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IMPLEMENTING OF ANALYTIC HIERARCHY PROCESS IN BANKING

Abstract. The Analytic Hierarchy Process (AHP) is a multicriteria decision support method created by Thomas L. Saaty. It provides both individual and group decision makers an objective way for reaching an optimal decision. The AHP is designed to select the best from a number of alternatives evaluated with respect to several criteria. It is taken by carrying out pairwise comparison judgements which are used to develop overall priorities for ranking the alternatives. This method allows for some level of inconsistency in judgements (that is unavoidable in practice) and provides some measures for limiting that. Our article describes classical Saaty solution to the AHP problem and shows the application of the AHP in establishing the price of the bank deposits.

1. PREFRACE

The Analytic Hierarchy Process (AHP) is a multiciriteria decision support method that provides both individual and group decision makers an objective way for reaching an optimal decision. The AHP is designed to select the best one from a number of alternatives evaluated with respect to several criteria. It is taken by carrying out pairwise comparison judgements which are used to develop overall priorities for ranking the alternatives. The method allows for some level of inconsistency in judgements (the is unavoidable in practice) and provides some measures for limiting that. Originally the AHP method was created by Thomas L. Saaty who is still deeply engaged in development of applications of this method.

In our paper we have used *Expert Choice For Windows 9.0* (E.C. 9.0) – a software developed by Ernest H. Forman for carrying out calculations.

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However, we checked these calculation via *Excel 97* reaching the same results. The E.C. 9.0 enables decision makers to sort out effectively the complexity and assist with the subjectivity that is inherent in many decisions. This software allows decision makers to build their models in *the Evaluation and Choice* component or use *Structuring* to visually organize the decision elements and build a hierarchy with drag and drop ease. After building the model, decision elements and build a hierarchy with drag and drop ease. After building the model, decision makers can choose two different measurement options depending on whether alternatives should be compared against each other (relative measurement) or rated against standards (absolute measurement). Often the *Ratings* approach is appropriate when large numbers of alternatives are involved. It is also within Evaluation and Choice that users will enter their judgements about the relative importance of the criteria and alternatives, synthesize to get results, and conduct sensitivity analyses.

2. ESTABLISHING THE PROBLEM

The AHP is a general theory of preference measurement with providing necessary information for choosing the best decision.

In the AHP process there are four main stages:

1. Building a hierarchy model.
2. Identifying the preferences of decision makers.
3. Synthesis.
4. Sensitivity analyses.

The basic AHP model consist of three levels: goal, criteria level and alternatives. Depending on complexity of the problem it is possible to add as many as necessary levels of subcriteria.

The most complex problem is identification of decision maker preferences. In AHP it is done by collecting information about pairwise judgements due to a goal (for criteria), a specified criterion (for alternatives or subcriteria) or a subcriterion (for alternatives). There are a few possible scales of converting collected information into numeric form – however it is not always necessary. Having one set of information we build a matrix of ration comparison for a given goal/criterion. It is possible to find many ways of converting the matrix **A** (matrix of ratio comparison) into the vector of priorities **w**. However, the need of consistency makes us choose the eigenvalue formulation $\mathbf{Aw} = n\mathbf{w}$. Assuming that the priorities $w = (w_1, ..., w_n)^T$ with respect to a single criterion are known, such as the weights of stones – we can examine what we have to do to recover them. Having the matrix **A**:

$$\mathbf{A} = \begin{bmatrix} {}^{w_1}\!/_{w_1} & {}^{w_1}\!/_{w_2} & \cdots & {}^{w_1}\!/_{w_n} \\ \vdots & \vdots & \vdots & \vdots \\ {}^{w_n}\!/_{w_1} & {}^{w_n}\!/_{w_2} & \cdots & {}^{w_n}\!/_{w_n} \end{bmatrix}$$

we multiply it on the right by $\mathbf{w}$

$$\mathbf{w} = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix},$$

to obtain $n\mathbf{w}$. Elements a_{ij} of the matrix of ratio comparison represent the importance of alternative i over alternative j. In order to guarantee the judgements to be consistent, relevant groups of the matrix elements have to follow the equation: $a_{ij}\, a_{jk} = a_{ik}$. In case, we do not have a scale at all, or do not have it conveniently as in the case of some measuring devices – we can only giver an estimation of w_i/w_j. It leads to the problem:

$$\mathbf{A}^*\mathbf{w}^* = \lambda_{\max}\mathbf{w}^*,$$

where $\lambda_{\max}$ is the principal eigenvalue of $\mathbf{A}^* = (a^*_{ij})$ the perturbed value $\mathbf{A} = (a_{ij})$ with the reciprocal $a^*_{ji} = 1/a^*_{ij}$ forced. The solution is obtained by raising the matrix to sufficiently large power – then summing over the rows and normalizing to obtain the priority vector $w^* = (w^*_1, ..., w^*_n)^T$. The above mentioned process is stopped when the difference between components of the priority vector obtained at k-th power and at the $(k+1)$-st power is less than some predermined small value. The vector of priorities is the derived scale associated with the matrix of comparisons. The value zero in this scale is assigned to an element that is not comparable with the elements considered. With the eigenvector for $n \leqslant 3$ normalizing the geometric means of the rows leads to an approximation to the priorities. In all the cases it is possible to get an approximation by normalizing the elements of each column of the judgement matrix and then averaging over ech row. However, it is important to remember that such stemps can lead to rank reversal (in spite of closeness of the eigenvector solution). A simple way to obtain the exact value (or an estimate) of $\lambda_{\max}$ when the exact value of $\mathbf{w}^*$ is availabe in normalized form is to add the columns of A^* and multiply the resulting vector by the priority vector $\mathbf{w}$.

After obtaining the principal eigenvector estimate $\mathbf{w}$ we should consider the question of consistency. The problem arises from the fact that the original matrix $\mathbf{A}$ need not to be transitive, for example A_1 may be preferred to A_2 and A_2 to A_3 but A_3 may be preferred to A_1. The solution to this problem is the consistency index (C.I.) of a matrix of comparison defined as:

$$\text{C.I.} = \frac{\lambda_{\max} - n}{n - 1}.$$

The consistency ration (C.R.) is obtained by comparing the C.I. with the appropriate one of the following set of numbers (Tab. 1) each of which is an average random consistency index derived from a sample of randomly generated reciprocal matrices. The study of the problem and revision of the judgements should be completed if $\frac{\text{C.I.}}{\text{R.I.}} \geqslant 0.10$.

Table 1

Average Random Consistency Index (R.I.) (Saaty, 1986, p. 9)

n	1	2	3	4	5	6	7	8	9	10
Random Consistency Index (R.I.)	0	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.49

The above solution to the problem is considered to be classical Saaty solution (Saaty, 1994, p. 7–9) and is used for reaching both local and global vectors of priorities – necessary for synthesis.

Hierarchic synthesis is obtained by a process of weighting and adding down the hierarchy leading to multilinear form. There are two possible modes of the synthesis:

- the distributive mode in which the principal eigenvector is normalized to yield a unique estimate of ratio scale underlying the judgements;
- the ideal mode in which the normalized values of alternatives for each criterion are divided by the value of the highest rate alternative.

The final step is sensitivity analysis that gives an answer to the question whether the alternative chosen as the best would be changed in case of modifying criteria/subcriteria preferences.

3. APLICATION OF THE AHP METHOD IN ESTABLISHING THE PRICE OF THE BANK DEPOSITS

In this chapter, we would like to describe the application of AHP in establishing the price of the bank deposits. By establishing the price we consider the change of present deposit rates. The below presented mechanism was experimentally implemented in one of the smallest Polish banks. According to the AHP methodology the first step was getting expert knowledge of present process of establishing the deposit rates. The next step was structuring the AHP hierarchy – the final version of the structure is presented in Fig. 1.

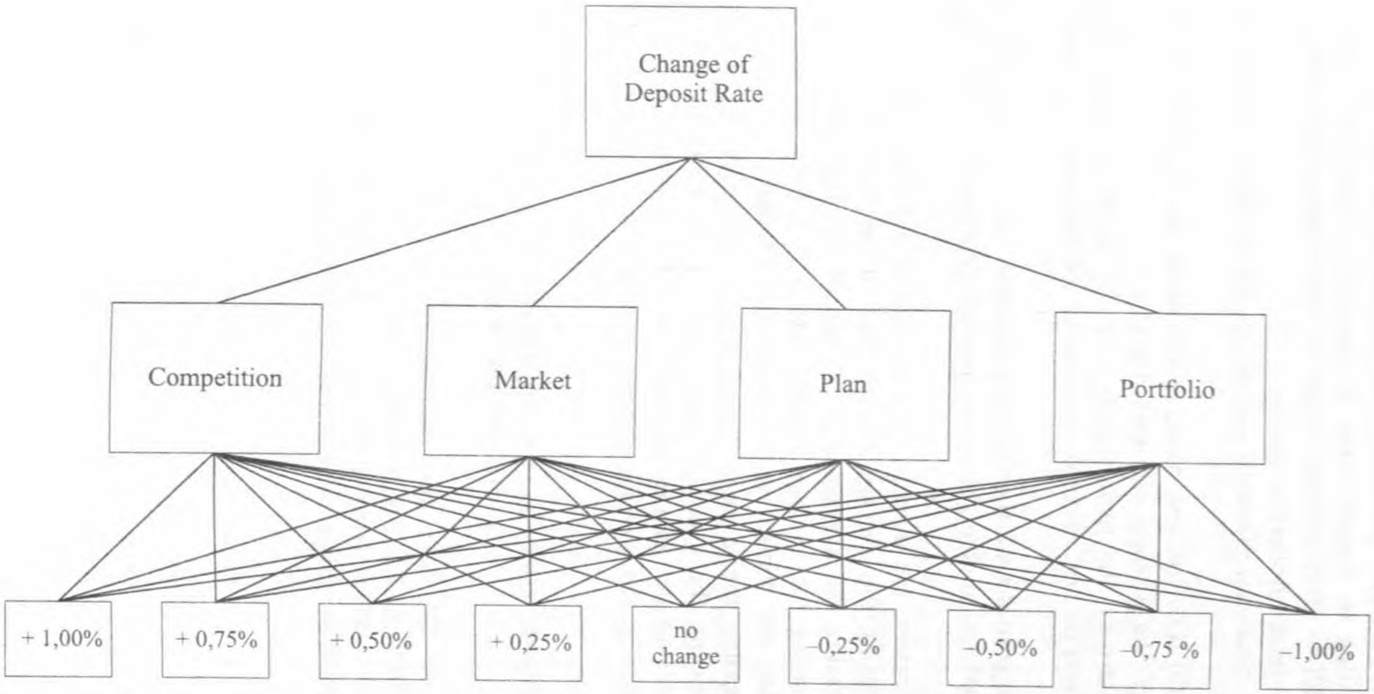

Fig. 1. The three level hierarchy used for changing deposit rate of the bank

• COMPETITION – precisely, it is marketing point of view on pricing deposits according to deposit rates of competitive banks of "our" bank;

• MARKET – it is treasury point of view, including possible buying bank deposits (and alternative costs);

• PLAN – financial planning and prognosis of future benefits and costs of the bank;

• PORTFOLIO – present assets portfolio of the bank as the measure of efficiency of the already acquired deposits.

In order to simplify the understanding of the graph we have decided to use short acronyms for alternatives instead of symbols (A_i) used in the next chapter.

Due to suggestions of the decision makers, we have decided to limit possible alternatives to changes of the average deposit rate, with alternatives as follows:

A_1 – incereasing the average deposit rate of the bank by 1.00%,
A_2 – increasing the average deposit rate of the bank by 0.75%,
A_3 – increasing the average deposit rate of the bank by 0.50%,
A_4 – increasing the average deposit rate of the bank by 0.25%,
A_5 – leaving the deposit rate without any change,
A_6 – decreasing the average deposit rate of the bank by 0.25%,
A_7 – decreasing the average deposit rate of the bank by 0.50%,
A_8 – decreasing the average deposit rate of the bank by 0.75%,
A_9 – decreasing the average deposit rate of the bank by 1.00%.

4. EXPERIMENTAL SOLUTION TO THE PROBLEM USING *EXPERT CHOICE FOR WINDOWS 9.0*

Primarly, all the data and calculations were collected using *Expert Choice For Windows 9.0*. In the next step the calculations were checked using *Excel 97*. Tables from 2 to 6 contain collected information about pairwise comparison judgements in the form described in Chapter 2.

Table 2

Pairwise comparison matrix of criteria

	Competition	Market	Plan	Portfolio
Competition	10/10	90/10	30/10	80/10
Market	10/90	10/10	10/80	10/20
Plan	10/30	80/10	10/10	50/10
Portfolio	10/80	20/10	10/50	10/10

Table 3

Pairwise comparison matrix of alternatives according to the criterion PLAN

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
A_1	10/10	10/20	10/30	10/40	10/60	10/70	10/80	10/90	10/90
A_2	20/10	10/10	10/20	10/30	10/50	10/70	10/80	10/90	10/90
A_3	30/10	20/10	10/10	10/20	10/40	10/60	10/80	10/90	10/90
A_4	40/10	30/10	20/10	10/10	10/15	10/35	10/55	10/75	10/80
A_5	60/10	50/10	40/10	15/10	10/10	10/15	10/30	10/45	10/50
A_6	70/10	70/10	60/10	35/10	15/10	10/10	10/15	10/30	10/30
A_7	80/10	80/10	80/10	55/10	30/10	15/10	10/10	10/20	10/20
A_8	90/10	90/10	90/10	75/10	45/10	30/10	20/10	10/10	10/10
A_9	90/10	90/10	90/10	80/10	50/10	30/10	20/10	10/10	10/10

Table 4

Pairwise comparison matrix of alternatives according to the criterion COMPETITION

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
A_1	10/10	12/10	14/10	16/10	18/10	20/10	22/10	23/10	30/10
A_2	10/12	10/10	12/10	15/10	18/10	20/10	22/10	22/10	25/10
A_3	10/14	10/12	10/10	12/10	16/10	18/10	19/10	19/10	21/10
A_4	10/16	10/15	10/12	10/10	12/10	16/10	17/10	18/10	20/10
A_5	10/18	10/18	10/16	10/12	10/10	12/10	14/10	15/10	17/10
A_6	10/20	10/20	10/18	10/16	10/12	10/10	12/10	13/10	15/10
A_7	10/22	10/22	10/19	10/17	10/14	10/12	10/10	13/10	15/10
A_8	10/23	10/22	10/19	10/18	10/15	10/13	10/13	10/10	14/10
A_9	10/30	10/25	10/21	10/20	10/17	10/15	10/15	10/14	10/10

Table 5

Pairwise comparison matrix of alternatives according to the criterion MARKET

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
A_1	10/10	10/20	10/25	10/30	10/40	10/50	10/60	10/90	10/90
A_2	20/10	10/10	10/20	10/25	10/30	10/40	10/60	10/80	10/80
A_3	25/10	20/10	10/10	10/20	10/30	10/45	10/65	10/80	10/85
A_4	30/10	25/10	20/10	10/10	10/20	10/35	10/50	10/65	10/70
A_5	40/10	30/10	30/10	20/10	10/10	10/20	10/30	10/50	10/52
A_6	50/10	40/10	45/10	35/10	10/10	10/10	10/20	10/30	10/35
A_7	60/10	60/10	65/10	50/10	30/10	20/10	10/10	10/15	10/17
A_8	90/10	80/10	80/10	65/10	50/10	30/10	15/10	10/10	10/15
A_9	90/10	80/10	85/10	70/10	52/10	35/10	17/10	15/10	10/10

Table 6

Pairwise comparison matrix of alternatives according to the criterion PORTFOLIO

	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
A_1	10/10	10/20	10/30	10/40	10/50	10/60	10/80	10/90	10/90
A_2	20/10	10/10	10/20	10/30	10/45	10/60	10/75	10/90	10/90
A_3	30/10	20/10	10/10	10/20	10/40	10/60	10/80	10/90	10/90
A_4	40/10	30/10	20/10	10/10	10/20	10/50	10/70	10/80	10/80
A_5	50/10	45/10	40/10	20/10	10/10	10/20	10/50	10/80	10/80
A_6	60/10	60/10	60/10	50/10	20/10	10/10	10/20	10/60	10/60
A_7	80/10	75/10	80/10	70/10	50/10	20/10	10/10	10/15	10/20
A_8	90/10	90/10	90/10	80/10	80/10	60/10	15/10	10/10	10/12
A_9	90/10	90/10	90/10	80/10	80/10	60/10	20/10	12/10	10/10

As we can see in Tab. 7, both local and global I.C./I.R. are lower than 0.10. It means that the matrices of pairwise comparison for all hierarchy levels allow us to complete synthesis.

Table 7

I.C./I.R. computed for local and global priorities

	Competition	Market	Plan	Portfolio
Local priorities I.C./I.R.	0.00	0.03	0.03	0.06
Global priorities I.C./I.R.	0.04			

Table 8

Summary of local and global priority vectors with necessary calculations leading to an optimal alternative (A_8) due to distributive mode

	Competition		Market		Plan		Portfolio			
global priorities wg_k	$wg_1 = \mathbf{0.593}$		$wg_2 = 0.044$		$wg_3 = 0.292$		$wg_4 = 0.071$		$swl_{ki}{*}wg_k$	Rank
local priorities wl_{ki}	wl_{1i}	$wl_{1i}{*}wg_1$	wl_{2i}	$wl_{2i}{*}wg_2$	wl_{3i}	$wl_{3i}{*}wg_3$	wl_{4i}	$wl_{4i}{*}wg_4$		
A_1	**0.181**	0.107	0.020	0.001	0.017	0.005	0.016	0.001	0.114	4
A_2	0.165	0.098	0.026	0.001	0.021	0.006	0.020	0.001	0.106	5
A_3	0.141	0.083	0.032	0.001	0.028	0.008	0.025	0.002	0.095	7
A_4	0.121	0.072	0.044	0.002	0.044	0.013	0.037	0.003	0.089	8
A_5	0.099	0.059	0.067	0.003	0.074	0.022	0.057	0.004	0.088	9
A_6	0.085	0.050	0.107	0.005	0.117	0.034	0.097	0.007	0.096	6
A_7	0.078	0.046	0.175	0.008	0.169	0.049	0.178	0.013	0.116	3
A_8	0.071	0.042	0.245	0.011	0.262	0.077	0.275	0.020	**0.149**	**1**
A_9	0.059	0.035	**0.284**	0.013	**0.269**	0.078	**0.295**	0.021	0.147	2

The results of the synthesis are presented in Tab. 8. The optimal alternative is decreasing the average deposit rate of the bank by 0.75%. We do not present the sensitivity analyses, however it is important to mention that decresing the importance of the COMPETITION criterion leads to changing optimal alternative to A_9 (decreasing the deposit rate by 1.00%).

5. CONCLUSIONS

The AHP is a good method to support decision makers especially when it is combined with understanding the problem of the judgement consistency. Due to its open characteristics, allowing combining quantitative and non-quantitative aspects of the preferences, the AHP may represent an interesting basis for development of combined optimisation methods.

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