A C T A U N I V E R S I T A T I S L O D Z I E N S I S
FOLIA OECONOMICA 235, 2010

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# PROPERTIES OF THE JARQUE-BERA TEST

**Abstract.** Developments in the theory of multivariate normality are mainly connected with solving practical problems in economy and social life.

The theory has become a useful tool for analyzing empirical data and statistical methods based on it offer mathematical inferences which are easy to interpret. The choice of the Jarque--Bera test results from the frequency of its application for analyses of capital market. The test uses skewness and oblateness measures based on Mahalanobis transformation. The paper examines size and power of the Jarque-Bera test and proposes empirical quantiles for the test.

**Key words:** power of test, empirical quantiles, size of a test.

## I. INTRODUCTORY REMARKS

In the literature of the subject numerous tests of multivariate normality and rules of constructing their test statistics have already been proposed. Here a number of questions arise; which tests are best when it comes to power, error of the first kind; which have the properties of the omnibus test; which are directed tests, and finally, which of them should be used in practice.

It is almost impossible to use the analytical method in order to examine which of the tests is best in the sense of power because introducing on alternative distribution does not solve the problem. Rejecting the $H_0$ hypothesis does not lead automatically to accepting $H_1$. We do not move in the set of points, as it happens in case of parametric tests, but in the set of functions.

It means that there is no possibility to formulate a non-trivial hypothesis which would be alternative to $H_0$ i.e. the $H_0$ hypothesis different from the non – $H_0$ hypothesis. In such a case we are only interested in the probability of the error of the first kind, and therefore we cannot speak about the most powerful tests.

The solution of the problem was made possible thanks to the fast development in computer technology. Empirical investigation of power of tests can be conducted by organizing experiments with the use of Monte Carlo method.

Simulation methods have been applied for over fifty years now. An English scholar – Karl Pearson was the first to discover that random numbers are extremely useful for solving some problems in the theory of mathematical statistics (cf. Rao 1994). However, the term the Monte Carlo method was

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introduced by Metropolis and Ulam (1949), and one of its first users was the co-author of the game theory and cybernetics – John von Neumann. In the field of statistics simulation enables to solve such problems as: searching for probability distributions, investigating properties of estimators and different statistical methods, as well as defining the power of tests.

We can distinguish several comprehensive investigations of power of tests of multivariate normality, yet none of them is fully versatile. Such a situation occurs because it would be pointless to investigate every existing method, and also impossible to test every likely deviation from normality. Majority of more comprehensive investigations limit the scope of their analyses, either to particular tests, or to the most popular or promising tests.

As the Jarque-Bera test is quite frequently applied in order to test normality of distributions, we decided to show its properties in the present paper.

## II. THE JARQUE – BERA TEST

Jarque and Bera (1987) gave skewness and kurtosis measures based on the Mahalanobis transformation.

The measure of multivariate skewness:

$$sk_i = \frac{\frac{1}{N}\sum_{n=1}^{N}\widetilde{W}_{in}^3}{\left(\frac{1}{N}\sum_{n=1}^{N}\widetilde{W}_{in}^2\right)^{3/2}} \tag{1}$$

The measure of multivariate kurtosis:

$$ku_i = \frac{\frac{1}{N}\sum_{n=1}^{N}\widetilde{W}_{in}^4}{\left(\frac{1}{N}\sum_{n=1}^{N}\widetilde{W}_{in}^2\right)^{2}} \tag{2}$$

where $\widetilde{W}_{in}$ denotes elements of matrix $\widetilde{W}$, defined as follows:

$$\widetilde{W} = \hat{U}S_{\hat{U}}^{-1} \tag{3}$$

where $\hat{U}$ is the matrix of reminders obtained through the estimation of variables with the use of the least square method, while $S_{\hat{U}}$ is a triangular upper matrix such that $\hat{U}'\hat{U} = S_{\hat{U}}^{'}S_{\hat{U}}$ and $(\hat{U}'\hat{U})^{-1} = S_{\hat{U}}^{-1}(S_{\hat{U}}^{-1})'$.

Thus $\tilde{W}$ can be interpreted as a standardized $\hat{U}$ matrix. In other words $sk_i$ and $ku_i$ are individual skewness and kurtosis measures based on standardized matrix of reminders.

Let us denote further:

$$SK = \left(sk_1,......,sk_p\right)' \left(sk_1,......,sk_p\right) \quad (4)$$

$$KU = \left(ku_1 - 3,......,ku_p - 3\right)'\left(ku_1 - 3,......,ku_p - 3\right) \quad (5)$$

The Jarque-Bera test was based on comparing how far the asymmetry and kurtosis measures diverge from values characteristic of the normal distribution.

$H_0$: random elements <u>are subject</u> to the normal distribution

$H_1$: random elements <u>are not subject</u> to the normal distribution

The Jarque – Bera test is based on the following statistic:

$$JB = \frac{N}{6} SK + \frac{N}{24} KU \quad (6\,)$$

The statistic has a $\chi^2$ distribution of *2p* degrees of freedom (Fig.1).

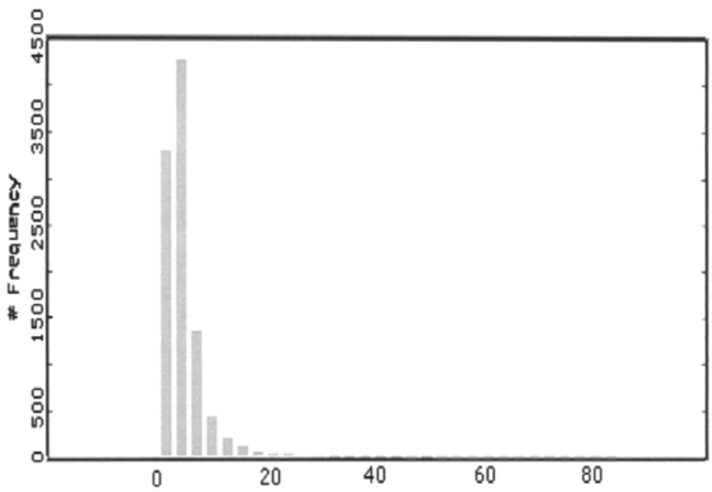


Fig. 1. Histogram of the distribution of the Jarque-Bera statistic $p = 3$ $n = 50$

If for the accepted significance level $x$ $JB \leq \chi^2_{2p,x}$ then there are no grounds to reject the $H_0$ hypothesis saying that random elements are subject to the normal distribution.

But if $JB > \chi^2_{2p,x}$ then we reject the $H_0$ hypothesis and we accept the $H_1$ hypothesis saying that random elements are subject to distribution which is different from the normal one.

## III. INVESTIGATION OF THE SIZE AND POWER OF THE JARQUE-BERA TEST

Due to the fact that Jarque – Bera test is one of the tests most frequently used by econometricians it was decided to test its power on the basis of Monte Carlo simulation experiments. Our experiment accounted for 10 000 repetitions of the multivariate normal distribution. The distribution was generated according to the method proposed by Wieczorkowski and Zieliński (1997) for n=20, 30, 40, 50, 60, 70, 80, 90, 100, 110, and 120; $p$=2,3,4,5; $\alpha$=0,1; 0,05;0,01 and 0,001. Then the size was checked again.

The obtained results are presented in Table 1 and in the diagrams. Under the assumption that the hypothesis is true, the numbers given in bold denote, that the multivariate distribution is normal and for a particular n, p and alpha it exceeds the given significance level. This means that in such cases the test does not "hold" the size. It can be clearly seen that for $\alpha \leq 0{,}05$ for a large number of cases the statistical inference based on this test is not congruent with the accepted significance level. For $\alpha = 0{,}01$ and $\alpha = 0{,}001$ these differences are even bigger, irrespective of the size of $p$ and $n$.

The experiment results give grounds to work out empirical critical values for the Jarque-Bera test which are presented in Table 2 for $p$ = 2, 3, 4, 5 and $n$ = 20, 30,…,120 and $\alpha$ = 0.1; 0.5; 0.01. We also investigated the size of the Jarque-Bera test taking into account the values presented in Table 2 (cf. Table 3).

We can notice that the number of values given in bold is significantly lower; moreover, these values only slightly diverge from the accepted significance levels. It can be stated that the differences are within the limits of the statistical error (cf. Fig .1.1, 1.2, 1.3, 1.4 – 4.1, 4.2, 4.3, 4.4). Therefore, it is recommended to apply these critical values for the Jarque-Bera test.

Table 3 The size of the Jarque-Bera test for the multivariate normal distribution for $p$ = 2, 3, 4, 5, $\alpha$ = 0,1; 0,05; 0,01 and $n$ = 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120. on the basis of empirical critical values.

The investigation of the power of the Jarque-Bera test was carried out for alternative Weibull distributions of the following parameters: $\lambda = 1{,}5$ and $k$ = 2 and Gamma $\theta = 2$ and $k$ = 2. The results of the experiments are presented in Table 4 for the Weibull distribution and in Table 5 for the Gamma distribution.

Table 1. The size of the Jarque-Bera test for the multivariate normal distribution for $p = 2, 3, 4, 5$, $\alpha = 0{,}1; 0{,}05; 0{,}01$ and $n = 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120$

| Dimension | Sample size ($n$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| $p = 2$ | 0.0416 | 0.055 | 0.0604 | 0.0659 | 0.0693 | 0.0733 | 0.0799 | 0.0764 | 0.0786 | 0.0786 | 0.0765 |
| $p = 3$ | 0.0359 | 0.0595 | 0.064 | 0.0668 | 0.0769 | 0.0788 | 0.0806 | 0.0857 | 0.0814 | 0.0868 | 0.0833 |
| $p = 4$ | 0.0396 | 0.0598 | 0.0655 | 0.0783 | 0.786 | 0.0843 | 0.0847 | 0.0881 | 0.0895 | 0.0867 | 0.0867 |
| $p = 5$ | 0.0383 | 0.575 | 0.0714 | 0.0747 | 0.082 | 0.0888 | 0.0905 | 0.0905 | 0.0911 | 0.0908 | 0.0926 |
| $\alpha = 0.05$ | | | | | | | | | | | |
| $p = 2$ | 0.028 | 0.0397 | 0.0419 | 0.045 | 0.0473 | **0.0505** | **0.055** | **0.0515** | 0.049 | 0.049 | 0.0487 |
| $p = 3$ | 0.0245 | 0.0399 | 0.0464 | 0.0464 | **0.0531** | **0.0534** | **0.0552** | **0.0578** | **0.0546** | **0.0575** | **0.054** |
| $p = 4$ | 0.0265 | 0.042 | 0.0474 | **0.0552** | **0.0562** | **0.059** | **0.0594** | **0.0593** | **0.061** | **0.0574** | **0.0592** |
| $p = 5$ | 0.026 | 0.0389 | 0.0484 | **0.052** | **0.0562** | **0.0616** | **0.063** | **0.0603** | **0.06** | **0.0616** | 0.0304 |
| $\alpha = 0.01$ | | | | | | | | | | | |
| $p = 2$ | **0.0135** | **0.0184** | **0.0216** | **0.0231** | **0.0211** | **0.0254** | **0.0266** | **0.0249** | **0.0246** | **0.0228** | **0.0237** |
| $p = 3$ | **0.0123** | **0.0207** | **0.0223** | **0.0246** | **00274** | **0.0288** | **0.028** | **0.0281** | **0.0292** | **0.0271** | **0.0262** |
| $p = 4$ | **0.0131** | **0.0214** | **0.0234** | **0.0296** | **0.0298** | **0.0284** | **0.0297** | **0.0281** | **0.0292** | **0.0271** | **0.0262** |
| $p = 5$ | **0.0122** | **0.0217** | **0.0268** | **0.0253** | **0.0307** | **0.0315** | **0.0319** | **0.0286** | **0.031** | **0.0272** | **0.0304** |
| $\alpha = 0.001$ | | | | | | | | | | | |
| $p = 2$ | **0.0054** | **0.0092** | **0.0109** | **0.0103** | **0.0099** | **0.012** | **0.0127** | **0.0118** | **0.01** | **0.0098** | **0.0097** |
| $p = 3$ | **0.0061** | **0.0089** | **0.0105** | **0.0131** | **0.0142** | **0.0127** | **0.0147** | **0.0124** | **0.0115** | **0.0121** | **0.0109** |
| $p = 4$ | **0.0062** | **0.0107** | **0.0116** | **0.0152** | **0.0136** | **0.0142** | **0.0135** | **0.0133** | **0.0148** | **0.0135** | **0.0125** |
| $p = 5$ | **0.0054** | **0.0113** | **0.0131** | **0.0122** | **0.0155** | **0.0141** | **0.014** | **0.0136** | **0.0142** | **0.0112** | **0.0127** |

Source: author's own calculations.

Table 2 Empirical critical values for the Jarque-Bera test of the multivariate normality

| | $p = 2$ | | | | $p = 3$ | | | |
|---|---|---|---|---|---|---|---|---|
| n \ α | 0.1 | 0.05 | 0.01 | 0.001 | 0.1 | 0.05 | 0.001 | 0.0001 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 20 | 4.7149 | 7.1136 | 14.9719 | 29.2787 | 6.795 | 9.3796 | 18.2066 | 37.1292 |
| 30 | 5.5409 | 8.1886 | 17.8437 | 38.5944 | 8.2325 | 11.4223 | 21.5195 | 46.072 |
| 40 | 6.0478 | 8.5392 | 19.0532 | 44.7221 | 8.5683 | 12.1575 | 23.2141 | 57.6788 |
| 50 | 6.2463 | 8.9559 | 18.7566 | 49.686 | 8.7004 | 12.171 | 24.3825 | 51.7526 |
| 60 | 6.3097 | 9.1922 | 18.2549 | 46.7586 | 9.454 | 12.9566 | 25.6538 | 60.0294 |
| 70 | 6.6219 | 9.5249 | 19.4723 | 46.3106 | 9.5232 | 13.0369 | 24.8628 | 47.6787 |
| 80 | 6.7595 | 9.9251 | 20.1868 | 47.3274 | 9.6454 | 13.0651 | 24.8934 | 54.943 |
| 90 | 6.779 | 9.672 | 20.0121 | 40.864 | 9.8776 | 13.4751 | 24.1215 | 44.1119 |
| 100 | 6.9162 | 9.4132 | 18.4509 | 43.2182 | 9.8419 | 13.1622 | 23.3812 | 49.4539 |
| 110 | 6.954 | 9.366 | 18.1392 | 44.1835 | 10.0245 | 13.3665 | 23.9224 | 54.1843 |
| 120 | 6.9025 | 9.3462 | 18.3104 | 41.0273 | 9.7303 | 13.1225 | 23.3423 | 45.706 |

Table 2 (cont.)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| $p = 4$ | | | | | $p = 5$ | | | |
| | 8.9629 | 12.2799 | 21.9012 | 41.298 | 11.3027 | 14.5242 | 24.9708 | 50.667 |
| 30 | 10.4656 | 14.3932 | 26.428 | 58.5881 | 12.7544 | 16.7787 | 30.9681 | 57.9245 |
| 40 | 11.1746 | 15.1718 | 27.8606 | 54.8902 | 13.9393 | 18.1532 | 32.2261 | 57.1135 |
| 50 | 11.9785 | 16.155 | 29.6787 | 52.5133 | 14.2774 | 18.5711 | 31.2221 | 65.2545 |
| 60 | 11.9537 | 16.1 | 28.969 | 59.3327 | 14.638 | 19.0588 | 34.1921 | 64.0857 |
| 70 | 12.49 | 16.4111 | 29.2745 | 68.4067 | 15.2974 | 19.5699 | 33.0439 | 69.8109 |
| 80 | 12.5137 | 16.7576 | 28.7735 | 65.4918 | 15.3275 | 19.7586 | 33.8174 | 62.2851 |
| 90 | 12.685 | 16.3703 | 28.988 | 56.2132 | 15.3018 | 19.3671 | 33.3226 | 67.1709 |
| 100 | 12.7914 | 16.7387 | 29.12 | 58.5273 | 15.4332 | 19.4606 | 32.6874 | 61.5109 |
| 110 | 12.6637 | 16.2865 | 29.2943 | 61.0495 | 15.4198 | 19.3993 | 30.4732 | 64.3796 |
| 120 | 12.6789 | 16.4345 | 27.7482 | 48.5365 | 15.6328 | 19.9717 | 31.5398 | 52.1149 |

Source: author's own calculations.

Table 3 The size of the Jarque-Bera test for the multivariate normal distribution for $p = 2, 3, 4, 5$, $\alpha = 0{,}1; 0{,}05; 0{,}01$ and $n = 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120$.

| Dimension | Sample size ($n$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| $p = 2$ | 0.0998 | **0.1018** | **0.1009** | **0.1014** | **0.1013** | **0.1004** | 0.0995 | **0.1006** | 0.0995 | 0.0992 | **0.1002** |
| $p = 3$ | 0.1011 | 0.0974 | 0.0984 | 0.103 | 0.0989 | 0.0976 | 0.0983 | 0.1014 | 0.0996 | 0.0996 | **0.1012** |
| $p = 4$ | 0.0996 | **0.1016** | **0.1** | **0.1007** | 0.1002 | 0.0985 | **0.1003** | 0.979 | 0.994 | **0.1016** | **0.1009** |
| $p = 5$ | 0.0978 | 0.0982 | 0.0994 | **0.1002** | 0.1005 | 0.0979 | 0.0998 | 0.0983 | 0.0992 | **0.1011** | **0.1003** |
| $\alpha = 0.05$ | | | | | | | | | | | |
| $p = 2$ | 0.0491 | 0.0497 | 0.0498 | **0.05** | **0.0512** | 0.0498 | 0.0488 | **0.0507** | 0.0488 | **0.0506** | **0.0506** |
| $p = 3$ | 0.0517 | 0.0492 | 0.0478 | **0.0522** | **0.0507** | 0.0489 | 0.0497 | **0.051** | 0.0488 | 0.0495 | **0.0516** |
| $p = 4$ | 0.0498 | **0.0501** | 0.0495 | 0.0499 | 0.0499 | 0.0491 | **0.051** | **0.0503** | 0.0492 | **0.051** | **0.0506** |
| $p = 5$ | 0.05 | **0.0582** | 0.0493 | 0.0495 | **0.0508** | 0.0478 | **0.0502** | 0.0485 | 0.0491 | **0.052** | **0.0507** |
| $\alpha = 0{,}01$ | | | | | | | | | | | |
| $p = 2$ | 0.0097 | **0.0101** | 0.0099 | **0.01** | **0.0105** | 0.0096 | 0.0097 | 0.01 | 0.0096 | 0.0096 | 0.0099 |
| $p = 3$ | 0.0105 | 0.0098 | 0.0097 | **0.0102** | **0.0105** | 0.0089 | 0.0093 | **0.011** | **0.0104** | 0.0095 | 0.0098 |
| $p = 4$ | 0.0097 | **0.0108** | **0.0101** | 0.0095 | **0.0104** | **0.0104** | 0.01 | 0.0099 | 0.0094 | 0.0093 | **0.0103** |
| $p = 5$ | 0.0099 | 0.0089 | 0.0097 | **0.0104** | **0.01** | 0.0098 | **0.0107** | **0.0103** | 0.0098 | **0.011** | **0.0103** |
| $\alpha = 0.001$ | | | | | | | | | | | |
| $p = 2$ | 0.0008 | **0.0012** | 0.0009 | 0.001 | 0.0009 | 0.001 | 0.001 | 0.0009 | **0.0011** | 0.0009 | 0.001 |
| $p = 3$ | 0.0009 | 0.001 | 0.001 | 0.001 | 0.001 | 0.0009 | 0.001 | **0.0011** | **0.0011** | 0.0009 | 0.001 |
| $p = 4$ | 0.001 | 0.0009 | **0.0012** | **0.0012** | **0.0011** | 0.0012 | 0.0009 | 0.001 | 0.0009 | 0.0009 | **0.0012** |
| $p = 5$ | 0.0009 | 0.0009 | **0.0011** | **0.0011** | **0.0011** | 0.001 | 0.001 | 0.001 | 0.0008 | 0.001 | **0.0011** |

Source: author's own calculations.

Table 4 Empirical power of tests under alternative empirical Weibull distribution $p = 2, 3, 4, 5$; $\alpha = 0{,}1$; 0,05; 0,01 and $n = 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120$

| Dimension | Sample size ($n$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| $p = 2$ | | | | | | | | | | | |
| 0.1 | 0.0592 | 0.0911 | 0.1224 | 0.146 | 0.177 | 0.2027 | 0.2291 | 0.2532 | 0.2826 | 0.3076 | 0.336 |
| 0.05 | 0.042 | 0.0656 | 0.881 | 0.1055 | 0.133 | 0.1483 | 0.1678 | 0.1898 | 0.2066 | 0.2257 | 0.2475 |
| 0.01 | 0.0207 | 0.356 | 0.0509 | 0.0606 | 0.0769 | 0.0854 | 0.0961 | 0.1061 | 0.1165 | 0.1249 | 0.139 |
| 0.001 | 0.0104 | 0.0182 | 0.0272 | 0.0335 | 0.0428 | 0.0429 | 0.0526 | 0.0577 | 0.06 | 0.0636 | 0.0712 |
| $p = 3$ | | | | | | | | | | | |
| 0.1 | 0.0546 | 0.084 | 0.1178 | 0.141 | 0.167 | 0.1923 | 0.2202 | 02399 | 0.2696 | 0.2898 | 0.3159 |
| 0.05 | 0.374 | 0.0577 | 0.0834 | 0.1035 | 0.1219 | 0.1449 | 0.1623 | 0.1802 | 0.1971 | 0.214 | 0.2328 |
| 0.01 | 0.019 | 0.0327 | 0.0454 | 0.0576 | 0.0682 | 0.0824 | 0.0904 | 0.1036 | 0.1116 | 0.1225 | 0.1238 |
| 0.001 | 0.0079 | 0.0163 | 0.0249 | 0.0299 | 0.0369 | 0.0431 | 0.0485 | 0.0537 | 0.0601 | 0.0642 | 0.071 |
| $p = 4$ | | | | | | | | | | | |
| 0.1 | 0.0539 | 0.0942 | 0.1249 | 0.1592 | 0.1916 | 0.22 | 0.2501 | 0.2811 | 0.3167 | 0.3497 | 0.3821 |
| 0.05 | 0.0362 | 0.0699 | 0.0934 | 0.1188 | 0.1403 | 0.1637 | 0.1873 | 0.2176 | 0.2442 | 0.2694 | 0.292 |
| 0.01 | 0.188 | 0.0398 | 0.0551 | 0.0703 | 0.0874 | 0.0988 | 0.1103 | 0.1278 | 0.1426 | 0.1544 | 0.1746 |
| 0.001 | 0.0092 | 0.0199 | 0.0295 | 0.0375 | 0.0475 | 0.0569 | 0.0612 | 0.0701 | 0.0782 | 0.0848 | 0.0927 |
| $p = 5$ | | | | | | | | | | | |
| 0.1 | 0.0568 | 0.1007 | 0.1476 | 0.186 | 0.227 | 0.2689 | 0.3112 | 0.3617 | 0.4048 | 0.4567 | 0.4982 |
| 0.05 | 0.0403 | 0.0744 | 0.1117 | 0.1414 | 0.1718 | 0.2098 | 0.2438 | 0.2768 | 0.3105 | 0.3605 | 0.3955 |
| 0.01 | 0.0209 | 0.0398 | 0.0681 | 0.0866 | 0.1067 | 0.1249 | 0.1474 | 0.1703 | 0.1906 | 0.22047 | 0.2437 |
| 0.001 | 0.0106 | 0.0228 | 0.0367 | 0.0513 | 0.0592 | 0.0692 | 0.081 | 0.0943 | 0.1073 | 0.1177 | 0.1362 |

Source: author's own calculations.

Table 5 Empirical power of tests under alternative empirical Gamma distribution $p = 2, 3, 4, 5$; $\alpha$=0,1; 0,05; 0,01 and $n$ =20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120

| Dimension | Sample size ($n$) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| $p = 2$ | | | | | | | | | | | |
| 0.1 | 0.1943 | 0.3374 | 0.4617 | 0.5634 | 0.6584 | 0.736 | 0.8089 | 0.8574 | 0.8936 | 0.93 | 0.9479 |
| 0.05 | 0.1517 | 0.276 | 0.3912 | 0.4888 | 0.5816 | 0.6616 | 0.7374 | 0.797 | 0.8439 | 0.8871 | 0.9133 |
| 0.01 | 0.0939 | 0.1896 | 0.2856 | 0.3692 | 0.4518 | 0.5347 | 0.6089 | 0.6758 | 0.7283 | 0.7785 | 0.8169 |
| 0.001 | 0.583 | 0.1241 | 0.2008 | 0.2669 | 0.339 | 0.4073 | 0.4721 | 0.5388 | 0.5898 | 0.6492 | 0.6883 |
| $p = 3$ | | | | | | | | | | | |
| 0.1 | 0.1775 | 0.318 | 0.4515 | 0.5654 | 0.6772 | 0.7466 | 0.8127 | 0.8546 | 0.8955 | 0.9276 | 0.951 |
| 0.05 | 0.1384 | 0.26 | 0.3808 | 0.4899 | 0.6057 | 0.6773 | 0.7555 | 0.8018 | 0.8536 | 0.8939 | 0.9194 |
| 0.01 | 0.0855 | 0.1806 | 0.2737 | 0.3742 | 0.4796 | 0.5529 | 0.6352 | 0.6912 | 0.7557 | 0.8053 | 0.8391 |
| 0.001 | 0.0478 | 0.1174 | 0.189 | 0.2643 | 0.3583 | 0.4166 | 0.5041 | 0.5607 | 0.6278 | 0.6763 | 0.7345 |
| $p = 4$ | | | | | | | | | | | |
| 0.1 | 0.1937 | 0.3773 | 0.5331 | 0.6641 | 0.7749 | 0.8369 | 0.8968 | 0.9349 | 0.9551 | 0.974 | 0.9843 |
| 0.05 | 0.1513 | 0.3162 | 0.4637 | 0.5926 | 0.7095 | 0.7835 | 0.8518 | 0.9012 | 0.9314 | 0.958 | 0.9706 |
| 0.01 | 0.0934 | 0.2229 | 0.3432 | 0.4698 | 0.5887 | 0.6695 | 0.7646 | 0.8182 | 0.867 | 0.9084 | 0.9318 |
| 0.001 | 0.0555 | 0.1479 | 0.2463 | 0.3501 | 0.4661 | 0.5397 | 0.6398 | 0.7086 | 0.7742 | 0.8258 | 0.862 |
| $p = 5$ | | | | | | | | | | | |
| 0.1 | 0.2242 | 0.4446 | 0.6389 | 0.7665 | 0.8622 | 0.9273 | 0.9625 | 0.979 | 0.9906 | 0.9953 | 0.9979 |
| 0.05 | 0.1798 | 0.3766 | 0.5667 | 0.7046 | 0.8148 | 0.8934 | 0.9398 | 0.9648 | 0.9831 | 0.9905 | 0.9959 |
| 0.01 | 0.1138 | 0.2743 | 0.4478 | 0.5833 | 0.7105 | 0.8129 | 0.8812 | 0.9224 | 0.9554 | 0.9735 | 0.9836 |
| 0.001 | 0.0678 | 0.1886 | 0.3351 | 0.4598 | 0.5924 | 0.6991 | 0.7859 | 0.8501 | 0.9014 | 0.9318 | 0.9562 |

Source: author's own calculations.

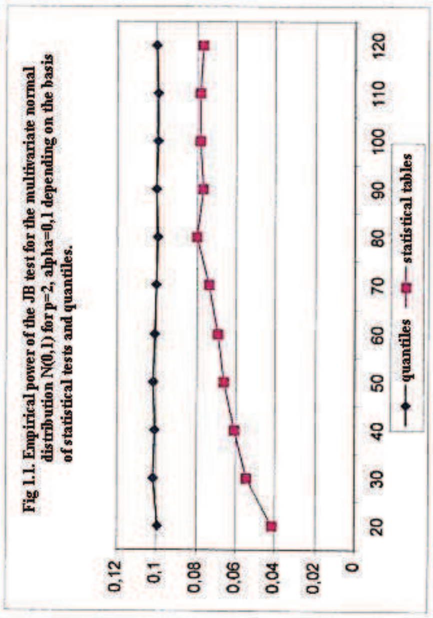


**Fig 1.1. Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=2, alpha=0,1 depending on the basis of statistical tests and quantiles.**

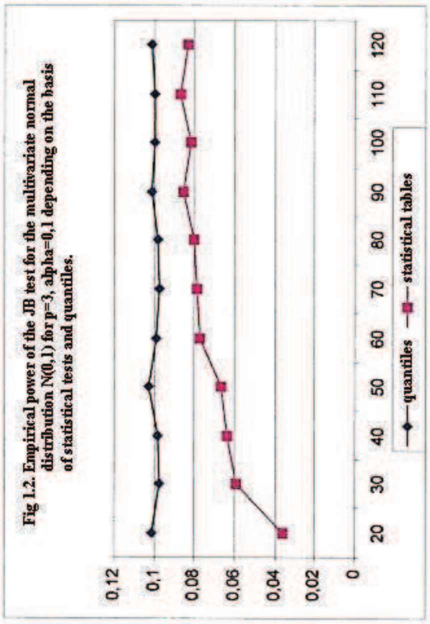


**Fig 1.2. Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=3, alpha=0,1 depending on the basis of statistical tests and quantiles.**

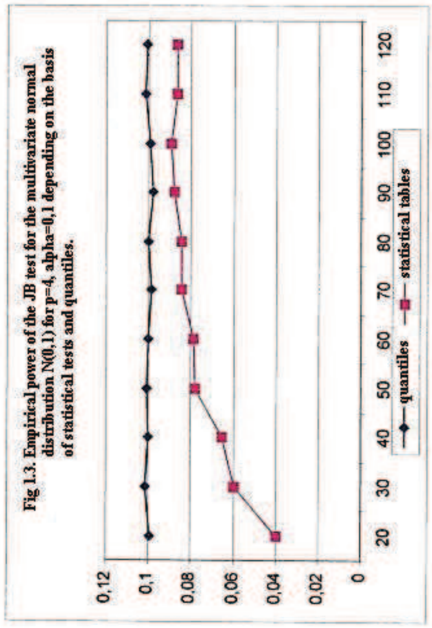


**Fig 1.3. Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=4, alpha=0,1 depending on the basis of statistical tests and quantiles.**

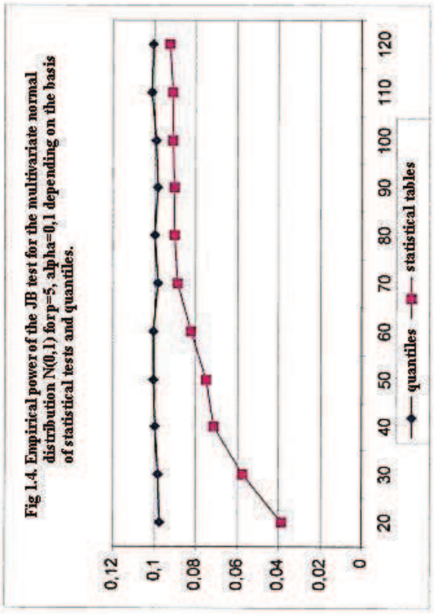


**Fig 1.4. Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=5, alpha=0,1 depending on the basis of statistical tests and quantiles.**


**Fig 2.1. Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=2, alpha=0,05 depending on the basis of statistical tests and quantiles.**

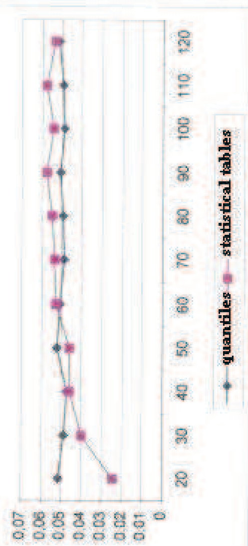


**Fig 2.2. Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=3, alpha=0,05 depending on the basis of statistical tests and quantiles.**


**Fig 2.3. Empirical power of the JB test for the multivariate nor,mal distribution N(0,1) for p=4, alpha=0,05 depending on the basis of statistical tests and quantiles.**

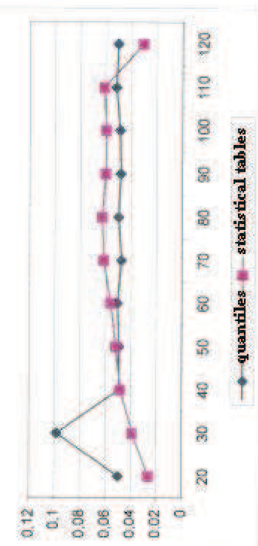


**Fig 2.4. Empirical power of the JB test for the multivariate nor,mal distribution N(0,1) for p=5, alpha=0,05 depending on the basis of statistical tests and quantiles.**

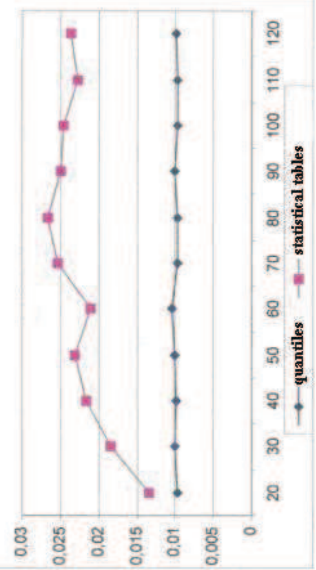


Fig 3.1. Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=2, alpha=0,01 depending on the basis of statistical tests and quantiles.

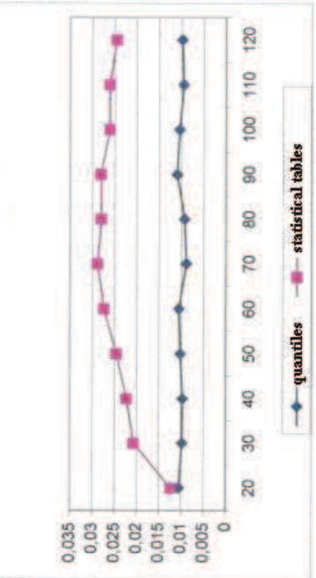


Fig 3.2. Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=3, alpha=0,01 depending on the basis of statistical tests and quantiles.

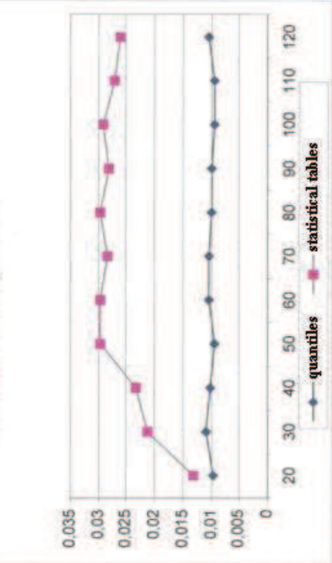


Fig 3.3. Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=4, alpha=0,01 depending on the basis of statistical tests and quantiles.

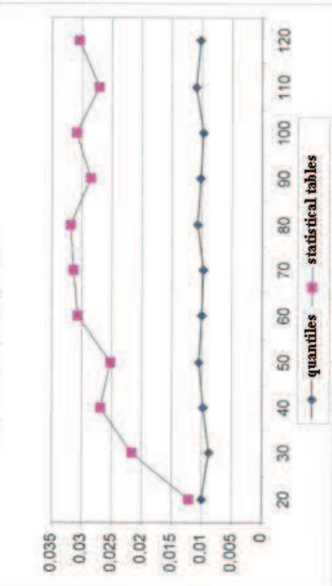


Fig 3.4. Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=5, alpha=0,01 depending on the basis of statistical tests and quantiles.

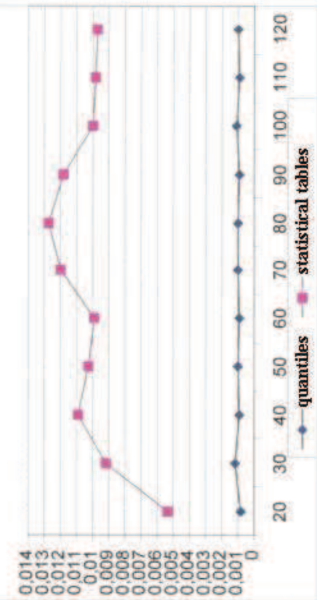


**Fig 4.1.** Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=2, alpha=0,001 depending on the basis of statistical tests and quantiles.

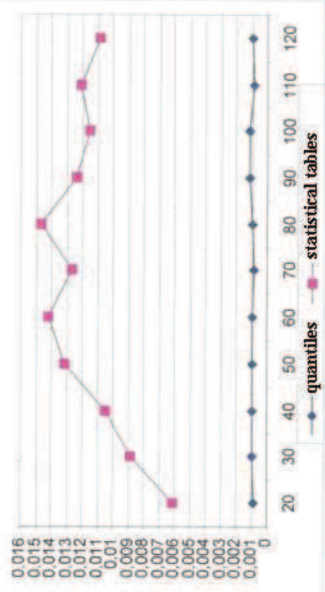


**Fig 4.2.** Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=3, alpha=0,001 depending on the basis of statistical tests and quantiles.


**Fig 4.3.** Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=4, alpha=0,001 depending on the basis of statistical tests and quantiles.

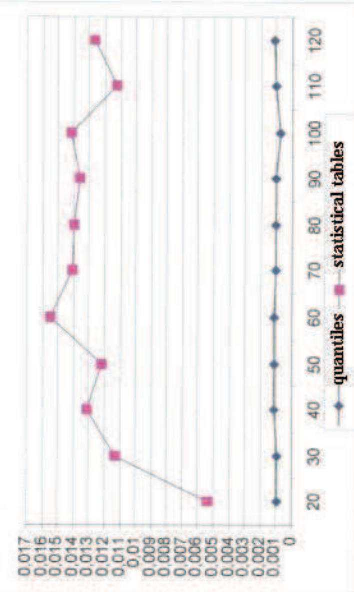


**Fig 4.4.** Empirical power of the JB test for the multivariate normal distribution N(0,1) for p=5, alpha=0,001 depending on the basis of statistical tests and quantiles.

In the light of the obtained results it should be stated that the Jarque-Bera test shows little power for the Weibull distribution, even for the relatively large samples, irrespective of the population size e.g. for $n = 5$, $\alpha = 0{,}05$, $n = 120$ the test power is equal to 0,3955.

For the Gamma distribution, which diverges significantly from the normal distribution, the Jarque-Bera test is more sensitive but only for $n \geq 100$.

For instance, for $p = 5$, $\alpha = 0{,}05$, $n = 100$ the test power amounts to 0,9831, and for $p = 2$ it amounts to 0,8439, respectively.

The power of the Jarque-Bera test grows together with the growing size of population for distributions distant from the normal distribution.

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## WŁASNOŚCI TESTU JARQUE-BERA

Zasadnicze kierunki rozwoju teorii wielowymiarowej normalności związane są z rozwiązywaniem praktycznych problemów z życia gospodarczego i społecznego.

Teoria ta stała się wygodnym instrumentem analizy danych empirycznych, a metody statystyczne oparte na tej teorii mają proste do interpretacji wnioski matematyczne. Wybór testu Jarque - Bera wynika z dużej częstotliwości jego zastosowania zwłaszcza w analizach rynku kapitałowego. Test ten wykorzystuje miary skośności i spłaszczenia, które oparte są na transformacji Mahalanobisa. W artykule badany jest rozmiar i moc testu Jarque – Bera oraz proponuje się empiryczne kwantyle dla tego testu.