Folia Oeconomica
Acta Universitatis Lodziensis
ISSN 0208-6018 e-ISSN 2353-7663
www.czasopisma.uni.lodz.pl/foe/
5(338) 2018

DOI: http://dx.doi.org/10.18778/0208-6018.338.03

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New Forecasting Technique for Intermittent Demand, Based on Stochastic Simulation. An Alternative to Croston's Method

Abstract: The main aim of the article is to present a new forecasting technique, applicable in case of intermittent demand. To present properties of this new technique, the accuracy of the predictions generated by the Croston's method and by the author's method (based on stochastic simulation) was analyzed. For comparison, methods such as moving average and simple exponential smoothing are as well used as a reference. Also the SBA method, a modification of Croston's method, is applied. Croston's method is an extension of adaptive methods. It separates the interval between the (non-zero) sales and the sales level. Its purpose is to better forecast intermittent (sporadic) demand. The second prognostic method is the author's proposal which relies on two stages. In the first stage, based on stochastic simulation, it determines if an event (sale) occurs in a given period. In the second stage, the sales level is estimated (if the previous stage shows that the sales will occur). Due to the strong asymmetry of the sales, the sales level is determined on the basis of the corresponding quantiles. The basis for forecasting are weekly sales series of about fourteen thousand products (real data). The analyzed time series can be defined as atypical, which is manifested by a small number of non-zero observations (high number of zeros), high volatility and randomness (randomness tests indicate white noise). Forecast error measures are used to characterize both the bias and the efficiency. The forecast error measures will be characterized so that they can be applied to a time series with a large number of zeros (including the author's forecast error measure proposal). Forecasts were evaluated with respect to the distributions of four ex post errors, such as mean error (ME), mean absolute deviation (MAD), mean absolute scaled error (MASE) and the author's proposal (error D). The proposed technique, based on stochastic simulation, seems to be the least biased and most efficient. The Croston's method gives positively biased predictions with rather low efficiency. The proposed forecasting technique might support decisions in enterprises facing the problem of forecasting intermittent demand. The more accurate forecasts could increase the quality of customer service and optimize the inventory level.

Keywords: intermittent demand forecasting, Croston's method, stochastic simulation, forecast error measures of intermittent demand

JEL: C53, E27, L81

1. Introduction

Many enterprises have to face the problem of intermittent demand forecasting. It is important, for instance, in the case of warehouse and distribution centers (Shukur, Doszyń, Dmytrów, 2017). In this type of business the number of products is usually very high. Some are sold with high frequency but most of them could be classified as slow moving items. The sales of such products are usually rare, which results in many periods with zero sales. Sales time series are not easy to analyze and predict because it is hard to find any stable regularities. Intervals between periods with non-zero sales as well as sales levels are usually random. In addition, the volatility of variables is very high. All of these features indicate a need for specific forecasting methods applicable in the case of intermittent demand.

The main aim of the article is to propose a new forecasting technique based on stochastic simulation. To present the advantages of proposed procedure, comparison of chosen forecasting methods and verification of their accuracy will be conducted. Methods like Croston's (and its modification – SBA), moving average and simple exponential smoothing are compared. To compare forecasts' accuracy, the distributions of ex post forecast error measures are analyzed, both with respect to biasedness and efficiency.

The presented analysis could be useful for the enterprises facing the problem of intermittent demand forecasting. It is important for the managers to know which forecasting method might be useful for given types of sales time series. It is important in the context of customer service quality and inventory control. The presented analysis could be also interesting from the theoretical perspective. The proposed, new method based on stochastic simulation (author's proposal) is simpler than Croston's method and has more desirable properties.

2. The problem

At first the following question should be answered: what is intermittent demand? There were many attempts in the literature to define this term but they are very general or operational. In most definitions, it is stated that the intermittent demand is infrequent (sporadic) with no reference to the distribution of sales' levels (Syntetos, 2001). The infrequency (sporadicity) means that the average time between transactions is longer than the unit time period. Sometimes it is stated that the average inter-demand interval should be longer than 1.25 periods, which is the same as the sales frequency lower than 0.8. So, if we have weekly data, the sales frequency, understood as a share of weeks with non-zero sale, should be lower than 0.8 to say that demand is intermittent. If it is greater than 0.8, we have a fast moving item.

As it was mentioned above, the intermittent demand definition does not imply any restrictions with regard to the level of demand (in period with non-zero sales)[1]. There are attempts in the literature to classify the intermittent demand with regard to sales' levels. If non-zero sales are unit sized, we are dealing with so-called low demand; if they are constant – we have clumped demand. Constant demand is of a non-zero, constant size, it does not contain zeros. If non-zero demand is highly variable, it is called lumpy. Highly variable demand is a demand with high coefficient of variation for non-zero values. In this article, as well as in the literature, terms "demand" and "sales" are used interchangeably, which is accurate, if there are enough products in the warehouse to always satisfy demand. A detailed discussion of issues connected with the definitions and classification of the intermittent demand can be found in (Syntetos, 2001).

3. The standard methods

Most methods used in the case of intermittent demand forecasting have their origins in Croston's ideas (Croston, 1972). Generally, Croston's method is based on exponential smoothing. It separates exponential smoothing of the sales size (z_t) from the time period between (non-zero) sales (p_t). The estimate of the demand per period is calculated as a ratio of these two components (the sales size and the interval between sales). If non-zero demand occurs, Croston's method updates the sales size and interval length. Otherwise (the sales is zero in a given period t) only the number of time periods since the last sale is increased by one.

The algorithm of Croston's method could be presented as follows[2]:

If $z_t = 0$, then

$$\hat{z}_t = \hat{z}_{t-1} \tag{1}$$

$$\hat{p}_t = \hat{p}_{t-1} \tag{2}$$

$$q_t = q_{t-1} + 1 \tag{3}$$

If $z_t > 0$, then

$$\hat{z}_t = \hat{z}_{t-1} + \alpha\left(z_t - \hat{z}_{t-1}\right) \tag{4}$$

$$\hat{p}_t = \hat{p}_{t-1} + \alpha\left(p_t - \hat{p}_{t-1}\right) \tag{5}$$

[1] If stock level if higher (or equal) to demand size, demand could be identified with sales' level.

[2] The algorithm was prepared on the basis of (Xu, Wang, Shi, 2012).

$$q_t = 1, \tag{6}$$

where
z_t – (non-zero) demand at time t,
$\hat{z}_t$ – forecast of the (non-zero) demand in period t,
$\hat{x}_t = \frac{\hat{z}_t}{\hat{p}_t}$ – (average) demand per period,
x_t – empirical sales in period t (both zero and non-zero demand),
p_t – number of time periods between two non-zero sales,
$\hat{p}_t$ – forecast of demand interval in period t,
q – number of periods since the last non-zero sales,
$\alpha \in (0, 1)$ – smoothing constant value.

In Croston's method, demand is assumed to follow Bernoulli's process, which is equivalent to assume that interdemand intervals are geometrically distributed with mean p. The next assumption is that (non-zero) demand sizes are normally distributed (with mean μ and variance σ^2) and that demand sizes and demand intervals are independent[3]. So, one of the Croston's method's assumptions is that a non-zero demand is normally distributed (in time) and intervals between non-zero sales are independent, according to interval lengths.

According to these assumptions, Croston thought that:

$$E\left(\hat{x}_t\right) = E\left(\frac{\hat{z}_t}{\hat{p}_t}\right) = \frac{E\left(\hat{z}_t\right)}{E\left(\hat{p}_t\right)} = \frac{\mu}{p}, \tag{7}$$

so the method is unbiased. Syntetos, Boylan (2005), Boylan, Syntetos (2007) have proven that Croston estimator is biased. If the demand size and the demand intervals are independent, it is true that

$$E\left(\frac{\hat{z}_t}{\hat{p}_t}\right) = E\left(\hat{z}_t\right) E\left(\frac{1}{\hat{p}_t}\right) \tag{8}$$

but

$$E\left(\frac{1}{\hat{p}_t}\right) \neq \frac{1}{E\left(\hat{p}_t\right)}. \tag{9}$$

[3] These assumptions have been challenged (Syntetos, 2001), however, it is not considered in this paper.

This makes Croston's method biased in case of intermittent demand. Therefore, Syntetos and Boylan (2005) proposed a new estimator of the average demand:

$$\hat{x}_t = \left(1 - \frac{\alpha}{2}\right)\frac{\hat{z}_t}{\hat{p}_t}, \tag{10}$$

where α is the smoothing constant value used for updating the interdemand intervals. This estimator (method) is called SBA (Syntetos-Boylan Approximation). In Croston's method usually one smoothing constant value is used but sometimes it is extended to two values, one for (non-zero) demand sizes and another – for demand intervals[4]. It is worth noticing that there is a problem with the SBA method. If we do not have periods with zero demand (the sale takes place in every period), the SBA method is biased by the factor: $\left(1 - \frac{\alpha}{2}\right)$. It is true for fast moving items with sales frequency (or average interdemand interval) equal to one.

4. Proposal of the new forecasting technique

Beyond Croston's and SBA methods, the author's forecasting procedure, based on stochastic simulation, will also be verified. This procedure consists of two stages (Doszyń, 2017).

At first, it is fixed if the sale of the *i*-th product in the week *t* appears. A random number w_{it} from the uniform distribution $U\langle 0, 1\rangle$ is generated. Next, it is checked whether the relative sales frequency of the *i*-th product (c_i) is higher than the generated random number: $c_i > w_{it}$. If it is true, it is assumed that the sale of the *i*-th product in week *t* will appear. At this stage it is assumed that the probability of sale is a realization of the binomial Bernoulli's process with the probability equal to the relative sales frequency. The relative sales frequency (c_i) is an estimator of the sales probability.

In the next stage, the sales level is generated. Forecasts are obtained as the median value of sale but only for weeks with a non-zero sale. A median calculated for all values (zero and non-zero) would be always equal to zero for items with the sales frequency lower than 0.5 (or the average interdemand interval greater than 2). If, instead of the median, average demand for non-zero values is calculated, then the forecast will be sensitive to outliers, which is inadvisable, due to stock management. The forecasts as a median of non-zero sales value are robust with respect to outliers.

[4] In this article one smoothing constant value is used. Smoothing constant value is the same for demand sizes and demand intervals.

To sum up, five forecasting methods have been verified. As a benchmarks, simple moving average (MA) and simple exponential smoothing (SES) is applied. Croston's (CR) and SBA methods are checked. Also, the author's forecasting procedure (MD) is applied.

In the simple moving average method (MA) forecasts are calculated as:

$$\hat{x}_{t+1} = \frac{1}{n}\sum_{t=1}^{n} x_t , \tag{11}$$

where n is a number of periods and x_t is a sale in period t.

In simple exponential smoothing (SES) forecasts are obtained as:

$$\hat{x}_{t+1} = F_t , \tag{12}$$

where

$$F_t = \alpha x_t + (1-\alpha)\hat{F}_{t-1} \tag{13}$$

and α is a smoothing constant value.

5. Forecast accuracy

In the case of forecast systems consisting of thousands of products, the accuracy of forecasts could be calculated by means of ex post forecast error measures distributions. In the article, the distributions of four ex post forecast error measures will be applied:

1) Mean Error (*ME*),
2) Mean Absolute Deviation (*MAD*),
3) Mean Absolute Scaled Error (*MASE*),
4) author's proposal (*D*).

ME is calculated as:

$$ME = \frac{\sum_{t=n+1}^{n+h}(x_{pt} - x_t)}{h}, \tag{14}$$

where

n – number of (in – sample) periods,

h – number of ex post forecasts (forecast horizon),

x_{pt} – ex post forecast in period t,

x_t – sale in period t.

The mean error (*ME*) shows biasedness of forecasts. The method is unbiased if the expected value of the forecast error (for given product), defined as a difference between forecasts and empirical values, is zero. Mean error's drawback is that it is scale dependent, because different products are sold in different quantities (the scale for products is different). Moreover, errors in different periods could cancel one another. Despite this, it seems reasonable to compare forecasting methods, dependant on the distributions of *ME*.

The next considered error is *MAD*:

$$MAD = \frac{\sum_{t=n+1}^{n+h} \left| x_{pt} - x_t \right|}{h}. \tag{15}$$

MAD is also scale dependent. It informs about forecast efficiency. In context of efficiency *MSE* (Mean Squared Error) is sometimes considered, however, *MAD* is more robust due to the outliers, which is important in case of lumpy demand[5]. This is why *MAD* is preferred in this case.

In terms of the intermittent demand, *MASE* (Mean Absolute Scaled Error) is strongly recommended (Hyndman, Koehler, 2006). The formula is as follows:

$$MASE = \frac{\frac{1}{h}\sum_{t=n+1}^{n+h} \left| x_{pt} - x_t \right|}{\frac{1}{n-1}\sum_{t=2}^{n} \left| x_t - x_{t-1} \right|}. \tag{16}$$

MASE is scale independent, relative (so forecast error measures for different products might be compared) and could be computed for almost all intermittent demand times series. The only exception is a situation when all x_t values are the same for $t = 1, 2, \ldots, n$. This makes the denominator equal to zero. If $MASE > 1$, then a given forecasting method is worse than the naïve method. Otherwise ($MASE < 1$), the analyzed forecasting method is better than the naïve method.

In context of intermittent demand it is not possible to compute typical relative ex post errors, such as *MPE*, *MAPE* or Theil coefficients because there are zeros in the denominator. *MASE* could be estimated in that kind of situations. It is also true in the case of another ex post forecast error measure (*D*), which is the author's proposal.

The proposed ex post forecast error measure is calculated for two cases (Doszyń, 2017):

$$D = \frac{1}{h}\sum_{t=n+1}^{n+h} \frac{x_{tp} - x_t}{max\left\{x_{tp},\, x_t\right\}}, \text{ if } x_{tp} \neq x_t, \tag{17}$$

[5] To remind, the lumpy demand is sporadic and highly variable.

$$D = 0 \text{, if } x_{tp} = x_t \,. \tag{18}$$

The error D has interesting informative properties (Table 1).

Table 1. Properties of error *D*

No.	Condition	*D*
1	$x_{tp} = x_t$	$D = 0$
2	$x_{tp} < x_t$	$D < 0$
3	$x_{tp} > x_t$	$D > 0$
4	$x_{tp} = 0, x_t > 0$	$D = -1$
5	$x_{tp} > 0, x_t = 0$	$D = 1$

Source: own elaboration

Generally, the error D is calculated as a relation of the difference between the forecast and the sale (nominator) and the higher one of these two values (denominator). Having a higher value in the denominator always makes the error (D) possible to calculate, even if a sale or a forecast are equal to zero. If the sale is equal to the forecast, the error is zero. If forecasts are lower than sales, the error is negative (forecasts are underestimated). If forecasts are higher than sales, the error is non-zero (forecasts are overestimated). We also have two extreme cases. If the forecasts are equal to zero and the sales are non-zero, then $D = -1$. If the sales are zero and the forecasts are non-zero, then $D = 1$. The error D is normalised: $D \in \langle -1.1 \rangle$.

6. The results

Five described forecasting methods were used:
1) MA – moving average,
2) SES – simple exponential smoothing,
3) MD – author's proposal based on stochastic simulation,
4) CR – Croston's method,
5) SBA – Syntetos-Boylan Approximation.

All of these methods were evaluated due to distributions of four ex post forecast error measures:
1) *D* – author's proposal (formula 17–18),
2) *ME* – mean error (14),
3) *MAD* – mean absolute deviation (15),
4) *MASE* – mean absolute scaled error (16).

The unit time period established for this research is one week. (Demand) interval is a number of weeks between non-zero sales. Sales of all products are measured in physical units. The forecasts were computed for real data containing weekly time series for 13 719 products. The number of observations was equal to 210 (210 weeks) or less, if a given product was introduced later. The average sales frequency (defined as a share of weeks with non-zero sale) was 0.18, so, approximately, one product was sold in each of the five weeks.

It was not possible to calculate forecasts for all products. In the case of Croston's (and SBA) method, there were 1931 such cases. It was mostly due to the fact that there was only one non-zero sale, so it was not possible to estimate even one interdemand interval. For MA (and MD), the number of omitted forecasts was 107, while in the case of SES – 131. It was due to a small number of observations. For all those cases it was assumed that forecasts were equal to zero, which is the practice applied in the analyzed company. So, in each analysis, the number of observations was as follows:

1) Croston's and SBA methods – 11 788,
2) MA, MD methods – 13 612,
3) SES – 13 588.

The smoothing constant value in SES, CR and SBA was equal to 0.1.

To compute ex post errors, forecasts were computed for the last five weeks. Therefore, the forecast horizon was equal to five ($h = 5$). Positional statistics computed for all forecast distributions are presented in Table 2. The positional statistics were used because they are robust due to normality assumption. Also, scale dependency of sales of different products is not problematic when positional statistics are used. Ex post forecast distributions for the three methods (CR, SBA, MD) are presented on Graphs 1–4. For clarity, the distributions of only the three forecasting methods are graphically presented. The MA and SES methods are treated in this research only as benchmarks. Besides, Croston's method (or its extensions) are commonly used for the forecasting of the intermittent demand.

Table 2. Summary statistics for ex post forecast error measures: $Q_{1.4}$ – first quartile, M – median, $Q_{3.4}$ – third quartile, Q – quartiles deviation, A_2 – (positional) asymmetry coefficient, K_p – positional kurtosis coefficient

	Statistics	$Q_{1.4}$	M	$Q_{3.4}$	Q	A_2	K_p
D	MA	0.000	0.607	1.000	0.500	–0.213	0.500
	SES	0.600	1.000	1.000	0.200	–1.000	0.219
	MD	–0.100	0.000	0.000	0.050	–1.000	0.114
	CR	0.381	0.816	1.000	0.310	–0.407	0.310
	SBA	0.360	0.807	1.000	0.320	–0.395	0.320

	Statistics	$Q_{1.4}$	M	$Q_{3.4}$	Q	A_2	K_p
ME	MA	–0.100	0.000	0.133	0.117	0.143	0.085
	SES	–0.134	0.007	0.143	0.139	–0.021	0.095
	MD	–0.200	0.000	0.000	0.100	–1.000	0.071
	CR	0.000	0.248	1.114	0.557	0.556	0.217
	SBA	0.000	0.218	1.033	0.517	0.577	0.213
MAD	MA	0.033	0.220	0.820	0.393	0.525	0.140
	SES	0.030	0.233	0.824	0.397	0.490	0.140
	MD	0.000	0.200	0.800	0.400	0.500	0.154
	CR	0.084	0.644	1.775	0.846	0.337	0.192
	SBA	0.080	0.619	1.700	0.810	0.334	0.189
MASE	MA	0.198	0.658	1.106	0.454	–0.012	0.225
	SES	0.177	0.650	1.288	0.556	0.149	0.217
	MD	0.000	0.237	1.010	0.505	0.530	0.253
	CR	0.628	1.479	2.656	1.014	0.161	0.232
	SBA	0.602	1.421	2.548	0.973	0.158	0.232

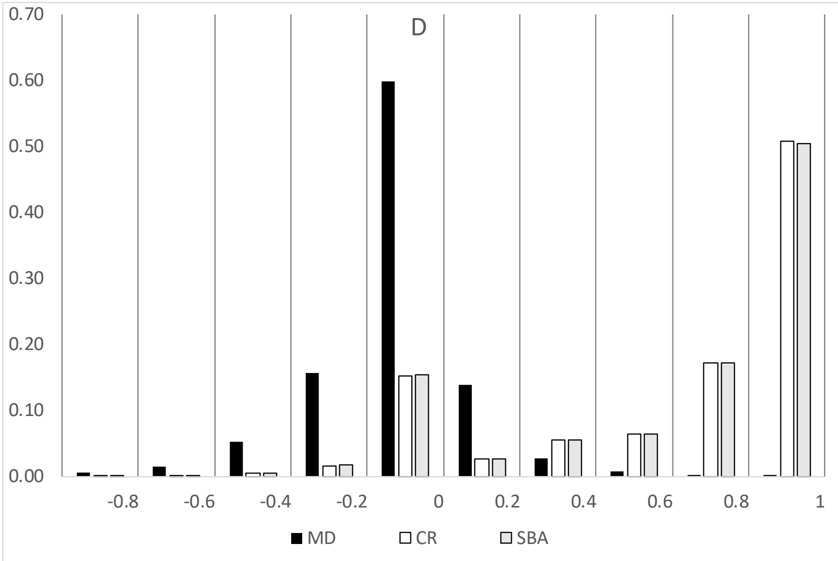
Source: own computations

The values of error D inform that MD method is the best in terms of forecast accuracy. The first quartile for this method $Q_{1.4}^{MD} = -0.100$ and it is the only negative value. The first quartiles for other methods are positive, with the highest value for SES $(Q_{1.4}^{SES} = 0.600)$. Also, for the CR and SBA methods the first quartile is positive, which leads to the conclusion that all methods (except for MD) are positively biased. It is an important conclusion for decision makers, especially in terms of inventory level.

The median is equal to zero only for the MD method. For the other methods it is very high, with $M^{SES} = 1$ for SES, which is a maximum possible value. The third quartile (of the error D distribution) is equal to zero for the MD method and one of the other ones, which confirms conclusions about high, positive biasedness of all methods with one exception – the MD method. All of these conclusions provide some important pieces of information for the managers. All forecasting methods (except for MD) could lead to high inventory levels.

The quartile variation (Q) is the smallest for the MD method, so in the narrowed area of variation the errors are close to median (which is equal to zero). This is a positive conclusion in terms of the forecasts efficiency. The error D distributions are negatively skewed (in the narrowed area of variation). The smaller the asymmetry (closer to zero), the better the situation in terms of biasedness. For MD and SES methods the distributions are leptokurtic, which supports the conclusions about higher efficiency.

The important fact for managers is that the method is unbiased. Positive bias could lead to high inventory levels. In this context the MD method is recommended.



Graph 1. Ex post forecast error measure distributions (*D*)

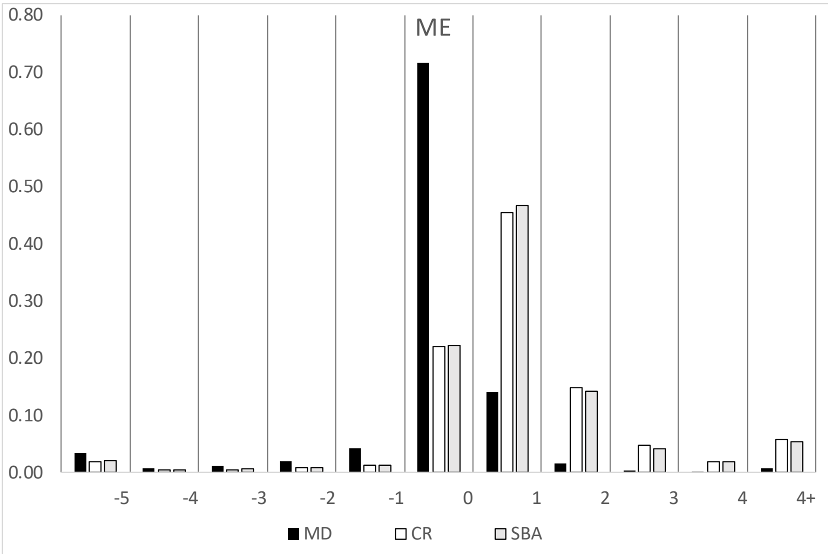
Source: own elaborations

Similar conclusions are valid in the case of the *ME* distributions. The first quartile is the lowest for the MD method. It is also negative for MA and SES methods. For CR and SBA the first quartile is equal to zero. All methods, despite MD and MA, are positively biased with respect to median. The third quartile of the ME distribution is zero for the MD method and it is above one for the CR and SBA methods. These conclusions are proving that the MD method is not positively biased, which is important in case of managing the inventory levels.

The variability of errors is the lowest for the MD method ($Q^{MD} = 0.100$) so this method is the most efficient. The *ME* distributions are least skewed for the SES and MA methods, which is positive in terms of unbiasedness. All *ME* distributions are leptokurtic, which is important with regard to forecast efficiency.

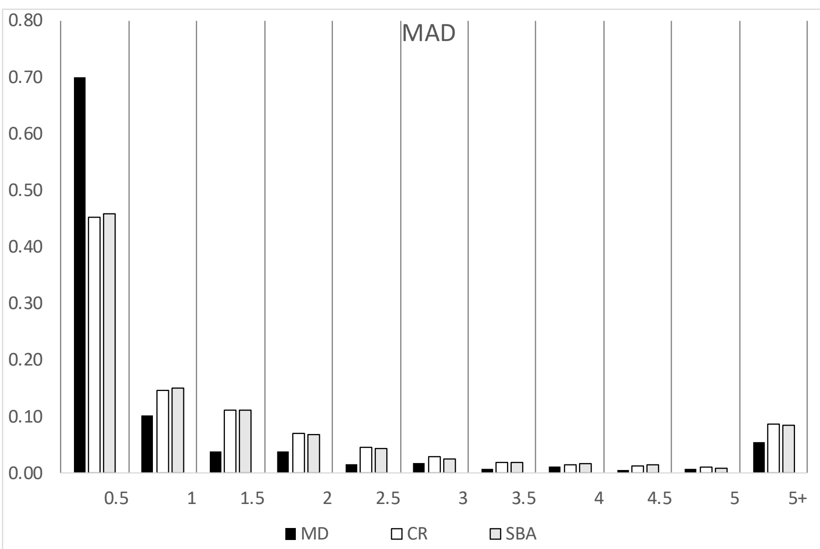
The general conclusion is the same as in case of error D. The only unbiased method is the MD method, which is important in terms of inventory levels.

The previous errors generally inform about the biasedness of forecasting methods. On the other hand, *MAD* tells us about variability. Variability is important due to safety levels management. With regard to the quantiles of *MAD*, we could say that the lower the value of a given quantile, the higher the efficiency of the forecasting method. The first and the third quantile, and the median are the lowest for the MD method and the highest for the CR method. The variability (due to *Q*) is the lowest for the MA, SES, MD methods and the highest for the SBA and CR methods. All *MAD* distributions are positively skewed and leptokurtic, which proves the forecast efficiency (there are many more small errors). If the forecast variability is lower, it is easier for the decision makers to manage the inventories. The safety levels could then be lower to satisfy demand.



Graph 2. Ex post forecast error measure distributions (*ME*)

Source: own elaborations

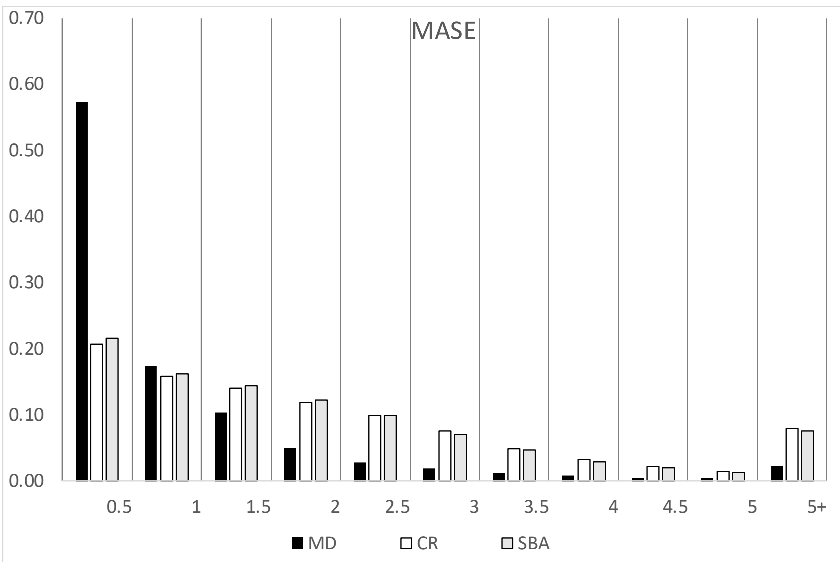


Graph 3. Ex post forecast error measure distributions (*MAD*)

Source: own elaborations

If *MASE* is below one, forecasts are better than naïve forecasts. Similarly, as in the case of *MAD*, all quartiles were the lowest for the MD method and the highest for the CR method. The variability was low for the MA, SES and MD methods and high for the CR and SBA methods. Most *MASE* distributions are positively skewed, so there are many low errors. The only exception is the *MASE* distribution for the MA method, which is slightly negatively skewed, so we could expect a higher number of bigger errors. All *MASE* distributions are leptokurtic (in the narrowed area of variation), which is favorable in terms of forecast efficiency.

The MD method is also the best in terms of efficiency, so it give the possibility of lower safety levels to satisfy demand.



Graph 4. Ex post forecast error measure distributions (*MASE*)

Source: own elaborations

7. Summary

The main aim of the article was to present a new forecasting technique based on stochastic simulation. Its quality was tested by comparing the accuracy of forecasting methods used in the case of intermittent demand. These methods include: moving average, simple exponential smoothing, Croston's method, Systetos-Boylan Approximation and the author's proposal. Forecasts were evaluated with respect to the distributions of four ex post error measures, such as mean error (*ME*), mean absolute deviation (*MAD*), mean absolute scaled error (*MASE*) and the author's proposal (error *D*). With respect to positional characteristics of error distributions, the MD method seems to be the least biased and the most efficient. The CR and SBA methods give positively biased predictions with rather low efficiency. The general conclusion is that the proposed forecasting technique might increase the quality of the decision making process in enterprises dealing with the problem of intermittent demand. More accurate forecasts could improve the quality of customer service. At the same time inventory level could be optimized. The proposed method fulfills these requirements in contrast to the other tested methods. The next step is to compare the MD method with other intermittent demand forecasting methods such as TSB (Teunter, Syntetos, Babai, 2011).

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Nowa technika prognozowania popytu nietypowego na podstawie symulacji stochastycznej. Alternatywa dla metody Crostona

Streszczenie: Głównym celem artykułu jest przedstawienie nowej techniki prognozowania popytu nietypowego (sporadycznego). Aby zaprezentować właściwości proponowanej techniki, przeanalizowano dokładność prognoz generowanych metodą Crostona i metodą autora (opartą na symulacji stochastycznej). Do porównań przyjęto również takie metody, jak średnia ruchoma i proste wygładzanie wykładnicze. Zastosowano również metodę SBA, która jest modyfikacją metody Crostona. Metoda Crostona jest rozszerzeniem metod adaptacyjnych. Oddzielnie analizowane są odstępy między niezerową sprzedażą oraz poziom sprzedaży. Jej celem jest lepsze prognozowanie nietypowego (sporadycznego) popytu. Drugą metodą prognostyczną jest propozycja autora, która opiera się na dwóch etapach. W pierwszym, w oparciu o symulację stochastyczną, określa się, czy zdarzenie (sprzedaż) w danym okresie wystąpi. W drugim etapie szacowany jest poziom sprzedaży (jeśli poprzedni etap pokazuje, że nastąpi sprzedaż). Ze względu na silną asymetrię sprzedaży jej poziom ustalany jest na podstawie odpowiednich kwantyli. Podstawą prognozowania są cotygodniowe serie sprzedaży około czternastu tysięcy produktów (dane rzeczywiste). Analizowane szeregi czasowe można zdefiniować jako nietypowe, co przejawia się niewielką liczbą niezerowych obserwacji (duża liczba zer), dużą zmiennością i losowością (testy losowości wskazują na biały szum). Stosowane miary błędów prognoz charakteryzują zarówno obciążenie, jak i efektywność prognoz. Błędy prognoz zostaną dostosowane do szeregów czasowych z dużą liczbą zer (w tym autorska propozycja miary błędu prognozy). Prognozy zweryfikowano ze względu na rozkłady czterech błędów *ex post*: błędu średniego (ME), średniego odchylenia bezwzględnego (MAD), średniego absolutnego błędu skalowanego (MASE) i błędu D (propozycja autora). Metoda prognozowania oparta na symulacji stochastycznej jest najmniej obciążona i najbardziej efektywna. Metoda Crostona daje dodatnio obciążone prognozy o raczej niskiej efektywności. Proponowana technika prognozowania może wspierać decyzje podejmowane w przedsiębiorstwach, które borykają się z problemem prognozowania sporadycznego popytu. Bardziej dokładne prognozy mogą zwiększyć poziom obsługi klienta i zoptymalizować poziom zapasów.

Słowa kluczowe: prognozowanie popytu nietypowego, metoda Crostona, symulacja stochastyczna, błędy prognoz

JEL: C53, E27, L81

Received: 2017-12-02; verified: 2018-04-03. Accepted: 2018-07-31