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With this letter I am responding to your (by previous dean) request for a review of the doctoral disseration of Anna Kimaczyńska, M. Sc., entitled *The differential operators in the bundle of symmetric tensors on a Riemannian manifold.* In summary, I find this dissertation very good and fully adequate for the doctoral degree; indeed, I think it is distinctive, giving new and interesting contributions to the theory of elliptic operators on natural bundles on Riemannian manifolds.

The dissertation consists of 59 pages in 4 sections, and a list of 17 references. The topic is Riemannian geometry on manifolds with emphasis on the construction of new natural differential operators in the bundle of symmetric tensors. The work extends previous work on skew-symmetric forms by introducing the two operators div and grad and their second-order composition div grad. For this operator, the author shows that it is strongly elliptic, and constructs 2^{k+1} elliptic and self-adjoint boundary conditions on the bundle of symmetric tensors of degree k.

Section 1, *Symmetric tensors,* discusses the basic operations on tensors on a Riemannian manifold, in particular the new operator $\mathfrak{a}$, and its dual $\mathfrak{a}^*$. Also two trace operators are defined, and together these form the basic operators of order zero. Section 2, *Differential operators,* gives the geometric differential operators of Levi-Civita and the basic symmetric derivative d^s on symmetric tensors (and their tensor product with the tangent bundle). This section has a good description of the geometry af tensors and these natural operators. Definition 2.10 gives the new and important gradient operator grad as $\mathfrak{a}d^s - d^s\mathfrak{a}$. This has an adjoint in the L^2 sense, called –div; and very explicit rules for the calculation with these new operators is carefully explained. Section 3, *Weitzenböck formula,* calculates the curvature terms entering the explicit expressions for the second-order operator div grad. The main result is Theorem 3.1, relating the Laplace operator on symmetric tensors with div grad and finds the explicit curvature operator of Ricci type. Section 4, *Natural boundary value conditions for div grad operator,* contains the main results of the dissertation; the operator div grad is shown to be strongly elliptic, and a careful study of natural boundary conditions is carried out. In fact, 2^{k+1} different boundary conditions are found to be self-adjoint (Proposition 4.2) and elliptic (Theorem

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4.2) on symmetric tensors of degree k. This is a very nice result, and should be compared to the case of skew-symmetric tensors, where only 4 natural such conditions are known. Finally some additional geometric conditions on the geometry of the boundary are investigated in detail, namely umbilical and totally geodesic boundary.

It should be stressed, that Anna Kimaczyńska in this dissertation has given a well-written presentation of basic and important constructions of natural differential operators associated a Riemannian structure on a smooth manifold. The strongest result is an original construction of a universal auxiliary bundle in the proof of Theorem 4.2 that simultaneously encompasses all the 2^{k+1} of the considered boundary conditions. The method seems to be applicable in much larger classes of bundles and elliptic operators. The result has several consequences by invoking the general theory of elliptic boundary value problems; indeed, the existence of an orthonormal basis of the corresponding L^2 – Hilbert space is a corollary (Proposition 4.6).

This dissertation fulfills the requirements for the doctoral degree; it is of an international research level, and I recommend it as distinctive.

Regards,

Bent Ørsted
Professor