam wanted to make his contribution to the war effort. In early autumn 1943 he received a reply, proposing a meeting at Union Station in Chicago during von Neumann's journey westward from Princeton. At the meeting, Von Neumann informed him of the existence of an important secret military project. Shortly afterwards Ulam received an official invitation from Los Alamos, which is about 40 miles north-east of Santa Fe in New Mexico, to join an unspecified military project. It was signed by the renowned physicist Hans Bethe (who would soon become the joint discoverer of the Bethe–Feynman formula, fundamental for calculations on yield of fission reactions). Stanisław Ulam and his pregnant wife found themselves at a secret laboratory in Los Alamos, where he was assigned to Edward Teller's group, working on a project to create a "superbomb". This was the first attempt to construct a hydrogen (thermonuclear) bomb. Apart from Teller's small group, all the scientists at Los Alamos were working on the atomic bomb project, using the energy released by the fission of uranium or plutonium nuclei. Work on this project involved many leading scientists: John von Neumann, Enrico Fermi, Hans Bethe, Niels Bohr, Richard Feynman, Edward Teller, Robert Oppenheimer, Otto Frisch, Victor Weisskopf, Emilio Segre. The intellectual potential of this group was astounding. In the entire history of science there is nothing that comes even close to such a concentration of great minds. It was Ulam's first encounter with practical problems of physics, directly relating to experimental data. A friend of Ulam's, Otto Frisch, in an article for the *Bulletin of the Atomic Scientists*, described his first impressions of Los Alamos, from which he had arrived from besieged Great Britain (quoted from Ulam (1976), p. 6), as follows: "I also met Stan Ulam early on, a brilliant Polish topologist with a charming French wife. At once he told me that he was a pure mathematician who had sunk so low that his latest paper actually contained numbers with decimal points!". The first task which Teller gave Ulam on his arrival involved investigating the exchange of energy between free electrons and radiation in the extremely hot gas which was expected to form during a thermonuclear bomb explosion. Ironically, it was this first problem given to Ulam in 1943 which would later be the main subject of his joint work with Cornelius Everett which proved Teller's version of the "superbomb" project to be impossible to realize.

Ulam's contribution to the work on construction of the atomic bomb involved performing statistical investigations on neutron branching and breeding. This process maintains the chain reaction and leads to the release of energy from uranium or plutonium. More precisely, in 1944 Ulam and David Hawkins were interested in the pure model problem of the "genealogical tree" of a neutron, which may produce zero (is absorbed and ends its life), one (simply continues to exist) or two, three or four neutrons (meaning that new ones

appear), each of these events having a defined probability. The task is to track the evolution of the system and the chain of possible events through many generations. Ulam and Hawkins quickly found a method which helped in the mathematical analysis of such branched chains. Laplace's characteristic functions, which were used in investigations of the distributions of sums of independent random variables, proved to be an ideal tool for investigation of multiplicative processes, later called branch processes. A theory of such processes was described by Ulam and Hawkins in the Laboratory's 1944 report. They achieved this earlier than Andrey Kolmogorov and other Russians.

A long discussion between Ulam and von Neumann at the beginning of 1944 revealed the need for a more accurate method – rather than the approximate method proposed by von Neumann – for calculating the hydrodynamic course of the implosion needed for ignition of a nuclear bomb. It was necessary to use “brute force” – mass numerical computations – which was impossible using the existing mechanical calculating devices. It was the need for these precise computations that began the development of electronic computers (initially based on vacuum tubes). These came into being as a result of a combination of scientific and technological advances, by way of analogy with the operations performed by the brain. In 1952 the MANIAC arrived at Los Alamos – the second model, after Princeton's, of the first reprogrammable computer. The idea of programming was invented by John von Neumann, deriving it from mathematical logic.

On 16 July 1945 the first atomic bomb test was carried out. Then came Hiroshima and the victory over Japan. The war ended, and the world re-emerged from the ashes. Many members of the team left Los Alamos to return to their universities or take up new academic posts.

In autumn 1945 the Ulams moved to Los Angeles, where Ulam became a professor at the University of Southern California. In January 1946 he became seriously ill with encephalitis, which he survived thanks to his skull being opened and his brain sprayed with penicillin. These were the early days of penicillin, which was used then without restriction. Ulam's first publication following his illness was an article dedicated to the memory of Stefan Banach, who had died of lung cancer in Lwów on 31 August 1945, aged just 53. The article appeared in the Bulletin of the American Mathematical Society, issue 52 (1946), pp. 600–603.

In mid 1946 Ulam returned to the Los Alamos National Laboratory. At a seminar shortly after his return he gave two lectures containing good, productive ideas. Further development of these concepts led to many successes. One of them related to what would come to be called the Monte Carlo method, and the second concerned certain new methods of calculation in hydrodynamics. Both lectures provided a foundation for very concrete applications of probability

theory and continuum mechanics. Ulam (1976, pp.196–199) said the following of the Monte Carlo method: “The idea for what was later called the Monte Carlo method occurred to me when I was playing solitaire during my illness. I noticed that it may be much more practical to get an idea of the probability of the successful outcome of a solitaire game (like Canfield or some other where the skill of the player is not important) by laying down the cards, or experimenting with the process and merely noticing what proportion comes out successfully, rather than to try to compute all the combinatorial possibilities which are an exponentially increasing number so great that, except in very elementary cases, there is no way to estimate it. This is intellectually surprising, and if not exactly humiliating, it gives one a feeling of modesty about the limits of rational or traditional thinking. In a sufficiently complicated problem, actual sampling is better than an examination of all the chains of possibilities.(...) The Monte Carlo method came into concrete form with its attendant rudiments of a theory after I proposed the possibilities of such probabilistic schemes to Johnny in 1946 during one of our conversations. It was an especially long discussion in a government car while we were driving from Los Alamos to Lamy.(...) After this conversation we developed together the mathematics of the method. It seems to me that the name Monte Carlo contributed very much to the popularization of this procedure. It was named Monte Carlo because of the element of chance, the production of random numbers with which to play the suitable games.”

However, Ulam’s most noteworthy accomplishment at Los Alamos was his contribution to post-war work on the hydrogen bomb. First he and Everett showed that Teller’s idea for the construction of a hydrogen bomb was impossible to realize, and then in February 1951 Ulam proposed a new method, using the propagation of the mechanical shock wave caused by the atomic explosion to bring about strong compression of the thermonuclear fuel, eventually causing a massive explosion. When Ulam told Teller about his idea for using an atomic bomb to compress deuterium just before ignition, Teller immediately appreciated its value. However he suggested that, instead of using the mechanical shock wave as proposed by Ulam, implosion could be achieved in a better way: by a so-called radiation implosion. The plan for the new hydrogen bomb, called the “Teller–Ulam design”, was quickly accepted by Los Alamos scientists and government officials. From that time on all thermonuclear bombs would use a mechanism based on the use of an atomic explosion to trigger a secondary thermonuclear explosion caused by implosion.

On 1 November 1952 the new weapon was tested for the first time. It made fission nuclear weapons look like pistol shots in comparison with a cannon salvo.

After completing theoretical work on the hydrogen bomb, Ulam considered his task complete, and decided to change his surroundings for a time. He took a sabbatical, and spent the winter semester of 1951/52 at Harvard. He was given another sabbatical in 1956/57, and became a visiting professor at the Massachusetts Institute of Technology. On returning to Los Alamos he took up the post of scientific adviser to the director of the Laboratory. In 1965 Ulam began to make regular visits to the dynamically developing University of Colorado in Boulder. In 1967 he retired from Los Alamos, staying on as consultant to the Laboratory for one dollar a year. In the same year he moved permanently to Boulder, where he became professor and dean in the faculty of mathematics. In 1968–1975 he was also professor of biomathematics at Colorado Medical School. In 1975 Ulam retired from the University of Colorado and returned to Santa Fe, close to Los Alamos. He continued to work there, using the Laboratory's computers and library. He lectured for several months at Harvard and at the Massachusetts Institute of Technology, and also visited Paris, as well as the University of California campuses in San Diego and Davis. Moreover, each year from 1974 to 1984 he spent two months at the University of Florida in Gainesville.

Stanisław Ulam was a member of the National Academy of Sciences in Washington, the American Academy of Arts and Sciences, the American Philosophical Society, the American Mathematical Society, the Polish Mathematical Society and several others societies. He was a member of the board of the Jurzykowski Foundation in New York. He was given honorary doctorates by universities in New Mexico, Wisconsin and Pittsburgh. He received the Polish mathematical August Medal. In 1973 he briefly visited Warsaw to lecture at the Banach Centre. From 1950 onwards the Ulams spent their annual vacations in France, where his brother-in-law lived.

Stanisław Ulam died suddenly from a heart attack on 13 May 1984 in Santa Fe, after returning from England, where he had been visiting the Polish mathematician Zbigniew Łomnicki. Françoise Ulam had her husband's ashes buried at Montmartre Cemetery in Paris.

Stanisław Ulam was a man gifted with an exceptionally fertile imagination and creativity, an almost visionary talent. He wrote more than 150 papers and three books. His work led to the emergence of many new areas of academic research.

Ulam provided a superb description of himself in the widely-read book *Adventures of a Mathematician*. I have a copy of this book with the following author's dedication: "To Mirosław Krzyśko with best wishes. Stanisław Ulam. Gainesville, 3, XII, 1980."

Thank you for your attention.

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