Folia Oeconomica
Acta Universitatis Lodziensis
ISSN 0208-6018 e-ISSN 2353-7663

www.czasopisma.uni.lodz.pl/foe/
3(335) 2018

DOI: http://dx.doi.org/10.18778/0208-6018.335.02

Grzegorz Kończak
University of Economics in Katowice, Faculty of Management, Department of Statistics, Econometrics and Mathematics, grzegorz.konczak@ue.katowice.pl

On Testing Significance of the Multivariate Rank Correlation Coefficient

Abstract: The Spearman's rho is a measure of the strength of the association between two variables. There are some extensions of this coefficient for the multivariate case. Measures of the multivariate association which are the generalisation of the bivariate Spearman's rho are considered in the literature. These measures are based on copula functions. This article presents a proposal of the testing for the multivariate Spearman's rank correlation coefficient. The proposed test is based on the permutation method. The test statistic used in the permutation test is based on the empirical copula function. The properties of the proposed method have been described using computer simulations.

Keywords: multivariate Spearman's rho, copula function, permutation tests, Monte Carlo study

JEL: C12, C14, C15

1. Introduction and basic notations

The Spearman's rho ρ_S is a well-known measure for the strength of the association between two random variables *X* and *Y*. Let us consider *n* objects ranked from 1 to *n*. Let R_x and R_y be the ranks of the variables *X* and *Y*. In this case, R_x and R_y are the permutations of the same set containing the numbers 1, 2, …, *n*. The Spearman rank correlation coefficient for the sample of size *n* has the form (Wywiał, 2004: 197):

$$R_s = 1 - \frac{6\sum_{i=1}^{n}(R_{x_i} - R_{y_i})^2}{n^3 - n}. \tag{1}$$

Let us consider the hypothesis

$$H_0 : \rho_S = 0$$

with the alternative

$$\mathrm{H}_1 : \rho_S \neq 0 \text{ or } \mathrm{H}_1 : \rho_S > 0 \text{ or } \mathrm{H}_1 : \rho_S < 0.$$

The hypothesis H_0 could be tested using the test statistic

$$t = R_s\sqrt{\frac{n-2}{1-R_s^2}}, \tag{2}$$

where R_S is the Spearman correlation coefficient based on the sample and $n > 10$ (see Sheskin, 2004). Under the null hypothesis, the test statistic (2) has *t* distribution with $n - 2$ degree of freedom (Zar, 1972: 578–579). Wywiał (2004: 197) pointed that for the sample of size $n \to \infty$ under the null hypothesis the distribution of the test statistic

$$z = R_s\sqrt{n-1} \tag{3}$$

could be approximated by the standard normal distribution.

The above presented Spearman's rho measures the strength of the association only for two variables. There are some extensions of this measure to the *d*-dimensional ($d > 2$) cases. The multivariate Spearman's rho extensions were considered by Joe (1990) and Schmid and Schmidt (2006). Bedő and Ong (2015) used this measure for aggregating ranks. Multivariate extensions of Spearman's rho are based on copula functions.

2. On the measuring of multivariate dependences

One of the statistical methods used to measure multivariate dependences are copulas. Copulas are very useful tools for describing and understanding the dependence between two or more random variables. A copula is a function which joins a multivariate function to its marginal distribution functions. It is a multivariate distribution function defined on the unit cube $[0, 1]^d$, with a uniformly distributed marginal. Formally, the definition of copulas could be written as follows (Nelsen, 1999: 8–9):

A d-dimensional copula is a function C with domain $[0, 1]^d$ such that

1. $C(\mathbf{u})$ is zero for all $\mathbf{u}$ in $[0, 1]^d$ for which at least one coordinate is equal to 0
2. $C(\mathbf{u}) = u_k$ if all coordinates of $\mathbf{u}$ are 1 except the k-th one
3. C is d-increasing in the sense that for every $\mathbf{a} \le \mathbf{b}$ ($a_i \le b_i$ for $i = 1, 2, \ldots, d$) in $[0, 1]^d$ the volume assigned by C to the d-box $[\mathbf{a}, \mathbf{b}] = [a_1, b_1] \times [a_2, b_2] \times \ldots \times [a_d, b_d]$ is nonnegative.

Let $(X_1, X_2, \ldots, X_d)$ and $(Y_1, Y_2, \ldots, Y_d)$ be two independent d-vectors with joint distributions $C_X(F(\boldsymbol{x}))$ and $C_Y(F(\boldsymbol{y}))$ where $F(\boldsymbol{x}) = (F_1(x_1), \ldots, F_d(x_d))$ and $F(\boldsymbol{y}) = (F_1(y_1), \ldots, F_d(y_d))$ are the marginal distributions and C_X, C_Y are the respective d copulas. Then the concordance function (see Bedő, Ong, 2015: 2) is given by

$$Q(C_X, C_Y) = P\left[\prod_{j=1}^{d}(X_j - Y_j) > 0\right] - P\left[\prod_{j=1}^{d}(X_j - Y_j) < 0\right] = 2^d \int_{[0,1]^d} C_X(v)dC_Y(u) - 1,$$

where $u = F(x)$ and $v = F(y)$.

There are methods of multivariate extensions for the Spearman's rho coefficient. Some of them are derived from multivariate dependence concepts (Nelsen, 1996: 223). The three following multivariate ($d \ge 2$) versions of Spearman's rho were analysed by Schmid and Schmidt (2006: 760)

$$\rho_1 = h(d)\left[2^d \int_{[0,1]^d} C(\mathbf{u})d\mathbf{u} - 1\right], \tag{4}$$

$$\rho_2 = h(d)\left[2^d \int_{[0,1]^2} \Pi(\mathbf{u})dC(\mathbf{u}) - 1\right], \tag{5}$$

$$\rho_3 = h(2)\left[2^2 \sum_{k<l} \binom{d}{2}^{-1} \int_{[0,1]^2} C_{kl}(u, v)dudv - 1\right], \tag{6}$$

where $h(d) = \frac{1}{Q(M,\Pi)} = \frac{d+1}{2^d - (d+1)}$, $M(\mathbf{u})$ is the upper Frėchet-Hoeffding bound given by $M(\mathbf{u}) = \max\{u_1 + u_2 + \ldots + u_d - (d-1), 0\}$ and $\Pi(\mathbf{u})$ is the independence copula given by $\Pi(\mathbf{u}) = \prod_{i=1}^{d} u_i$, $\mathbf{u} \in [0, 1]^d$.

The measures ρ_1, ρ_2 and ρ_3 are multivariate extensions of two-dimensional Spearman's rho, because for $d = 2$ there is (Schmid, Schmidt, 2006: 761) $\rho_1 = \rho_2 = \rho_3 = \rho_s$. For $d > 2$, the values of ρ_1, ρ_2 and ρ_3 are different in general.

Empirical copula

Let us consider a random sample $\mathbf{X}_j = (X_{1j}, X_{2j}, \ldots, X_{dj})$ ($j = 1, 2, \ldots, n$) from a d-dimensional random vector $\mathbf{X}$ with the joint distribution function F and the copula C which are unknown. The distribution function F could be estimated as follows:

$$\hat{F}_i(x) = \frac{1}{n}\sum_{j=1}^{n} \mathbf{1}_{\{X_{ij} \le x)\}}, \text{ for } i = 1, 2, \ldots, d \text{ and } x \in R.$$

The copula function C could be estimated by (Schmid, Schmidt, 2006):

$$\hat{C}_n(u) = \frac{1}{n}\sum_{j=1}^{n}\prod_{i=1}^{d} \mathbf{1}_{\{\hat{U}_{ij} \le u_i\}}, \text{ for } \mathbf{u} = (u_1, u_2, \ldots, u_d) \in [0,1]^d,$$

where $\hat{U}_{ij} = \hat{F}_i(X_{ij})$ for $i = 1, 2, \ldots, d$, $j = 1, 2, \ldots, n$ and $\hat{\mathbf{U}}_j = (\hat{U}_{1j}, \hat{U}_{2j}, \ldots, \hat{U}_{dj})$.

Empirical copulas will be used to estimate the multivariate ($d > 2$) Spearman's rho correlation coefficient.

Let $R_1, R_2, \ldots, R_d$ be the rankings of d experts. Then the ranking R_i ($i = 1, 2, \ldots, d$) is an n-dimensional vector. This vector is the permutation of the numbers 1, 2, …, n. The normalised ranks (Bedő, Ong, 2015) are calculated as follows: $\tilde{R}_{ij} = \frac{R_{ij}}{n+1}$ ($i = 1, 2, \ldots, d$). Using the empirical copula (Schmid, Schmidt, 2007) expression in the Spearman's formula, we obtain an empirical expression of multivariate Spearman's correlation coefficient (Bedő, Ong, 2015: 2; Schmid, Schmidt, 2007: 410)

$$\hat{\rho}_1 = h(d)\left[\frac{2^d}{n}\sum_{j=1}^{n}\prod_{i=1}^{d}(1 - \tilde{R}_{ij}) - 1\right], \tag{7}$$

$$\hat{\rho}_2 = h(d)\left[\frac{2^d}{n}\sum_{j=1}^{n}\prod_{i=1}^{d}\tilde{R}_{ij} - 1\right], \tag{8}$$

$$\hat{\rho}_3 = \frac{12}{n}\binom{d}{2}^{-1}\sum_{k<l}\sum_{j=1}^{d}(1-\tilde{R}_{kj})(1-\tilde{R}_{lj}). \tag{9}$$

The formulas for ρ_1, ρ_2 and ρ_3 are different in general. In this paper, formula (4) is considered as well as the estimator given by (7). This formula will be used for testing the significance of the multivariate Spearman's rho.

3. The properties of two- and multivariate Spearman's rho

The Spearman's rho is a nonparametric measure of rank correlation. It assesses how well the relationship between two variables can be described using a monotonic function. This coefficient takes values from –1 to 1. The Spearman's coefficient is equal to 1 if the rankings are identical, for example:

Ranking 1: 1, 2, …, n.

Ranking 2: 1, 2, …, n.

This coefficient is equal to –1 if they are in reverse order, for example:

Ranking 1: 1, 2, …, n.

Ranking 2: n, n – 1, …, 1.

Typical histograms for R_s for samples of the size $n = 5$, $n = 10$ and $n = 20$ for independent rankings ($\rho_s = 0$) are presented in Figure 1.

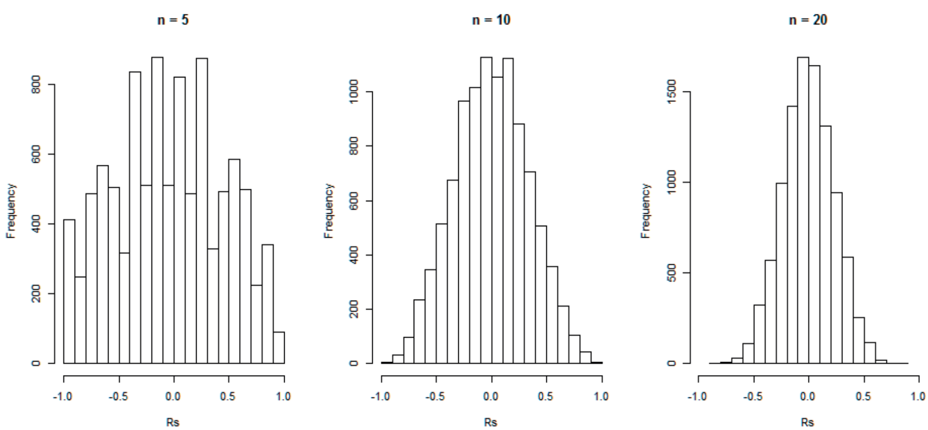

Figure 1. Empirical distributions of R_s for independent rankings (n = 5, 10 and 20)

Source: own elaboration

The exact distributions of Spearman's R_S for independent rankings for the sample of size 5, 8 and 10 are presented in Figure 2.

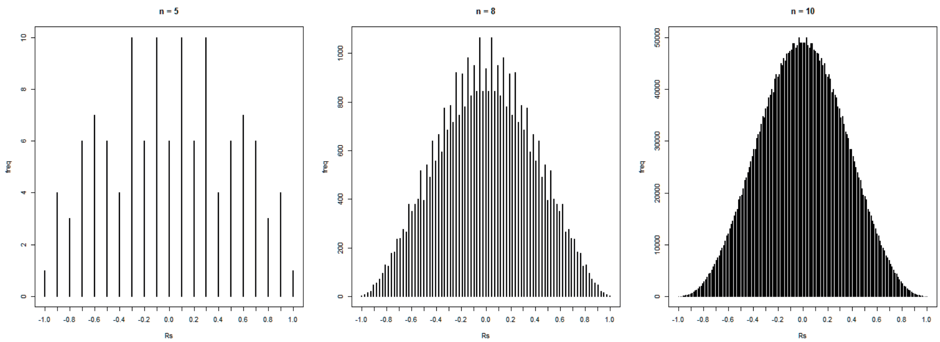

Figure 2. Theoretical distributions of R_s (n = 5, 8 and 10) for independent rankings

Source: own elaboration

For $n = 5$, there are 5! = 120 permutations of the second variable (the first variable is fixed). The Spearman's ρ_S for the sample of the size $n = 5$ can take 21 following variants of the values:

–1.0; –0.9; …; –0.1; 0.0; 0.1; …; 0.9 and 1.0.

The values of the potential variants of Spearman's ρ_S are presented in Table 1.

Table 1. The number of variants of Spearman's ρ_S values for the sample size n = 5, 6, …, 10

Sample size n	5	6	7	8	9	10
No. of variants	21	36	57	85	121	166

Source: own elaboration

Domański and Pruska (2000: 115) described difficulties in constructing tables with critical values for the Spearman's rho due to the number of possible permutations of variables. For $n = 10$, there are 10! = 3,628,800 permutations of the ranking. For the multivariate extension of the Spearman's coefficient ($d > 2$), calculations are much more complicated. The number of possible permutations grows radically for the dimension $d > 2$. The number of permutations of the 2, 3, …, d variable (the first ranking is fixed) is $N_p = \prod_{i=2}^{d} n!$. There are 216 variants of different values of ρ_1 in the 3-dimensional case and 1,194 variants of different values in 4-dimensional case for the sample size of $n = 5$. The number of permutations of $d - 1$ variables for the sample sizes n = 5, 6, …, 10 are presented in Table 2.

Table 2. The number of possible permutations (the first variable is fixed)

Sample size *n*	***d* = 2**	***d* = 3**	***d* = 4**
5	120	14 400	1 728 000
6	720	518 400	373 248 000
7	5 040	25 401 600	128 024 064 000
8	40 320	1 625 702 400	65 548 320 768 000
9	362 880	131 681 894 400	c.a. $47.78*10^{15}$
10	3 628 800	13 168 189 440 000	c.a. $47.78*10^{18}$

Source: own elaboration

The empirical distributions of the multivariate Spearman's coefficient ρ_1 for $d = 3$ are presented in Figure 3 and for $d = 4$ in Figure 4.

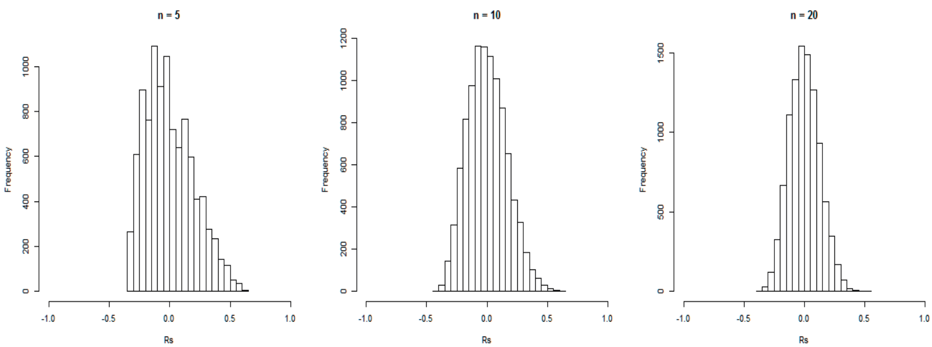

Figure 3. The empirical distributions of the multivariate ($d = 3$) Spearman's coefficient $\hat{\rho}_1$ for independent rankings ($n = 5$, 10 and 20)

Source: own elaboration

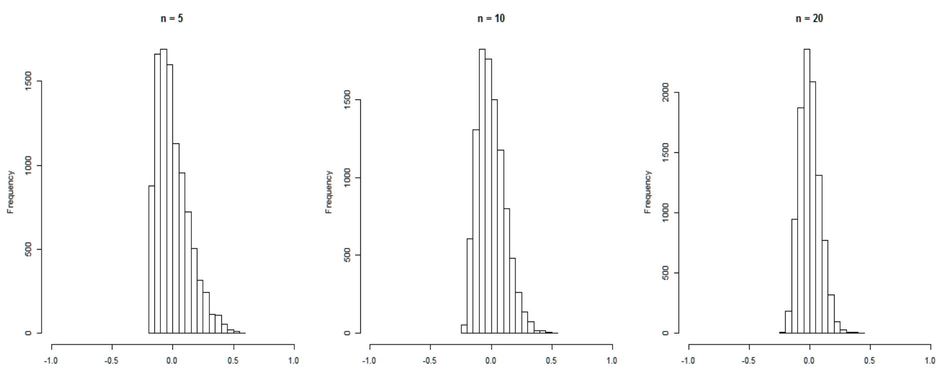

Figure 4. The empirical distributions of the multivariate ($d = 4$) Spearman's coefficient $\hat{\rho}_1$ for independent rankings ($n = 5$, 10 and 20)

Source: own elaboration

The distribution of the multivariate Spearman's coefficient for independent rankings for $d > 2$ in general is not symmetric. The exact distributions of the multivariate Spearman's $\hat{\rho}_1$ for the sample of the size $n = 5$ for $d = 3$ and $d = 4$ are presented in Figure 5.

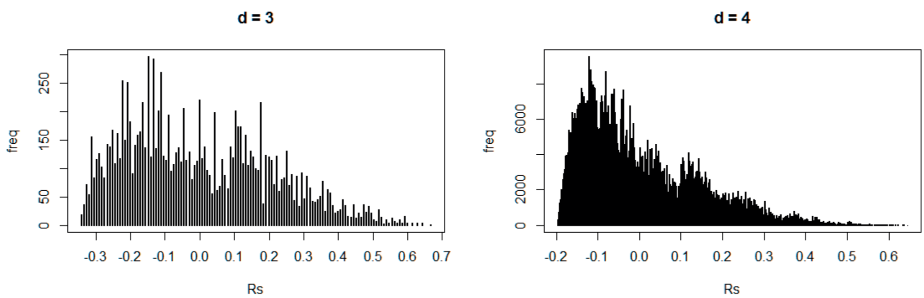

Figure 5. The exact distributions of the multivariate ($d = 3$ and $d = 4$) Spearman's coefficient for independent rankings ($n = 5$)

Source: own elaboration

The value of Spearman's rank correlation coefficient varies between –1 and 1. The maximum, minimum and estimated values of quantiles of the multivariate Spearman's ρ_1 are presented in Table 3 ($n = 5$) and Table 4 ($n = 10$). These values were obtained in series of computer simulations. In each case, there were generated 1000 times d ($d = 2, 3, \ldots, 10$) independent rankings and the value of the multivariate Spearman's coefficient ρ_1 was calculated using formula (7).

Table 3. The estimated quantiles of the multivariate Spearman's $\hat{\rho}_1$ for the sample of the size $n = 5$

Dim	Quantile							
d	Min	0.01	0.025	0.05	0.95	0.975	0.99	Max*
2*	–1.000	–0.900	–0.900	–0.800	0.800	0.900	0.900	1.000
3	–0.341	–0.319	–0.304	–0.281	0.363	0.437	0.496	0.667
4	–0.195	–0.182	–0.172	–0.162	0.257	0.314	0.388	0.644
5	–0.117	–0.109	–0.104	–0.098	0.180	0.228	0.304	0.610
6	–0.070	–0.066	–0.063	–0.060	0.117	0.162	0.222	0.568
7	–0.042	–0.040	–0.038	–0.036	0.077	0.106	0.151	0.524
8	–0.025	–0.023	–0.023	–0.022	0.049	0.072	0.105	0.478
9	–0.014	–0.014	–0.013	–0.013	0.030	0.044	0.068	0.433
10	–0.008	–0.008	–0.008	–0.007	0.018	0.028	0.044	0.389

* Exact values.

Source: computer simulation

Table 4. The estimated quantiles of the multivariate Spearman's $\hat{\rho}_1$ for the sample of the size n = 10

Dim	**Quantile**							
d	**Min**	**0.01**	**0.025**	**0.05**	**0.95**	**0.975**	**0.99**	**Max***
2*	–1.000	–0.733	–0.636	–0.552	0.552	0.636	0.733	1.000
3	–0.424	–0.319	–0.284	–0.250	0.286	0.343	0.409	0.828
4	–0.236	–0.189	–0.172	–0.154	0.201	0.247	0.302	0.804
5	–0.139	–0.115	–0.106	–0.096	0.145	0.182	0.230	0.782
6	–0.082	–0.069	–0.064	–0.059	0.099	0.129	0.169	0.755
7	–0.048	–0.042	–0.039	–0.036	0.068	0.091	0.124	0.725
8	–0.028	–0.025	–0.023	–0.022	0.044	0.062	0.086	0.693
9	–0.016	–0.014	–0.014	–0.013	0.027	0.040	0.060	0.661
10	–0.009	–0.008	–0.008	–0.008	0.017	0.025	0.038	0.628

* Exact values.

Source: computer simulation

The examples of the complete agreement in the rankings for 4-dimension and the highest discrepancy for the sample of the size n = 5 are presented in Figure 6 and Figure 7. If the first ranking is fixed, then there exists the one and only combination of three other rankings which gives the maximum $\hat{\rho}_1$ = 0.644 (see Figure 6). In this case, there are 288 rankings with the minimum $\hat{\rho}_1$ = –0.195. One of these combinations is presented in Figure 7.

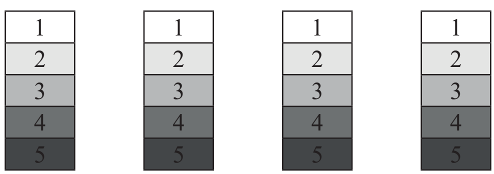

Figure 6. The complete agreement in 4 rankings

Source: own elaboration

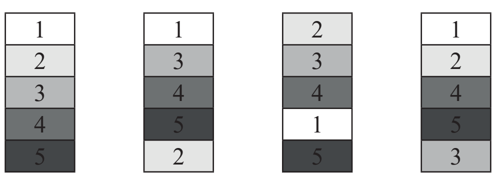

Figure 7. One of the highest discrepancy in 4 rankings

Source: own elaboration

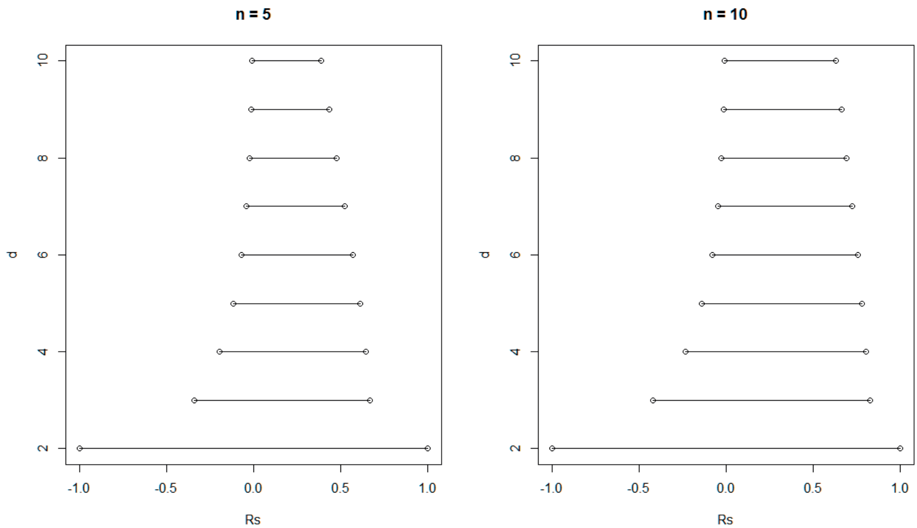

Figure 8. Intervals of variations for the *d*-dimensional correlation coefficient for the samples of the size $n = 5$ and $n = 10$

Source: own elaboration

The area of variations of the *d*-dimensional ($d = 2, 3, \ldots, 10$) Spearman's ρ_1 for the sample sizes of $n = 5$ and $n = 10$ is presented in Figure 8. The distribution of the multivariate Spearman's ρ_1 for the dimension greater than 2 is not symmetric. Due to the asymmetry of the distribution, the critical region for the H_0 should be asymmetric. To test the significance of the multivariate Spearman coefficient, the permutation test will be proposed.

4. Testing multivariate dependences

Zar (2010: 773) presented tables of critical values of the Spearman's ranked correlation coefficient. These tables could be used only for the two-dimensional version of Spearman's rank coefficient. For the case where $d > 2$, the permutation test could be used.

Permutation tests were introduced by R.A. Fisher and E.J.G. Pitman in 1930s (Berry, Johnston, Mielke, 2014: 20). Lehmann (2009: 439) shows that permutation tests are generally asymptotically as good as the best parametric ones. The concept of permutation tests is simpler than that of tests based on normal distribution. Efron and Tibshirani (1993: 202) point out that the main application of these tests is a two-sample problem. In permutation tests, the observed value of the test statistic (T_0) is compared with the empirical distribution of this statistic under the null hypothesis. The following steps are taken in dealing with permutation tests (Good, 2005: 8; Kończak; 2016: 29):

1. Assume the significance level α.
2. Identify the null hypothesis and the alternative hypothesis.
3. Choose a form of the test statistic T.
4. Calculate the value T_0 of the test statistic for the sample data.
5. Determine by a series of permutations the frequency distribution of the test statistic under the null hypothesis ($T_1, T_2, \ldots, T_N$, where $N \geq 1000$).
6. Make a decision using this empirical distribution as a guide.

The ASL (*Achieved Significance Level*) has the following form:

$$ASL = P\left(T \geq T_0\right). \tag{1}$$

The ASL is unknown and could be estimated by the following formula:

$$ASL \approx \frac{card\{i : T_i \geq T_0)}{N}. \tag{2}$$

This notation applies where the H_0 rejection area is right-sided. In the case of the left-sided rejection area in the above notation, inequality should be changed. If the value of ASL is lower than the assumed level of significance α, then H_0 should be rejected.

The significance of the described multivariate Spearman's rank coefficient will be tested. The sample multivariate Spearman's rank coefficient given by (7) as a test statistic will be used in Monte Carlo study. The empirical distribution of this coefficient will be obtained in the procedure of permutation testing. The null hypothesis will be rejected for ASL < α.

5. The test procedure – Monte Carlo study

Let us consider the null hypothesis that all rankings are independent. This hypothesis could be written as follows:

$$H_0 : \rho_S = 0$$

with the alternative

$$H_1 : \rho_S > 0.$$

There were considered hypotheses for three-, four- and five- dimensional rankings. Two following variants were considered:

1) H_0 is true – there is no correlation between vectors $R_1, R_2, \ldots, R_d$.

2) H_0 is false – two rankings are identical, and the others were no correlated.

The probabilities of rejection of H_0 were estimated by a sequence of 1000 computer simulations of permutation tests. In each permutation test, there were 1000 permutations considered. In all the simulations, the significance level $\alpha = 0.05$ was assumed. The estimated probabilities of rejection of H_0 are presented in Table 5 (H_0 true – the size of the test) and in Table 6 (H_0 false).

Table 5. Estimated probabilities of H_0 rejection (H_0 true)

Dimension *d*	**Sample size** *n*					
	5	**6**	**7**	**8**	**9**	**10**
3	0.047	0.042	0.049	0.044	0.043	0.046
4	0.048	0.050	0.039	0.052	0.053	0.051
5	0.056	0.045	0.052	0.036	0.040	0.042

Source: own elaboration

Table 6. Estimated probabilities of H_0 rejection (H_0 false)

Dimension *d*	**Sample size** *n*					
	5	**6**	**7**	**8**	**9**	**10**
3	0.294	0.341	0.358	0.414	0.418	0.481
4	0.183	0.219	0.223	0.255	0.262	0.298
5	0.156	0.153	0.158	0.162	0.177	0.189

Source: own elaboration

The size of the test is close to the assumed significance level $\alpha = 0.05$ (see Table 5). For the greater size of the sample in the case of false $H_{0,}$ there is a greater probability of H_0 rejection. For the smaller dimension *d* in the case of false H_0, there is a greater probability of H_0 rejection (see Table 6).

6. Conclusions

This article presents a proposal of the testing for multivariate extensions of Spearman's rho. There are some variants of such extensions. In the paper, one of them given by formula (4) was considered. The properties of these multivariate measures were described. These multivariate Spearman's correlations could be used for measuring the rankings agreement. The test for the significance of the multivariate Spearman's rho was proposed. The proposed testing procedure is based on the permutation method.

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O testowaniu istotności wielowymiarowego współczynnika korelacji rang

Streszczenie: Współczynnik korelacji rang Spearmana pozwala na badanie siły zależności między dwiema zmiennymi, dla których dokonano pomiaru na skali porządkowej. W literaturze są prezentowane rozszerzenia tego współczynnika na przypadek wielowymiarowy. W tych konstrukcjach wykorzystywane są zwykle funkcje łączące (kopule). W artykule przedstawiono propozycję testowania istotności zależności wielowymiarowej dla danych mierzonych na skali rangowej. Przedstawiony test dla istotności wielowymiarowego współczynnika korelacji rang wykorzystuje metodę permutacyjną. Własności proponowanego testu scharakteryzowano z wykorzystaniem symulacji komputerowych.

Słowa kluczowe: wielowymiarowy współczynnik rang Spearmana, kopuła, test permutacyjny, symulacja Monte Carlo

JEL: C12, C14, C15

Received: 2016-12-16; verified: 2017-12-11. Accepted: 2018-03-07