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Małgorzata Aniol, Wiesław Wagner***

PROBABILITY PROGNOSIS OF THE STATES OF RESULTS DURING VOLLEYBALL MATCH SETS

Abstract. One of characteristic features of a volleyball game is playing consecutive sets and deciding the current score according to the results of particular sets. In this way we can examine volleyball game in many stages, but the results of a game are fixed in the third, fourth or fifth stage. Each set creates a temporary state, which occurs with definite probability. It is immediately dependent on accepted foundations of probability of winning a set in every game stage.

In the research we analysed a simplified model. For the needs of this model we draw a tree figure, which describes states of passing sets in a volleyball game. We described also a theoretical model and illustrated its helpfulness for interpretation of the results of female I-league from the starting season 1998/1999 for the team Augusto Kalisz, the winner of principle season.

Key words: probability model, binomial distribution, graph, prognosis, match volleyball.

1. INTRODUCTION

During a volleyball match, players play successive sets and the state of a match is determined depending on the results of the completed sets. This means that we should treat such a match as a certain multistage game, where the states of results are determined at the third, fourth or fifth stage of the game. Each stage generates a transient state occurring with a determined probability. The probability is directly dependent on the assumed probability of winning a set. This leads to a certain theoretical model with a constant probability of winning a set at each stage of a match. The above-mentioned model lets us prognosticate the wins and defeats in a match as well as lets us make a prognosis of the states of results.

* Master of Science, Department of Sports Games, Academy of Physical Education, Poznań.

** Professor, Department of Statistics, Academy of Physical Education, Poznań.

The issue of probability models has been discussed in many researches. The contextual studies of the following authors deserve our attention: Hsi and Burych (1971), Wołyniec J. (1989), Wagner and Majewska (1996), Bennett (1998), as well as Anioł and Wagner (2000).

The study discusses the use of the simplified model in volleyball with the aim of making prognosis. In order to describe such a model, researchers drew a graph in the form of a tree, depicting transient states during a volleyball match, appropriate formulas of probability of results in a match and the aspects of match results prognosis. The results discussed in the study were illustrated on the basis of the results of women's first division in the starting season 1999/2000.

2. GRAPH OF TRANSIENT STATES IN MATCH SETS

A graph is a useful way of presenting multistage actions in a developed form. In order to describe transient states of volleyball sets, researchers use graphs called trees. Such trees include edges and vertices, such as:

(a) starting vertex with no edges directed inwards;

(b) transient vertices with edges directed inwards and outwards;

(c) final vertices with no outward edges.

The edges are attributed to one of the following states: win (w) or defeat (nw) in a set. The set of successive edges and vertices drawn from the starting to the final vertex is defined as a path. In case of a volleyball match, the lengths of such paths include 3, 4 or 5 edges, which corresponds with the 3^{rd}, 4^{th} or 5^{th} stage (set) of a match. The states of results, i.e. final vertices in the graph are marked with a symbol of double underlining (=). Figure 1 shows a completed graph of transient states of a volleyball match set with a successive number of vertices (from 1 to 14) and edges (from 1 to 18).

The graph presented in Figure 1 includes:

– 1 starting vertex (0),

– 14 vertices of the states in a match (numbers 1, 2, ..., 14),

– 18 edges including 8 inner (numbers 4, 5, 8, 9, 10, 11, 14, 15) and 10 outer edges (numbers 1, 2, 3, 6, 7, 12, 13, 16, 17, 18),

– 6 final vertices (numbers 6, 9, 10, 12, 13, 14), all the remaining vertices are transient.

If 1(w) and 1(nw) stand for the numbers of wins and defeats, a given edge occurs only if $1(w) \neq 0$ or $1(nw) \neq 0$. If $1(w) = 0$ or $1(nw) = 0$ a given edge does not occur and we obtain a reduced graph, characterized by a smaller number of edges and vertices. The shortest graph will include only three vertices of the states in a match – vertices (1, 3, 6) – when a team wins all meetings by 3:0, or vertices (2, 5, 9) – when a team loses all games 0:3.

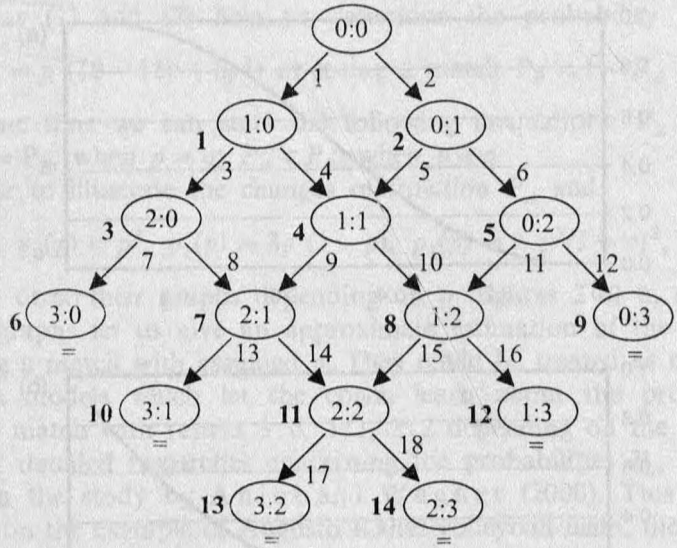

Fig. 1. The tree of transient states – a complete graph

3. PROBABILITY MODEL OF THE STATES OF RESULTS DURING A MATCH

Let letter Z represent a certain team playing a series of volleyball matches in a starting season. If we assume that the probability p of winning a set is known in case of team Z, $q = 1 - p$ expresses the probability of defeat. The state of the match is represented by $a:b$, where we can distinguish two variants:

– variant 1: team Z wins a match by 3:0, 3:1 or 3:2, then $a = 3$, $b = 0, 1, 2$

– variant 2: team Z loses a match by 0:3, 1:3 or 2:3, then $b = 3$, $a = 0, 1, 2$.

The probability of the states of match results (transition to the final vertices) equals:

(a) for variant 1

$$p_b = \binom{b+2}{b} p^3 q^b, \quad b = 0,\ 1,\ 2, \tag{1}$$

(b) for variant 2

$$p_a = \binom{a+2}{a} p^a q^3, \quad a = 0,\ 1,\ 2. \tag{2}$$

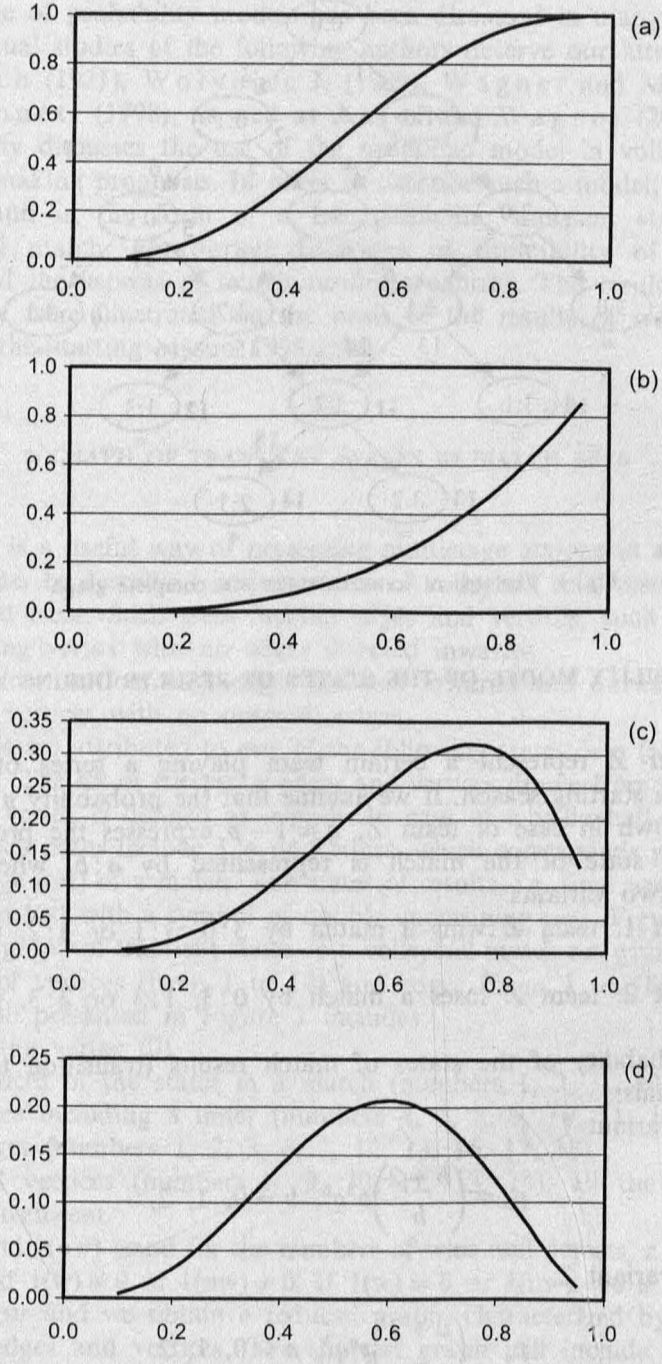

Fig. 2. Graphs of functions: P_A (a), $p_0(p)$ (b), $p_1(p)$ (c) and $p_2(p)$ (d)

Formulas (1) and (2) help us determine the probability of winning $P_A = \sum_{b=0}^{2} p_b = p^3(10 - 15p + 6p^2)$ or losing a match $P_B = 1 - P_A$ by team Z. At the same time we can state the following inequalities: $P_A > P_B$, when $p > q$; $P_A = P_B$, when $p = q$; $P_A < P_B$, when $p < q$.

In order to illustrate the changes of function P_A and

$$p_0(p) = p^3,\ p_1(p) = 3p^3(1-p),\ p_2(p) = 6\ p^3(1-p)^2, \tag{3}$$

researchers drew their graphs depending on p (figures 2 a, b, c, d).

These graphs let us give an approximate estimation of the probability of finishing a match with assumed p. They could be treated as nomographs i.e. certain models which let the coach learn about the probability of finishing a match with results 3:0, 3:1, 3:2 depending on the assumed p.

Further detailed researches concerning the probabilities P_A and P_B are included in the study by A n i o ł and W a g n e r (2000). This study also illustrates, on the example of Augusto Kalisz volleyball team, the assessment of the probability of transient states between successive stages of a game, given in Figure 1.

4. PROGNOSIS OF THE STATES OF MATCH SETS

The above mentioned probability model lets us prognosticate the states of match sets. Let's describe such a prognosis for a certain team Z. Let's assume that this team play matches in successive rounds k. Let's assume that the team have played at least $k \geqslant 3$ matches, where they won $sw(k)$ sets or lost $sp(k)$ sets, that is the team have played $s(k) = sw(k) + sp(k)$ sets altogether. These data let us estimate the empirical probability $p(k) = sw(k)/s(k)$ of winning a set by team Z after completing k rounds. It is used in order to prognosticate the states of matches for $(k + 1)$ round of matches, according to the following formulas:

$$\begin{gathered} P_A(k+1) = p(k)^3[10 - 15p(k) + 6p(k)],\ P_B = 1 - P_A, \\ p_0(k+1) = p(k),\ p_1(k+1) = 3p(k),\ p_2(k+1) = 6p(k)^3q(k)^2 \\ \tilde{p}_0(k+1) = q(k)^3,\ \tilde{p}_1(k+1) = 3q(k)^3p(k),\ \tilde{p}_2(k+1) = 6q(k)^3p(k)^2. \end{gathered} \tag{4}$$

The practical use of the formula (4) will be illustrated on an example. Let us assume that team Z have played 4 rounds of matches, where they won 12 sets and lost 1 set. The result of the assumptions is that $sw(4) = 12$, $sp(4) = 1$, and thus $p(4) = 12/13 = 0.923$ and $q(4) = 1 - 0.923 = 0.077$. By inserting the obtained probability values $p(4)$ and $q(4)$ into the formula (4), we can make the following prognosis for the 5th round:

$$P_A(5) = (923)^3[10 - 15 \cdot 0.923 + 6(0.923)^2] = 0.986,\ P_B(5) = 1 - 0.986 = 0.014,$$

$$p_0(5) = 0.923^3 = 0.787,\ p_1(5) = 3(0.923)^3 0.077 = 0.182,$$

$$p_2(5) = 6(0.923)^3(0.077)^2 = 0.028,\ \tilde{p}_0(5) = 0.077^3 = 0.000,$$

$$\tilde{p}_1(5) = 3(0.077)^3 0.923 = 0.001,\ \tilde{p}_2(5) = 6(0.077)^3(0.923)^2 = 0.002.$$

These estimations show that the prognosis for team Z for the $(k+1)^{st}$ round provides for the following probabilities of the states of matches: winning – 0.986, losing – 0.014, finishing with result 3:0 – 0.787, 3:1 – 0.182, 3:2 – 0.028, 0:3 – 0.000, 1:3 – 0.001, 2:3 – 0.002.

5. ILLUSTRATED EXAMPLE

In order to illustrate the prognosis concerning the states of matches, researchers used the data obtained in the starting season 1999/2000 by the women's first division of volleyball. The teams played 18 rounds, 9 matches in each round, there were 274 match sets altogether. The order of teams after the play-off was as follows: 1 – Nafta Gaz Piła, 2 – Melnox Autopart Stal Mielec, 3 – Stal Bielsko-Biała, 4 – Skra Warszawa, 5 – Gedania Gdańsk, 6 – Wisła Solidex Kraków, 7 – BOŚ Stoen Nike Węgrów, 8 – Centrostal AMT Bydgoszcz, 9 – Calisia Kalisz.

The state $k = 3$ was assumed as the initial state. Researchers made prognoses of wins for selected teams, which is presented in Table 1.

Table 1

Prognoses of the states of wins and defeats

k	Nafta		Melnox		Stal		Skra		Calisia	
1	2	3	4	5	6	7	8	9	10	11
4	12	0.996	4	0.210	6	0.317	12	0.977	6	0.275
	1	0.004	8	0.790	9	0.683	2	0.023	10	0.725
5	15	0.986	7	0.384	9	0.407	12	0.977	7	0.235
	2	0.014	9	0.616	11	0.593	2	0.023	13	0.765
6	18	0.977	10	0.500	12	0.500	13	0.865	7	0.169
	3	0.023	10	0.500	12	0.500	5	0.135	16	0.831
7	21	0.984	13	0.578	12	0.500	13	0.715	9	0.193
	3	0.016	11	0.422	12	0.500	8	0.285	19	0.807
8	21	0.984	16	0.632	13	0.433	16	0.749	10	0.180
	3	0.016	12	0.368	15	0.567	9	0.251	22	0.820
9	24	0.977	18	0.585	14	0.384	19	0.738	10	0.180
	4	0.023	15	0.415	18	0.616	11	0.262	22	0.820
10	27	0.982	18	0.585	17	0.448	22	0.790	11	0.171
	4	0.018	15	0.415	19	0.552	11	0.210	25	0.829
11	30	0.977	21	0.625	18	0.407	24	0.736	12	0.163
	5	0.023	16	0.375	22	0.593	14	0.263	28	0.837
12	32	0.942	24	0.632	21	0.438	24	0.657	12	0.137
	8	0.058	18	0.368	24	0.562	17	0.343	31	0.863

Table 1 (contd.)

1	2	3	4	5	6	7	8	9	10	11
13	35	0.952	27	0.683	24	0.500	27	0.683	12	0.115
	8	0.048	18	0.317	24	0.500	18	0.317	34	0.885
14	38	0.948	30	0.725	27	0.536	27	0.683	15	0.171
	9	0.052	18	0.275	25	0.464	18	0.317	34	0.829
15	41	0.956	33	0.740	30	0.585	27	0.616	16	0.166
	9	0.044	19	0.260	25	0.415	21	0.384	37	0.834
16	44	0.963	36	0.736	30	0.585	27	0.555	16	0.145
	9	0.037	21	0.264	25	0.415	24	0.445	40	0.855
17	44	0.963	37	0.694	33	0.593	30	0.603	16	0.127
	9	0.037	24	0.306	27	0.407	24	0.397	43	0.873
18	45	0.934	39	0.667	36	0.616	33	0.610	16	0.127
	12	0.066	27	0.333	28	0.384	26	0.390	43	0.873

Table 1 presents the first four teams (Nafta. Melnox. Stal. Skra) as well as the last team (Calisia) after the games in the basic season. The probability of winning for Nafta was very high throughout the entire season and equalled from 0.934 (prognosis at the end of the season) to 0.996 (prognosis for the 4th round). Such values were the result of 14 wins in 16 matches, including 8 wins by 3:0. The probability of defeat for Melnox, in turn, was fairly high after the first five rounds, however, beginning from the 6th round, the prognoses of winning exceeded the prognoses of defeat, and relatively the biggest difference in these prognoses occurred for the 15th round (prognosis of winning – 0.740, defeat – 0.260). The prognoses for Stal varied between 0.4:0.6 for the 12th round, and beginning with the 14th round this ratio changed to 0.6:0.4. The high prognoses for Skra up to the 16th round were based on the fact that that team had obtained high results in the initial phase of the season, and after the defeats in rounds 15, 16 and 17, those prognoses fell to 0.6:0.4. The high prognoses of defeats for Calisia, the team that was placed in the last position, were confirmed by the fact that that team had won only one match and only 16 sets in the entire season.

The series of wins (+) and defeats (–) that agree with the assumed prognoses (Table 2) confirm the prognoses given in Table 1. The symbol has been used to mark the fact that the final result of a match did not correspond with the initial prognosis.

Table 2

Series of matches for selected teams

Round	Team				
	Nafta	Melnox	Stal	Skra	Calisia
4	+	+#	–#	+	–
5	+	+#	+#	X	–
6	+	+#	+#	–#	–
7	+	+	X	–#	–
8	X	+	–#	+	–
9	+	–#	–	+	X
10	+	X	+#	+	–
11	+	+	–	–#	–
12	–#	+	+#	–#	–
13	+	+	+#	+	–
14	+	+	+	X	+#
15	+	+	+	–#	–
16	+	+	X	–#	–
17	X	–#	+	–#	–
18	–#	–#	+	+	X

+ – win (defeat) consistent with the prognosis.
+# –# – win (defeat) inconsistent with the prognosis.

Table 3

Prognoses of match results for selected teams

Prognosis and result	Team									
	Nafta		Melnox		Stal		Skra		Calisia	
Prognosis after the 3rd round	3:0 3:1	0.73 0.22	0:3 1:3	0.63 0.27	3:1 3:2	0.19 0.19	3:0 3:1	0.55 0.30	1:3 0:3	0.28 0.26
Result of the 4th round	3:0		3:2	0.10	0:3	0.12	3:0		2:3	0.21
Prognosis after the 4th round	3:0 3:1	0.79 0.18	0:3 1:3	0.30 0.30	1:3 0:3	0.26 0.22	3:0 3:1	0.63 0.27	1:3 0:3	0.27 0.24
Result of the 5th round	3:1		3:1	0.14	3:2	0.17	X		1:3	
Prognosis after the 5th round	3:0 3:1	0.69 0.24	1:3 2:3	0.23 0.20	1:3 2:3	0.22 0.20	3:0 3:1	0.63 0.27	1:3 0:3	0.29 0.27
Result of the 6th round	3:1		3:1	0.14	3:1	0.15	1:3	0.04	0:3	
Prognosis after the 6th round	3:0 3:1	0.63 0.27	3:1 3:2	0.19 0.19	3:1 3:2	0.19 0.19	3:0 3:1	0.38 0.32	0:3 1:3	0.34 0.31
Result of the 7th round	3:0		3:1		X		0:3	0.05	2:3	0.19
Prognosis after the 7th round	3:0 3:1	0.67 0.25	3:1 3:2	0.22 0.20	3:1 3:2	0.19 0.19	3:1 3:0	0.27 0.24	0:3 1:3	0.31 0.30
Result of the 8th round	X		3:1		1:3	0.19	3:1		1:3	
Prognosis after the 8th round	3:0 3:1	0.67 0.25	3:1 3:2	0.24 0.21	1:3 2:3	0.21 0.20	3:1 3:0	0.28 0.26	0:3 1:3	0.32 0.30
Result of the 9th round	3:1		2:3	0.17	1:3		3:2	0.20	X	
Prognosis after the 9th round	3:0 3:1	0.63 0.27	3:1 3:2	0.22 0.20	1:3 2:3	0.23 0.20	3:1 3:0	0.28 0.25	0:3 1:3	0.33 0.30
Result of the 10th round	3:0		X		3:1	0.14	3:0		1:3	

Prognosis after the 10th round	3:0 3:1	0.66 0.25	3:1 3:2	0.22 0.20	1:3 2:3	0.21 0.20	3:1 3:0	0.30 0.30	0:3 1:3	0.33 0.31
Result of the 11th round	3:1		3:1		1:3		2:3	0.09	1:3	
Prognosis after the 11th round	3:0 3:1	0.63 0.27	3:1 3:2	0.24 0.21	1:3 2:3	0.22 0.20	3:1 3:0	0.28 0.25	0:3 1:3	0.34 0.30
Result of the 12th round	2:3	0.01	3:2		3:2		0:3	0.05	0:3	
Prognosis after the 12th round	3:0 3:1	0.51 0.31	3:1 3:2	0.24 0.21	1:3 2:3	0.21 0.20	3:1 3:2	0.25 0.21	0:3 1:3	0.37 0.31
Result of the 13th round	3:0		3:0	0.19	3:0		3:1		0:3	
Prognosis after the 13th round	3:0 3:1	0.54 0.30	3:1 3:0	0.26 0.22	3:1 3:2	0.19 0.19	3:1 3:0	0.26 0.22	0:3 1:3	0.40 0.32
Result of the 14th round	3:1		3:0		3:0	0.12	X		3:0	0.02
Prognosis after the 14th round	3:0 3:1	0.53 0.30	3:1 3:0	0.27 0.24	3:1 3:2	0.20 0.19	3:1 3:0	0.26 0.22	0:3 1:3	0.33 0.31
Result of the 15th round	3:0		3:1		3:0	0.11	0:3	0.06	1:3	
Prognosis after the 15th round	3:0 3:1	0.55 0.30	3:1 3:0	0.28 0.26	3:1 3:0	0.22 0.20	3:1 3:2	0.23 0.20	0:3 1:3	0.34 0.31
Result of the 16th round	3:0		3:2	0.20	X		0:3	0.08	0:3	
Prognosis after the 16th round	3:0 3:1	0.57 0.29	3:1 3:0	0.28 0.25	3:1 3:2	0.22 0.20	3:1 3:2	0.21 0.20	0:3 1:3	0.36 0.31
Result of the 17th round	X		0:3	0.05	3:2		0:3	0.10	0:3	
Prognosis after the 17th round	3:0 3:1	0.57 0.29	3:1 3:0	0.26 0.22	3:1 3:2	0.22 0.20	3:1 3:2	0.23 0.20	0:3 1:3	0.39 0.31
Result of the 18th round	1:3	0.01	2:3	0.13	3:1		3:2		X	

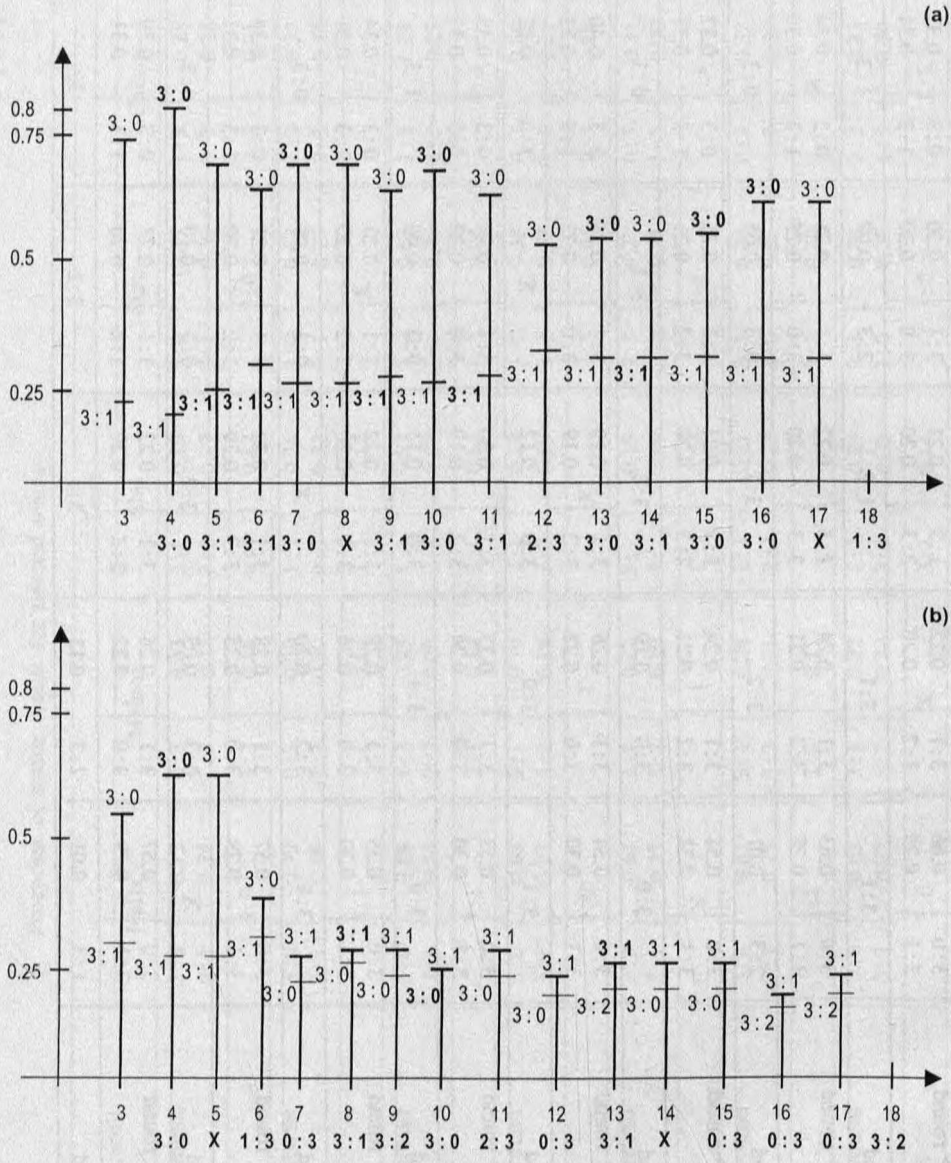

Fig. 3. The diagram of the dynamism of prognoses for teams: (a) Nafta Gaz Piła, (b) Skra Warszawa

In order to evaluate the formation of the prognoses of the final results of matches, researchers made appropriate prognoses for selected teams (Table 3).

Table 3 shows two highest prognoses of the states of matches for each team and after each round, which has already been graphically illustrated for two teams (Figure 3). In case a match result was not consistent with the prognoses, such a prognosis has been marked. The interpretation of these prognoses is similar to that of the prognoses depicted in Table 1.

Thanks to the empirical analysis, the assessment model of the probability of results in volleyball matches confirms its practical usefulness. In many cases, the theoretical prognoses are confirmed in practice. In order to be more precise in future, one should qualify the model where the constant probability of winning a set would not be taken into account, such a probability should be determined at successive stages of a game. This issue will affect further researches into the prognoses that have been discussed in this study.

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Małgorzata Aniol, Wiesław Wagner

PROGNOZOWANIE PROBABILISTYCZNE STANÓW MECZOWYCH W PIŁCE SIATKOWEJ

(Streszczenie)

W meczu piłki siatkowej rozgrywane są kolejne sety i wraz z tym ustalany jest stan meczu w zależności od rezultatywnie zakończonych setów. Oznacza to traktowanie meczu jako pewnej gry wieloetapowej, przy czym stany rezultatywne są ustalane na trzecim, czwartym lub piątym etapie gry. Każdy z etapów tworzy stan przejścia występujący z określonym prawdopodobieństwem. Jest ono bezpośrednio zależne od przyjętego prawdopodobieństwa wygrania seta. Prowadzi to do pewnego modelu probabilistycznego wygrania seta na każdym etapie przebiegu

meczu. Wspomniany model pozwala prowadzić prognozę zarówno wygrania, jak i przegrania meczu, a także dla pośrednich stanów meczowych.

W pracy zajęto się wykorzystaniem modelu probabilistycznego w piłce siatkowej do celów prognozowania. Dla potrzeb opisu takiego modelu podano graf w postaci drzewa opisującego stany przejścia setów w meczu piłki siatkowej, odpowiednie wzory prawdopodobieństwa dla wyników rezultatywnych meczu oraz przedstawiono aspekty prognozowania wyników meczowych. Rozważane zagadnienia zilustrowano na wynikach I ligi żeńskiej z sezonu startowego 1999/2000.