FOLIATIONS 2016

Paweł Walczak's 50 years in mathematics

Będlewo, July 11-17, 2016

ABSTRACTS & PROGRAM

FOLIATIONS 2016
PAWEŁ WALCZAK'S 50 YEARS IN MATHEMATICS

PROGRAM & ABSTRACTS

Będlewo, July 11–17, 2016

7th European Congress of Mathematics
satellite conference

Faculty of Mathematics and Computer Science
University of Łódź

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Dear Collegues,

The international conference Foliations 2016 is in the series of conferences on foliations organized in Poland (1990, 1995, 2000, 2005, 2012). This time we celebrate one of the most recognizable person in the field Professor Paweł Walczak from Uniwersytet Łódzki who coorganized all the previous events. The conference is hosted by the Research and Conference Centre in Będlewo, Poland – a part of Mathematical Institute which belongs to the Polish Academy of Sciences, and takes place in the period of July 11-17, 2016. Foliations 2016 is a satellite of the 7th European Congress of Mathematics (July 18-22, 2016, Berlin, Germany). We wish you a pleasant stay in Będlewo.

Organizers

CONTENTS

PAWEŁ WALCZAK'S 50 YEARS IN MATHEMATICS

List of publications

1. P. Walczak, *A proof of some theorem on the C^∞-functions of one variable which are not analytic*, Demonstratio Math. **4** (1972), 209–213.
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78. M. Lużyńczyk & P. Walczak, *New integral formulae for two complementary orthogonal distributions on Riemannian manifolds*, Ann. Global Anal. Geom. (2015), DOI 10.1007/s10455-015-9465-1.

Ph.D. students:

1. Ryszard Hołubowicz (Uniwersytet Łódzki) 1987
2. Jacek Rogowski (Uniwersytet Łódzki) 1990
3. Mariusz Frydrych (Polska Akademia Nauk) 1991
4. Andrzej Biś (Uniwersytet Łódzki)1995
5. Anna Waliszewska (Uniwersytet Łódzki) 1997
6. Konrad Blachowski (Uniwersytet Łódzki) 2000
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9. Krzysztof Andrzejewski (Polska Akademia Nauk) 2010
10. Magadalena Lużyńczyk (Uniwersytet Łódzki) 2015
11. Tomasz Zawadzki (Uniwersytet Łódzki) 2015

Distinguished Colleague!
Highly Honorable Professor!
Dearest Paweł!

On the occasion of the 50th anniversary of your collaboration with the Queen of all Sciences – Mathematics, we wish to thank you with all ours hearts for making the courageous, very important and valuable decision to work in this field, half a century ago. Owing to this decision, the mathematical community in Łódź gained a mathematician who has been indispensable for Polish Science, not only due to his research but also due to his enormous didactic and organizational achievements.

We cordially congratulate you on all of your professional successes. We are proud of the fact that we have the opportunity to work with you as our Mentor, Director and a Friend, who knows more than others know, who sees more than others see, who understands more than others understand, and gives more than others give.

Your great knowledge, and your ability to share that knowledge, are appreciated by all your colleagues from the Faculty of Mathematics and Computer Science. These talents are also appreciated by your students, who claim that at the beginning of your lectures they have a Euclidean vision, then an elliptic vision, and then a projective one, and by the end they can imagine everything – even great grades on final exams. It's a pity that mainly these are only imaginations ...

We wish you many healthy years, full of serenity. We wish that your dreams come true. We hope for the completion of all your life plans and professional projects, and energy for further activities.

Personally, I hope that this celebration – the Foliations 2016 conference – will remain in your memory as a tribute to you; you have the gratitude of the academic community, in particular the academic community of Łódź. We sincerely and appreciatively recognize your significant contribution to the development of science, and the creation of fundamentals for research for many mathematicians.

Ryszard Pawlak
Dean of the Faculty of Mathematics and Computer Science
University of Łódź

PROGRAM

Monday, July 11

13:00 – 14:00	Lunch (optional)
14:00 – 19:00	Arrival
from 19:00	Dinner

Tuesday, July 12

08:00 – 09:00	Breakfast
09:30 – 10:00	Openning
10:00 – 11:00	Remi Langevin, *Entropy, a "functional" viewpoint*
11:00 – 11:30	Coffee break
11:30 – 12:30	Kathryn Mann, *Group orders, dynamics and rigidity*
13:00 – 14:00	Lunch
15:00 – 15:30	Vladimir Rovenski, *Integral formulae for codimension-one foliated Finsler manifolds*
15:30 – 16:00	Kamil Niedziałomski, *Frame bundle approach to generalized minimal submanifolds*
16:00 – 16:30	Coffee break
16:30 – 17:30	Sebastian Hurtado Salazar, *Burnside problem on diffeomorphism groups*
17:30 – 18:00	Vladimir Slesar , *Vaisman manifolds, canonical foliations and the associated spectral sequence*
18:00 – 18:30	Tomasz Zawadzki , *Variations of total mixed scalar curvature*
19:00 – 20:00	Dinner

Wednesday, July 13

08:00 – 09:00	Breakfast
09:30 – 10:30	Daniel Peralta-Salas , *Helicity is the only integral invariant of volume-preserving diffeomorphisms*

10:30 – 11:00	Coffee break
11:00 – 11:30	Aleksy Tralle, *Smale-Barden manifolds with K-contact and Sasakian structures*
11:30 – 12:00	Takashi Inaba, *Producing compact invariant sets in Reeb flows*
12:00 – 12:30	Icaro Gonçalves, *The Euler class of an umbilic foliation*
13:00 – 14:00	Lunch
15:00 – 15:30	Steven Hurder, *On the dynamics of derived from Kuperberg flows*
15:30 – 16:00	Yoshifumi Matsuda, *Rotation number and lifts of a Fuchsian action on the circle*
16:00 – 16:30	Coffee break
16:30 – 17:30	Juliette Bavard , *About a big mapping class group*
17:30 – 18:00	Ryszard J. Pawlak, *On $\mathcal{A}$-focal entropy points*
18:00 – 18:30	Coffee break
18:30 – 19:00	Andrzej Biś, *Foliations, fractals and dynamics*
from 20:00	Banquet

Thursday, July 14

08:00 – 09:00	Breakfast
09:30 – 10:00	Sergio Fenley, *Quasi–geodesic pseudo–Anosov flows in hyperbolic 3-manifolds*
10:00 – 11:00	Steven Frankel , *Quasigeodesic and pseudo-Anosov flows*
11:00 – 11:30	Coffee break
11:30 – 12:30	Emmanuel Militon, *Distortion and Tits alternative for big mapping class groups*
13:00 – 14:00	Lunch
14:30 – 18:30	Excursion
19:00 – 20:00	Dinner

Friday, July 15

08:00 – 09:00	Breakfast
09:30 – 10:30	Joaquín Pérez Muñoz, *Minimal laminations in $\mathbb{R}^3$ and the Hoffman-Meeks conjecture*
10:30 – 11:00	Coffee break

11:00 – 11:30	Gilbert Hector, *Generic properties of foliations and laminations*
11:30 – 12:00	Jesús A. Álvarez López, *Topological Molino's theory*
12:00 – 12:30	Ramón Barral Lijó, *Leaves of laminations and colorings of graphs*
13:00 – 14:00	Lunch
15:00 – 15:30	Shigenori Matsumoto, *Dynamics of the geodesic and horocycle flows for laminations by hyperbolic surfaces*
15:30 – 16:00	Antoni Pierzchalski, *A short story on the ellipticity of the Stein-Weiss gradients*
16:00 – 16:30	Coffee break
16:30 – 17:30	Joanthan Bowden, *Approximating C^0-foliations by contact structures*
17:30 – 18:00	Wojciech Kozłowski, *Natural boundary value problems for weighted form Laplacians*
18:00 – 18:30	Anna Kaźmierczak, *Some estimates for the product of modules of foliations*
19:00 – 20:00	Dinner

Saturday, July 16

08:00 – 09:00	Breakfast
09:30 – 10:30	Olga Lukina, *Invariants for equicontinuous group actions on Cantor sets*
10:30 – 11:00	Coffee break
11:00 – 11:30	Taro Asuke, *A Chern–Weil construction for derivatives of characteristic classes*
11:30 – 12:00	Hiraku Nozawa, *Independent variation of secondary characteristic classes of Riemannian foliations*
12:00 – 12:30	Yuri A. Kordyukov, *A trace formula for codimension one foliations with simple foliated flows*
13:00 – 14:00	Lunch
15:00 – 15:30	Robert Wolak, *Geometric structures on foliated manifolds*
15:30 – 16:00	Yoshihiko Mitsumatsu, *Reeb components with complex leaves and their symmetries*
16:00 – 16:30	Coffee break

16:30 – 17:30	HIROKAZU MARUHASHI, *Parameter rigidity of the action of AN on $\Gamma\backslash G$ for higher rank semisimple Lie groups*
17:30 – 18:00	PAUL A. SCHWEITZER, S.J., *Exotic open 4-manifolds which are non-leaves*
18:00 – 18:30	CLOSING
19:00 – 20:00	DINNER

SUNDAY, JULY 17

08:00 – 09:00	BREAKFAST
09:00 – 13:00	DEPARTURE
13:00 – 14:00	LUNCH (optional)

ABSTRACTS

Topological Molino's theory

Jesus A. Alvarez Lopez
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Molino's description of Riemannian foliations on compact manifolds is generalized to the setting of compact equicontinuous foliated spaces, in the case where the leaves are dense. In particular, a structural local group is associated to such a foliated space. As an application, we obtain a partial generalization of results by Carrière and Breuillard-Gelander, relating the structural local group to the growth of the leaves.

This is joint work with Manuel F. Moreira Galicia.

A Chern–Weil construction for derivatives of characteristic classes

Taro Asuke

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Secondary characteristic classes for foliations are usually constructed by using Bott connections via the Chern–Weil (and the Chern–Simons) construction. Given an infinitesimal deformation of a foliation, we can define the derivatives of characteristic classes with respect to it [6], [7]. They are defined by means of differential forms, however, the construction involves combinatorial arguments and seemingly different from that of their primitives, namely, usual characteristic classes [7], [2]. In this talk, I will introduce a certain vector bundle which is an analog of 2-tangent bundles TTM for manifolds. Once an infinitesimal deformation is given, one can define a connection on the bundle with which a characteristic homomorphism can be constructed in the usual way. The homomorphism gives not only derivatives but some exotic classes such as the Fuks–Lodder–Kotschick class '$\dot{h}_1 h_1 c_1^q$' [5], [9], [8]. If we deal with the Godbillon–Vey class or the Bott class, the derivatives are known to be represented by projective Schwarzians [10], [1]. This is also explained in a similar framework as above [3]. I will discuss it if the time allows.

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Leaves of laminations and colorings of graphs

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Consider triples (M, f, x), where M is an n-dimensional manifold, $x \in M$ and $f \in C^\infty(M, \mathfrak{H})$, where $\mathfrak{H}$ a separable Hilbert space. Two triples (M, f, x) and (N, g, y) are declared to be equivalent if there is an pointed isometry $\phi \colon (M, x) \to (N, y)$ such that $f = \phi^* g$. The set of equivalent classes of these triples can be endowed with a Polish topology such that certain subspaces are canonically foliated. Using these foliated structure, it is shown that any Riemannian manifold M of bounded geometry can be isometrically realized as a leaf of a compact Riemannian foliated space X with trivial holonomy groups. Moreover X can be chosen to be minimal if M is repetitive. The reciprocal statements are elementary. To get the trivial holonomy groups of X and its minimality, an appropriate C^∞ function $f : M \to \mathfrak{H}$ must be chosen. The properties of bounded geometry allow to discretize this problem, reducing it to the following result about graph colorings. For any graph with an upper bound of its vertex degrees, it is proved that there is a limit-aperiodic vertex coloring by finitely many colors. Moreover the coloring can be chosen to be repetitive if the graph is repetitive.

About a big mapping class group

Juliette Bavard
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The mapping class group of the complement of a Cantor set in the plane arises naturally in dynamics. More precisely, the study of this group is motivated by potential obstructions that it could give on group actions on the plane. To get informations about this ”big mapping class group”, we can look at its action on a Gromov-hyperbolic space: the ray graph. In this talk, I will give general motivations and explain why this ray graph has infinite diameter and is Gromov-hyperbolic.

Foliations, fractals and dynamics

Andrzej Biś
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One of the most fundamental invariants of the dynamics of a continuous map $f : X \to X$ is its topological entropy which measures the complexity of f. When the entropy is positive, it reflects some chaotic behavior of the map f. There exists a corresponding notion of the topological entropy for a group or pseudogroup action. For any foliated space (M, F), the action of the holonomy pseudogroup H on the complete transversal T contains complete information about the dynamics of (M, F).

Ghys, Langevin and Walczak in the celebrated paper [4] proved that a foliation can be considered as a generalized dynamical system. A codimension one foliation with positive geometric entropy admits a resilient leaf with very complicated geometry and its exceptional minimal set is a Cantor set.

The theory of foliations of codimension greater than one starts with the paper by Thurston [5], there are many particular foliations in higher codimension studied by Molino, Epstein, Blummental and others. However for foliations of codimension greater than one there are very few results on its dynamics (see [1], [2], [3]) and much more open problems. From the dynamical point of view there are many differences between codimension one and higher codimension foliations.

Most fractals can be realized as minimal sets of codimension greater than one foliated spaces. The geometry of leaves, fractals being transversals and dynamics are interrelated. In the talk I will present old and new results related with the dynamics of codimension greater than one foliated spaces.

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Approximating C^0-foliations by contact structures

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There is fundamental relationship between foliation theory and contact topology that was discovered by Eliashberg and Thurston in the late 90's: they showed that any cooriented foliation $\mathcal{F}$ (of class C^2) on an orientable, closed 3-manifold can be approximated by contact structures. More precisely, the tangent distribution $T\mathcal{F}$ can be approximated by contact structures in the C^0-sense. Amongst other things this played a central role in Mrówka and Kronheimer's proof of the Property P conjecture, which shows that surgery on a knot K yields a homotopy sphere if and only if K is the unknot, the surgery coefficient is $\mathcal{P}(M)1$ and the trace of the surgery is S^3.

The power of their theory comes from the following corollary to their approximation theorem (which does not require any regularity of the foliation):

Corollary 1. *Let $\mathcal{F}$ be a taut coorientable foliation on a closed orientable 3-manifold. Then any sufficiently close contact structure is (universally) tight.*

The regularity assumption in Eliashberg-Thurston's result is *a priori* quite strong and many constructions (surgery, blow-up, gluing...) yield foliations that are only of class C^0, in the sense that the leaves are smooth, but the tangent distribution is only continuous. For example on rational homology spheres that are not graph manifolds it is not known in general that any taut foliations of class C^2-exist, in the case that there are taut C^0-foliations. Moreover, one cannot approximate C^0-foliations by ones of class C^2 in general, with obstructions coming from Kopell's Lemma for example.

Eliashberg and Thurston already noted in their book on Confoliations that one can approximate foliations that are smooth away from a finite collection of compact leaves and they also write

> "However it is feasible that the result holds without any assumptions about the smoothness of the foliation."

In this talk we will discuss the following generalisation of Eliashberg-Thurston result to C^0-foliations, which confirms Eliashberg and Thurston's comment above and was also independently proven by Kazez-Roberts:

Theorem 2 (Bowden, Kazez-Roberts). *Let $\mathcal{F}$ be a C^0-foliation on a closed 3-manifold that is neither the foliation by spheres on $S^2 \times S^1$ nor a non-minimal foliation by planes on T^3. Then $T\mathcal{F}$ can be C^0-approximated by positive and negative contact structures.*

Quasi–geodesic pseudo–Anosov flows in hyperbolic 3-manifolds

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We obtain a simple topological and dynamical systems condition which is necessary and sufficient for an arbitrary pseudo-Anosov flow in a closed, hyperbolic three manifold to be quasigeodesic. Quasigeodesic means that orbits are efficient in measuring length up to a bounded multiplicative distortion when lifted to the universal cover. We prove that such flows are quasigeodesic if and only if there is an upper bound, depending only on the flow, to the number of orbits which are freely homotopic to an arbitrary closed orbit of the flow.

Quasigeodesic and pseudo-Anosov flows

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We will discuss two kinds of flows on 3–manifolds: quasigeodesic and pseudo–Anosov. Quasigeodesic flows are defined by a tangent condition, that each flowline is coarsely comparable to a geodesic. In contrast, pseudo–Anosov flows are defined by a transverse condition, where the flow contracts and expands the manifold in different directions.

When the ambient manifold is hyperbolic, there is a surprising relationship between these apparently disparate classes of flows. We will show that a quasigeodesic flow on a closed hyperbolic 3-manifold has a ”coarsely contracting–expanding” transverse structure, and use this to show that every such flow has closed orbits. We will also illustrate an approach to Calegari’s conjecture, that every quasigeodesic flow can be deformed into a pseudo-Anosov flow.

The Euler class of an umbilic foliation

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The main idea of this manuscript is to compute the Euler class of a foliation $\mathcal{F}$, assuming it admits a compact and umbilic leaf. Besides the umbilicity of the leaf, the geometrical assumptions considered are the sectional curvatures of the ambient manifold restricted to the leaves of $\mathcal{F}$, and they are the key to write explicitly this class. Translating geometrical hypothesis into topological ones implies obstructions to the existence of these foliations by looking at the cohomology of the ambient manifold as well as by asking for positiveness of sectional curvatures of M along $\mathcal{F}$.

Theorem A. *Let $\mathcal{D}^{2k}$ be a distribution on a Riemannian manifold M^{2k+p} with pure curvature form. Let L be a compact umbilic submanifold of M, with dimension $2k$, and suppose the sectional curvatures of M are nonnegative along L. If $\mathcal{D}$ is tangent to L, then $\epsilon(\mathcal{D}) \neq 0$.*

In order to remove the "pure curvature form" hypothesis, we consider Milnor's proof of Hopf conjecture on dimension four,

Theorem [Milnor]. *Let M be a compact orientable Riemannian manifold of dimension 4. If its sectional curvatures always have the same sign, $\chi(M) \geqslant O$. If the sectional curvature is always positive or always negative, $\chi(M) > 0$.*

Theorem B. *Let $\mathcal{D}^4$ be a distribution on a Riemannian manifold M^{4+p}. Let L be a compact umbilic submanifold of M, with dimension 4, and suppose the sectional curvatures of M are positive along L. If $\mathcal{D}^4$ is tangent to L, then $\epsilon(\mathcal{D}) \neq 0$.*

On the other hand Alain Connes introduced the Euler characteristic $\chi(\mathcal{F}, \nu)$ for a foliation endowed with a transverse measure. The particular case where $\mathcal{F}$ is determined by a closed and global form ν of its normal distribution, which is called SL-foliation, $\chi(\mathcal{F}, \nu)$ is shown to be nonnegative, provided the sectional curvatures of the leaves of $\mathcal{F}$ always have the same sign. It reads

Theorem C. *Let $\mathcal{F}$ be a SL-foliation of dimension 4 on a closed Riemannian manifold M^{4+p}. If the sectional curvatures of the leaves always have the same sign, then $\chi(\mathcal{F}, \nu) = \int_M \epsilon(\mathcal{F}) \wedge \nu \geqslant 0$.*

Applications of characteristic classes to foliations date back to theorems of J. Milnor and J. Wood, dealing with the Euler class as an obstruction to the existence of foliations transverse to the fibers of overly twisted circle bundles over surfaces. Regarding obstructions to integrability, a theorem of R. Bott asserts that given a codimention p

distribution on the tangent bundle, a necessary condition for its integrability is the vanishing of all Pontryagin classes (associated to the normal bundle) of degrees higher than $2p$. In addition, J. Pasternack lowered the condition to degrees above p, assuming that the distribution is tangent to a Riemannian foliation.

Foliations are integrable subbundles of the tangent bundle, and although in the literature characteristic classes are constructed on the normal bundle, there are interesting consequences when they are computed on the tangent distributions themselves. In this context, geometrical and topological hypothesis on the foliations and on the ambient manifold are assumed in order to explicitly determine properties of the classes.

For example, if the foliation is totally geodesic and of odd dimension n, a theorem of A. Naveira asserts the $(n+p)$-th Pontryagin class of $\mathcal{F}$ vanishes. If the leaves are surfaces, and the normal distribution is a minimal foliation, then from F. Brito, the Euler class of $\mathcal{F}$ is different from zero when $\mathrm{Ric}(M) > 0$.

Umbilic foliations were studied from the perspective of conformal geometry by R. Langevin and P. Walczak. Their approach includes properties of local and global invariants, the question whether a Riemannian manifold admits an umbilic or a foliation with weaker conditions, such as Dupin foliations, as well as asking how far from umbilic a foliation is by defining a conformal invariant quantity. In dimension 3, they were classified in the light of transversely holomorphic fields by M. Brunella and E. Ghys.

Euclidean spheres do not admit totally geodesic nor umbilic foliations of codimension one. However, for codimention greater than one, those are far from being geometrically classified. The geometrical abundance is made explicit already in the codimension 2 case of S^3,

THEOREM [GLUCK-WARNER]. *A submanifold of $\widetilde{G}_2(\mathbb{R}) \cong S^2 \times S^2$ corresponds to a fibration of S^3 by oriented great circles if and only if it is the graph of a certain distance decreasing map $f : S^2 \to S^2$.*

Umbilic foliations of S^3 and other odd spheres S^{2k+1} are obtained by taking a smooth positive function f constant on the leaves of a totally geodesic foliation and making a conformal change of the induced metric, $\langle \cdot, \cdot \rangle \mapsto f\langle \cdot, \cdot \rangle$, or by considering small deformations of all planes which give great circle fibrations, in order to obtain affine nonlinear planes intersecting the sphere.

This work is part of I. Gonçalves' thesis. Supported by a scholarship from CNPq, number 141113/2013-8. This is a joint work with Fabiano G.B. Brito.

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Generic properties of foliations and laminations

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On the dynamics of derived from Kuperberg flows

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We consider the dynamical properties of C^∞-variations of the flow on an aperiodic Kuperberg plug $\mathbb{K}$. Our main result is that there exists a smooth 1–parameter family of plugs $\mathbb{K}_\epsilon$ for $\epsilon \in (-a, a)$ and $a < 1$, such that:

1. The plug $\mathbb{K}_0 = \mathbb{K}$ is a generic Kuperberg plug;
2. For $\epsilon < 0$, the flow in the plug $\mathbb{K}_\epsilon$ has two periodic orbits that bound an invariant cylinder, all other orbits of the flow are wandering, and the flow has topological entropy zero;
3. For $\epsilon > 0$, the flow in the plug $\mathbb{K}_\epsilon$ has positive topological entropy, and an abundance of periodic orbits.

Burnside problem on diffeomorphism groups

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Suppose G is a finitely generated group such that every element has finite order. Must G be a finite group?

This is known as the burnside problem, it was formulated around 1902 by Burnside himself and it was central in the development of group theory during the 20th century. The answer in general turned out to be negative, G might be infinite. Nonetheless, if one restricts G to be a linear group (group of matrices), the answer is positive (Schur, 1911).

The problem remains open if we assume G is a group of homeomorphisms of a surface or a manifold in general. I will talk about the case where G is a group of diffeomorphisms of a surface.

Producing compact invariant sets in Reeb flows

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Let (M, ξ) be a contact manifold. Then for each contact form α with $\operatorname{Ker}\alpha = \xi$ one can associate a unique flow, say ψ^α, on M called the Reeb flow of α. I am interested in the following question: Given (M, ξ), to what extent can we vary the dynamics of the Reeb flow by a change of α? In this talk, I exclusively consider $(\mathbb{R}^{2n+1}, \xi_{\text{std}})$, where $\xi_{\text{std}} = \operatorname{Ker}\alpha_{\text{std}}$ and $\alpha_{\text{std}} = dz + \frac{1}{2}\sum_{j=1}^{n} {r_j}^2 d\theta_j$. Remark that the Reeb flow of α_{std} is generated by $\partial/\partial z$. I want to modify it so that it contains a compact invariant set of various type. Recently, Geiges-Röttgen-Zehmisch [2014] have realized an n-dimensional torus ($n \geqslant 2$) with an irrational linear flow as an invariant set of a Reeb flow of ξ_{std}. We generalize their result as follows.

Theorem. *Let φ be any flow on T^n which is obtained by a suspension of a diffeomorphism of T^{n-1}, and let $\mathcal{A} \subset T^n$ be any compact invariant set of φ. Then, we can find an embedding $T^n \subset \mathbb{R}^{2n+1}$ and a contact form α on $\mathbb{R}^{2n+1}$ with $\operatorname{Ker}\alpha = \xi_{\text{std}}$ such that :*

(1) *$\alpha = \alpha_{\text{std}}$ outside a small neighborhood of $\mathcal{A}$.*

(2) *The Reeb flow ψ^α restricted to $\mathcal{A}$ is orbit equivalent to φ.*

(3) *All orbits of ψ^α outside $\mathcal{A}$ are unbounded.*

For instance, when $n \geqslant 2$, one can take as $\mathcal{A}$ a transversely Cantor minimal set etc. Some subsets of $\mathbb{R}^{2n+1}$ other than subsets of T^n are also realizable:

Proposition. *The generalized Hopf flows on S^{2n-1} are realizable in the sense that S^{2n-1} (instead of $\mathcal{A}$) satisfies the three properties in the above theorem. $S^{2k_1-1} \times \cdots \times S^{2k_p-1}$ ($k_1 + \cdots + k_p = n$) and $S^{2n-1} \times I^2$ are also realizable.*

Problem. What flows on what manifolds can be realized as a compact invariant sets of a Reeb flow in $(\mathbb{R}^{2n+1}, \xi_{\text{std}})$? Find many more examples, or, develop a powerful method of realization.

This is a joint work with T. Arai and Y. Kano.

Some estimates for the product of modules of foliations

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Referring to the recent results obtained by A. Romanov for pairs of planar condensers defined by opposite arcs of a curvilinear quadrilateral and then to the pair of foliations defined by the external functions of such condensers, we investigate on which conditions the product of modules of a pair of orthogonal foliations is greater, equal or less than one. We also formulate sufficient conditions that enable us to obtain similar estimates for the product of modules of more than two mutually orthogonal foliations on a Riemannian manifold of any dimension.

This is a joint work with Antoni Pierzchalski.

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A trace formula for codimension one foliations with simple foliated flows

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Let $\mathcal{F}$ be a smooth, transversely oriented, codimension one foliation on a compact smooth manifold M and ϕ a foliated flow on $(M, \mathcal{F})$. Denote by $\mathrm{Fix}(\phi)$ the fixed point set of ϕ. Let M^0 be the $\mathcal{F}$-saturation of $\mathrm{Fix}(\phi)$, and $M^1 = M \setminus M^0$. We will assume that ϕ is simple, which means that all of its fixed points and closed orbits are simple, and its orbits in M^1 are transverse to the leaves. In this case, M^0 is a finite union of compact leaves, and M^1 has finitely many connected components, denoted by M_l. One can construct a bundle-like metric g^1 on M^1 such that each M_l with respect to this metric is a manifold of bounded geometry, and the restriction $\mathcal{F}_l$ of $\mathcal{F}$ to M_l is a foliation of bounded geometry. In addition, without loss of generality, we can assume that g^1 has a particular form in a neighborhood of M^0. Denote by $d_{\mathcal{F}_l}$ and $\delta_{\mathcal{F}_l}$, respectively, the leafwise derivative and the leaf- wise coderivative, acting in $C^\infty\left(\bigwedge T\mathcal{F}_l^*\right)$, and set $D_{\mathcal{F}_l} = d_{\mathcal{F}_l} + \delta_{\mathcal{F}_l}$. Let $\mathcal{A}$ be the Fréchet algebra of functions $\psi : \mathbb{R} \to \mathbb{C}$ that can be extended to entire functions on $\mathbb{C}$ such that, for each compact subset K of $\mathbb{R}$, the set $\{x \mapsto \psi(x+iy) | y \in K\}$ is bounded in the Schwartz space $\mathcal{S}(\mathbb{R})$. For any $\psi \in \mathcal{A}$ and $f \in C_c^\infty(\mathbb{R})$, consider the operator $P : C_c^\infty\left(M^1; \bigwedge T\mathcal{F}^{1*}\right) \to C^\infty\left(M^1; \bigwedge T\mathcal{F}^{1*}\right)$, whose restriction to $C_c^\infty\left(M_l; \bigwedge T\mathcal{F}_l^*\right)$ is given by

$$P_l = \int_{-\infty}^{\infty} \phi^{t*} \psi\left(D_{\mathcal{F}_l}\right) f(t) dt.$$

One can show that the Schwartz kernel of P extends to a smooth function on $M \times M^1 \cap M^1 \times M$ and has singularity at $M^0 \times M^0$. In particular, the operator P is not of trace class in $L^2\left(M; \bigwedge T\mathcal{F}^*\right)$. To define a trace of P, we use the machinery of pseudodifferential b-calculus on manifolds with boundary developed by R. Melrose in his book on the Atiyah-Patodi-Singer theorem. For each l, M_l is the interior of a connected compact manifold M_l^c with boundary and the foliation $\mathcal{F}_l$ extends to a smooth foliation $\mathcal{F}_l^c$ on M_l^c tangent to the boundary. We prove that each P_l defines an operator of the class $\Psi_b^{-\infty}\left(M_l^c; \bigwedge T\mathcal{F}_l^{c*}\right)$ of b-pseudodifferential operators of order $-\infty$. R. Melrose constructed an extension ${}^{\mathrm{b}}\mathrm{Tr}$ of the trace functional $\Psi_b^{-\infty}\left(M_l^c; \bigwedge T\mathcal{F}_l^{c*}\right)$, called b-trace. These facts allow us to introduce the Lefschetz distribution of ϕ and study the associated trace formula. In my talk, I will report on the recent progress in this direction. This is joint work with Jesús A. Álvarez López and Eric Leichtnam.

Natural boundary value problems for weighted form Laplacians

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Gradients in the sense of Stein and Weiss are $\mathsf{O}(n)$-irreducible parts of ∇, the covariant derivative of an Riemannian manifold M of dimension n and of Riemannian metric g. For example, the bundle of differential p-forms is $\mathsf{O}(n)$-irreducible. But the target bundle of ∇ acting p-forms splits.

As a result, we obtain three $\mathsf{O}(n)$-gradients: d, δ and S. The first two are the familiar exterior derivative and coderivative. The third operator S completing the list, and defined just by the splitting, seems to be at least equally important. It is the only one of the three that has, like ∇, an injective symbol. And that means the ellipticity. Roughly speaking, we can say that S is carrying the ellipticity of ∇. S is called to be the *Ahlfors operator*.

In the particular case $p = 1$, the operator S, being the symmetric and trace free part of ∇, is one of the most important operators in conformal geometry: conformal Killing forms, or - by duality- vector fields, constitute its kernel. It is worth to notice that, in the case of $M = \mathbb{R}^2 = \mathbb{C}$, the Ahlfors' operator becomes the Cauchy-Riemann one, so S may be treated as its higher (even odd) dimensional extension.

Recall that Ahlfors studied S as an operator acting on vector fields X in $\mathbb{R}^n$:

$$SX = \frac{1}{2}(DX + DX^t) - \frac{1}{n}\mathrm{trace}\,(DX)I,$$

where $DX = (\partial X_i/\partial x_j)$, DX^t is the transpose of DX and $I = (\delta_{ij})$.

The adjoint operator is

$$(S^\star \phi)_i = \sum_{j=1}^{n} \frac{\partial}{\partial x_j}\phi_{ij}.$$

So, the resulting differential operator $S^\star S$ maps vector fields into vector fields.

In the case of an arbitrary Riemannian manifold it is more convenient to replace vector fields by their duals: one forms. $S^\star S$ may be then written in its invariant shape

$$S^\star S = \frac{n-1}{n}d\delta + \frac{1}{2}\delta d - \mathrm{Ric},$$

where Ric is the Ricci action on one-forms.

$S^\star S$ is strongly elliptic second order differential operator. In the case of Ricci flat manifold (and such is $M = \mathbb{R}^n$) it reduces to

$$L_{a,b} = a d\delta + b\delta d,$$

where a and b are positive constants. The operators of this form will be called the *weighted form Laplacains*. These operators give a subclass of the class of so called *non-minimal operators* Notice that the last formula enables getting an extension of the action of $L_{a,b}$ onto skew-symmetric forms of any degree p.

The extended operator $L_{a,b}$ is just the subject of the talk. It would seem that $L_{a,b}$ theory were just a version of that one for the Laplace-Beltrami operator $\Delta = \delta d + d\delta = L_{1,1}$. But, when $a \neq b$, this is not the case. In contrast to the situation which pertains for Δ, the symbol of $L_{a,b}$, is no longer given by the metric tensor, so the situation is more subtle.

In the dimension three a version of $L_{a,b}$ acting on vector fields in a bounded domain was investigated in the context of an elastic body by H. Weyl. In particular, the boundary problem under three different, physically motivated, boundary conditions were solved there. Ahlfors solved the Dirichlet boundary problem for $S^{\star}S$ in the n-dimensional hyperbolic ball. He used there the fact that the group of Möbius transformations of the unit ball (i.e., the group of isometries with respect to the hyperbolic metric) acts transitively. A Poisson type centre formula he derived there enabled therefore getting the value of a solution at any point of the ball. In the case of the Euclidean ball there is no such a tool. Reimann solved the Dirichlet problem for $S^{\star}S = 0$ on vector fields in this case. In analogy to the classical procedure for the Dirichlet problem for the Laplace operator, consisting in expanding the functions on the sphere into a series of spherical harmonics, he decomposed the space of vector fields into some suitably chosen $\mathsf{O}(n)$-invariant subspaces. Then he found a nice basis in each of the summands.

We are also going to adopt the Reimann's method here though his way of defining the Ahlfors operator of a higher order was passing to the space of trace-free symmetric tensors. Our way, in contrast to the Reimann one, is passing the space of skew-symmetric forms of an arbitrary degree p. The splitting onto $\mathsf{O}(n)$-invariant subspaces is then essentially different.

It seems to be interesting to find all the solutions for a complete list of some natural boundary conditions.

All conditions from the list are self-adjoint, and, in the case of elliptic gradients they constitute so called *elliptic boundary conditions* in the sense of Gilkey and Smith.

Let us describe shortly that rule. For any gradient G (one can think for a while that G is, e.g., d, δ or S but the formula is really very general) we have

$$(G^{*}G\omega_1, \omega_2) - (\omega_1, G^{*}G\omega_2) = \int_{\partial M} \left[g(\iota_\nu G\omega_1, \omega_2) - g(\omega_1, \iota_\nu G\omega_2) \right] \tag{1}$$

where $\iota_\nu G\omega$ is the contraction of $G\omega$ with the unit vector ν normal to the boundary

∂M. To make G^*G self-adjoint we have to accept boundary conditions annihilating the right hand side of (1), or stronger, annihilating each of the summands under the boundary integral. Now the unit normal ν will play its role. The original bundle we are dealing with (in our case the bundle of p-forms) is $\mathsf{O}(n)$-irreducible. But, it reduces at the boundary under the action of the subgroup $\mathsf{O}(n-1)$ of $\mathsf{O}(n)$ keeping ν invariant. As a result, by the Branching Rule, the original bundle splits at the boundary onto, say s, $\mathsf{O}(n-1)$-invariant subbundles. Denote by $\pi^1, \ldots, \pi^s$ the projections defined by the splitting. Then, by the orthogonality, $g(\omega_1, \iota_\nu G\omega_2)$ is equal to the sum

$$g(\pi^1\omega_1, \pi^1\iota_\nu G\omega_2) + \ldots + g(\pi^s\omega_1, \pi^s\iota_\nu G\omega_2).$$

Now, there are 2^s candidates for elliptic boundary conditions, constructed as follows: For each $b = 1, \ldots, s$, we choose *exactly* one of $\pi^b\omega_1$ and $\pi^b\iota_\nu G\omega_2$ and require it to vanish. For example, if we require to vanish the first multiplier in each summand we get the Dirichlet condition; if we require to vanish the other one we get the Neumann one. By other choices we get the whole their variety. The boundary conditions obtained that way seems to be in some sense "basic", at least from the point of view of the representation theory. Of course, we realize that the list may not contain some other geometrically or physically important conditions like Robin one etc. Of course, for different purposes or for some physical applications, we may always perturb by lower order operators. When we do so, we need to worry about losing the symmetry condition for the boundary integrand in (1), i.e., about loosing the self-adjointness. These perturbations will possibly take the form of order 0 operators, added either to the interior operator G^*G, or to the boundary operator $\omega \mapsto \iota_\nu G\omega$.

According to Branson and Pierzchalski, there are four such conditions on the list in the case of $\mathsf{O}(n)$-gradients acting on the space of differential forms of any degree on a Riemannian manifold M with an nonempty boundary ∂M:

Dirichlet boundary condition ($\mathcal{D}$):

$$\omega^{\mathrm{T}} = 0 \quad \text{and} \quad \omega^{\mathrm{N}} = 0 \quad \text{on} \quad \partial M.$$

Absolute Boundary condition ($\mathcal{A}$):

$$\omega^{\mathrm{N}} = 0 \quad \text{and} \quad (d\omega)^{\mathrm{N}} = 0 \quad \text{on} \quad \partial M.$$

Relative boundary condition ($\mathcal{R}$):

$$(\delta\omega)^{\mathrm{T}} = 0 \quad \text{and} \quad \omega^{\mathrm{T}} = 0 \quad \text{on} \quad \partial M.$$

The fourth boundary condition ($\mathcal{B}$):

$$(\delta\omega)^{\mathrm{T}} = 0 \quad \text{and} \quad (d\omega)^{\mathrm{N}} = 0 \quad \text{on} \quad \partial M.$$

Here ω^{T} and ω^{N} denote the tangent and the normal parts of ω at the boundary, respectively. The first three conditions are known to geometers. The fourth one seems to be unknown. But, being natural, it should have a geometric or physical meaning.

Observe also a surprising symmetry with respect to the Hodge star operator $\star$. Namely, by the following known relations:

$$\star\star = \mathcal{P}(M)1, \quad (\star\omega)^{\mathrm{T}} = \mathcal{P}(M)\star(\omega^{\mathrm{N}}), \quad (\star\omega)^{\mathrm{N}} = \mathcal{P}(M)\star(\omega^{\mathrm{T}})$$

and

$$\delta\omega = \mathcal{P}(M)\star d\star\omega, \quad d\omega = \mathcal{P}(M)\star\delta\star\omega$$

it follows easily that the set of all the four boundary conditions $\{\mathcal{D}, \mathcal{A}, \mathcal{R}, \mathcal{B}\}$ is star-invariant. More precisely, each of the conditions $\mathcal{D}, \mathcal{B}$ is star-invariant, while the conditions $\mathcal{A}$ and $\mathcal{R}$ are star-symmetric each to the other.

We are going to solve all the four boundary problems $\mathcal{D}, \mathcal{A}, \mathcal{R}, \mathcal{B}$ for the operators $L_{a,b}$ acting onto differential forms of arbitrary degree p in the Euclidean unit ball in $\mathbb{R}^n$.

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Entropy, a "functional" viewpoint

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Entropy (with respect to a probability measure μ) is associated to a transformation preserving the measure μ, topological entropy to a continuous map of a compact space X into itself. Using a metric defined on a compact foliated space, one can also define the entropy of a foliation. In the conference I'll recall some definitions and examples and present an attempt (work in progress) to obtain a common definition using ϵ-separated sets of the intersection of finite dimensional subspaces with unit balls of some spaces of functions. A by-product will be a definition of the entropy of a foliation with respect to a measure.

Invariants for equicontinuous group actions on Cantor sets

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Equicontinuous group actions on compact metric spaces have appeared, in various contexts, in many areas of dynamics and foliation theory. For example, the transverse dynamics of a Riemannian foliation is equicontinuous. In continuum theory, actions on fibres of generalized solenoids provide examples of equicontinuous group actions on Cantor sets with many counterintuitive properties. Group actions on topological spaces and, in particular, their enveloping (Ellis) semigroup, have long been a topic of interest in topological dynamics.

In this talk, we concentrate on equicontinuous group actions on Cantor sets. We obtain a 'coordinate representation' of the Ellis group, associated to such action, and apply it to the study of geometric and dynamical properties of foliated spaces with totally disconnected transversals. In particular, for certain classes of such foliated spaces we obtain results relating the Ellis semigroup, the growth properties of the leaves, and the automorphism group of the transverse dynamical system.

Based on recent results joined with Clark and Fokkink, Dyer and Hurder.

Group orders, dynamics and rigidity

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1 Introduction

Let G be a group. A *left-order* on G is a total order invariant under left multiplication, i.e. such that $a < b$ implies $ga < gb$ for all $a, b, g \in G$. It is well known that a countable group is left-orderable if and only if it embeds into the group of orientation-preserving homeomorphisms of $\mathbb{R}$, and each left-order on a group defines a canonical embedding up to conjugacy, called the *dynamical realization*. For this reason, left-orders appear in the study of dynamics and foliations.

A *circular order* on G is defined by a *cyclic orientation cocycle* $c : G^3 \to \{\mathcal{P}(M)1, 0\}$ satisfying the following conditions:

i) (non degeneracy) $c(g_1, g_2, g_3) = 0$ if and only if $g_i = g_j$ for some $i \neq j$

ii) (cocylce condition)
$c(g_2, g_3, g_4) - c(g_1, g_3, g_4) + c(g_1, g_2, g_4) - c(g_1, g_2, g_3) = 0$ for all $g_1, g_2, g_3, g_4 \in G$.

iii) (left invariance) $c(g_1, g_2, g_3) = c(hg_1, hg_2, hg_3)$ for all $g_i, h \in G$.

For countable groups, there is also a *dynamical realization* associating to each circular order a canonical conjugacy class of embedding $G \to \mathrm{Homeo}_+(S^1)$. This correspondence is the starting point for a rich relationship between the algebraic constraints on G imposed by orders, and the dynamical constraints on G–actions on S^1 or $\mathbb{R}$.

Spaces of orders and actions. For fixed G, we let $\mathrm{LO}(G)$ denote the set of all left-orders on G, and $\mathrm{CO}(G)$ the set of circular orders. These spaces have a natural topology; that on $\mathrm{CO}(G)$ comes from its identification with a subset of the infinite product $\{\mathcal{P}(M)1, 0\}^{G\times G\times G}$. Left orders are a special case of circular orders (degenerate cocycles) so $\mathrm{LO}(G) \subset \mathrm{CO}(G)$ can be given the subspace topology. This agrees with the topology previously studied by Sikora [5] and others. For this reason we focus primarily on circular orders here, treating left orders as a special case.

Due to the relationship, via dynamical realization, between circular orders on a countable group G and actions of G on S^1, it is natural to ask about the relationship between the two spaces $\mathrm{CO}(G)$ and $\mathrm{Hom}(G, \mathrm{Homeo}_+(S^1))$, where $\mathrm{Hom}(G, \mathrm{Homeo}_+(S^1))$ is the space of actions of G on S^1, with the compact open topology. Similarly, there should be some relationship between $\mathrm{LO}(G)$ and $\mathrm{Hom}(G, \mathrm{Homeo}_+(\mathbb{R}))$. In particular, one hopes to use the topology of $\mathrm{CO}(G)$ to study that of $\mathrm{Hom}(G, \mathrm{Homeo}_+(S^1))$, and vice versa.

Following Sikora [5], we have that $\mathrm{LO}(G)$ and $\mathrm{CO}(G)$ are both compact, totally disconnected and, for countable groups G, metrizable. Consequently, if $\mathrm{LO}(G)$ or $\mathrm{CO}(G)$ has no isolated points, then it is homeomorphic to a cantor set, and an important first question is thus to identify its isolated points. Even this can be highly nontrivial. We propose an approach by studying the realizations of isolated points in $\mathrm{Hom}(G, \mathrm{Homeo}_+(S^1))$.

2 Main results

Dynamical realization gives a map $\mathrm{CO}(G) \to \mathrm{Hom}(G, \mathrm{Homeo}_+(S^1))/\sim$, where $\sim$ is the conjugacy relation. One can also define a partial inverse to this map. To what extent are these spaces related? As a first guess, one might (naively) propose the following.

Naive conjecture 2.1 *Let G be a countable group. $\mathrm{CO}(G)$ has no isolated points if (or perhaps if and only if) $\mathrm{Hom}(G, \mathrm{Homeo}_+(S^1))$ is connected.*

A supportive example is the case $G = \mathbb{Z}^2$. It is not difficult to show both that $\mathrm{Hom}(\mathbb{Z}^2, \mathrm{Homeo}_+(S^1))$ is connected and that $\mathrm{CO}(\mathbb{Z}^2)$ has no isolated points. In the case of the free group F_2 on two generators,
$\mathrm{Hom}(F_2, \mathrm{Homeo}_+(S^1)) \cong \mathrm{Homeo}_+(S^1) \times \mathrm{Homeo}_+(S^1)$, which is also connected. This may have motivated the following conjecture stated in [1].

Conjecture 2.2. [1] . *$\mathrm{CO}(F_2)$ has no isolated points.*

It was also shown by Rivas in [4] that $\mathrm{LO}(F_2)$ has no isolated points, giving further evidence. However, we prove the following.

Theorem 2.3. [2] . *$\mathrm{CO}(F_2)$ has has infinitely many isolated points. In fact, for any $n \geqslant 1$, the space $\mathrm{CO}(F_{2n})$ has infinitely many distinct classes of isolated points under the natural conjugation action of F_{2n} on $\mathrm{CO}(F_{2n})$.*

This answers a question of [3] in the negative. The construction of the isolated orders in Theorem 2.3 is explicit and elementary, especially when described via their dynamical realizations – these are geometrically motivated “ping-pong” actions. The difficulty is in showing that these orders are indeed isolated points.

We remark that, since dynamical realizations are faithful, one might try to improve naive conjecture 2.1 by restricting to the subspace of *faithful* actions of a group on S^1. However, it is possible to show that the subset of faithful representations in $\mathrm{Hom}(F_2, \mathrm{Homeo}_+(S^1))$ is also connected. In fact, the relationship between $\mathrm{Hom}(G, \mathrm{Homeo}_+(S^1))$ and $\mathrm{CO}(G)$ is much more subtle. The aim of our work in [2] is to bring this relationship to light.

Dynamical characterization of isolated points. Our main theorem is a complete characterization of isolated points in $\mathrm{CO}(G)$ in terms of the dynamics of their dynamical realization.

Theorem 2.4. [2]. *Let G be a countable group. A circular order on G is isolated if and only if its dynamical realization ρ is* rigid *in the following strong sense: for every action ρ' sufficiently close to ρ in $\mathrm{Hom}(G, \mathrm{Homeo}_+(S^1))$ there exists a continuous, degree 1 monotone map $h : S^1 \to S^1$ fixing the* basepoint $x_0 \in S^1$ *of the dynamical realization, and such that $h \circ \rho'(g) = \rho(g) \circ h$ for all $g \in G$.*
There is an analogous statement for left orders and rigid actions on $\mathbb{R}$.

Theorem 2.4 is the main ingredient in the proof of Theorem 2.3, indeed, the isolated orders on free groups can be seen as an application. A major tool in the proof of Theorem 2.4 is an extension of work of Navas [3] on "maximal minimality" of dynamical realizations of left orders.

Further applications. We also give a detailed description of which faithful actions of a countable group G on S^1 can arise as dynamical realizations, leading to a new notion of the *linear part* of a circular order – a maximal, convex subgroup.

One can also move from isolated circular orders to isolated linear orders via central extensions by $\mathbb{Z}$. As a particular example, lifts of the rigid actions of F_2 on S^1 to actions on the real line give isolated left-orderings on the central extension $\mathbb{Z} \times F_2$, obtaining

Corollary 2.5. *The pure braid group $P_3 \cong F_2 \times \mathbb{Z}$ has infinitely many distinct conjugacy classes of isolated left-orders.*

3 Further questions

1. Give examples of other groups with isolated circular orders.
2. For $n > 1$ odd, do there exist (infinitely many?) isolated circular orders on F_n?
3. Are the examples given in the proof of Theorem 2.3 the only isolated circular orders on F_{2n}?

This is joint work with Cristóbal Rivas (Universidad de Santiago de Chile).

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4. C. Rivas, *Left-orderings on free products of groups*. Journal of Algebra **350** (2012), 318-329.
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Parameter rigidity of the action of AN on $\Gamma\backslash G$ for higher rank semisimple Lie groups

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Let $M \overset{\rho_0}{\curvearrowleft} S$ be a C^∞ locally free right action of a connected simply connected solvable Lie group S on a closed C^∞ manifold M. Recall that ρ_0 is said to be locally free if the isotropy subgroup of any point of M is a discrete subgroup of S. The set $\mathcal{F}$ of all the orbits of ρ_0 is a C^∞ foliation of M, which is called the orbit foliation of ρ_0. We say ρ_0 is *parameter rigid* if any C^∞ locally free action $M \overset{\rho}{\curvearrowleft} S$ whose orbit foliation coincides with $\mathcal{F}$ is *parameter equivalent* to ρ_0, that is, there exist an automorphism Φ of S and a diffeomorphism F of M such that $F(\rho_0(x,s)) = \rho(F(x), \Phi(s))$ for all $x \in M$ and $s \in S$ and F preserves each leaf of $\mathcal{F}$ and is C^0 homotopic to the identity map of M through C^∞ maps preserving each leaf.

For example a linear flow on a torus is parameter rigid if and only if the velocity vector of the flow at a point satisfies the Diophantine condition.

In two papers published in 1994, A. Katok and R. J. Spatzier proved the following theorem.

Theorem 1. *Let G be a connected semisimple Lie group with finite center of real rank at least 2 without compact factors or simple factors locally isomorphic to $\mathrm{SO}_0(n,1)$ $(n \geqslant 2)$ or $\mathrm{SU}(n,1)$ $(n \geqslant 2)$ and Γ be an irreducible cocompact lattice in G. Let $G = KAN$ be an Iwasawa decomposition. Then the action $\Gamma\backslash G \curvearrowleft A$ by right multiplication is parameter rigid.*

Recently I proved the following.

Theorem 2. *Let G be a connected semisimple Lie group with finite center of real rank at least 2 without compact factors or simple factors locally isomorphic to $\mathrm{SO}_0(n,1)$ $(n \geqslant 2)$ or $\mathrm{SU}(n,1)$ $(n \geqslant 2)$ and Γ be an irreducible cocompact lattice in G. Let $G = KAN$ be an Iwasawa decomposition. Then the action $\Gamma\backslash G \curvearrowleft AN$ by right multiplication is parameter rigid.*

The major difference in the proof comes from the noncommutativity of AN. I will explain how to prove Theorem 2 in the talk.

The proof is basically a combination of a sufficient condition for parameter rigidity of Maruhashi, cohomology vanishing results of Katok–Spatzier and Kanai and rigidity theorems of quasiisometries of symmetric spaces of Pansu, Kleiner–Leeb, Farb–Mosher and Reiter Ahlin.

Rotation number and lifts of a Fuchsian action on the circle

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TBA

Dynamics of the geodesic and horocycle flows for laminations by hyperbolic surfaces

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Most of the results reported here is taken from a joint work **[1]** with Matilde Martinez and Alberto Verjovsky.

Throughout the talk, $(M, \mathcal{F})$ is to be a *minimal* lamination by hyperbolic surfaces on a compact metrizable space M. Let $\hat{M}$ be the leafwise unit tangent bundle of M: $\hat{M} = \cup_{x \in M} T^1_x L_x$, where L_x is the $\mathcal{F}$-leaf at x. Then $\hat{M}$ has a 3-dimensional lamination $T^1\mathcal{F}$ obatined by the decomposition $\hat{M} = \cup_{L \in \mathcal{F}} T^1 L$. Since $\mathcal{F}$ is minimal, $T^1\mathcal{F}$ is also minimal. In fact, $T^1\mathcal{F}$ is the orbit foliation of a right $PSL(2, \mathbb{R})$ action. Let D, U and B be the subgroups of $PSL(2, \mathbb{R})$ consisiting, respectively, of diagonal, unipotent upper triangular, and upper triangular matrices. The flow of D (U) is called leafwise geodesic (horocycle) flow. We discuss dynamical properties of these flows as well as the B-actions. Let $\Pi : \hat{M} \to M$ be the canonical projection, X a closed B-invariant subset of $\hat{M}$, and μ any ergodic harmonic measure of $\mathcal{F}$ (a probability measure on M). Denote $M_{X,1} = \{x \in M \mid \sharp(\Pi^{-1}(x) \cap X) = 1\}$ and $M_{X,>1} = M \setminus M_{X,1}$. By the ergodicity either $\mu(M_{X,1}) = 1$ or $\mu(M_{X,>1}) = 1$. By an argument using the leafwise Brownian motion, suggested by É. Ghys, we get:

Lemma 1. If $\mu(M_{X,>1}) = 1$, then $X = \hat{M}$

Corollary 2. (1) There is a unique minimal set for the B-action. (2) There is a dense orbit for the B-action.

Theorem 3. The following (1) and (2) are equivalent. (1) There is a closed B-invariant subset X for which $M_{X,1} = M$. (2) The lamination $\mathcal{F}$ is the orbit foliation of a continuous locally free B-action on M.

It is not the case that $\mu(M_{X,1}) = 1$ implies $M_{X,1} = M$. An example is constructed on a 4-manifold M, using the Thurston hyperbolization of the mapping torus of a pseudo-Anosov homeomorphism.

Theorem 4. The B-action is minimal under (1) or (2): (1) $\mathcal{F}$ admits a holonomy invariant transverse measure. (2) $(M, \mathcal{F})$ is the foliated Z bundle over a closed surface $\Gamma \setminus \mathbb{H}^2$ given by a minimal and indisrete homomorphism $\phi : \Gamma \to \mathrm{Homeo(Z)}$ for some compact metrizable space Z. Here indiscrete means that there are elements $g_n \in \Gamma \setminus \{e\}$ such that $\phi(g_n) \to \mathrm{id}_Z$.

Theorem 5 ([2]). The U-flow is minimal under (1) or (2): (1) $\mathcal{F}$ is a Riemannian foliation which admits a nonplanar leaf. (2) $\mathcal{F}$ is a codimension one foliation and the B-action on $\hat{M}$ is minimal.

Theorem 6. Assume M is a closed manifold. The D-flow (leafwise geodesic flow) is structurally stable in the sense that any $T^1\mathcal{F}$-leaf preserving leafwise C^1-perturbation is topologically equivalent to the D-flow by a $T^1\mathcal{F}$-leaf preserving homeomorphisms.

1. M. Matilde, S. Matsumoto & A. Verjovsky, *Horocycle flows for laminations by hyperbolic Riemann surfaces and Hedlund's thoerem*, J. Modern Dynamics, **10** (2016) 113-134.
2. S. Matsumoto, *Remarks on the horocycle flows for foliations by hyperbolic surfaces*, To appear in Proc. A. M. S.

Distortion and Tits alternative for big mapping class groups

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The setting of this talk is the study of actions of finitely generated groups on surfaces. When such an action has a Cantor set as minimal invariant set, we are naturally led to study groups of mapping classes which preserve a Cantor set. I will introduce some results of a joint work with Sebastian Hurtado on these big mapping class groups.

REEB COMPONENTS WITH COMPLEX LEAVES AND THEIR SYMMETRIES

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Introduction and rough statement of results

We compute the automorphism group of a Reeb component with complex leaves, assuming that the leaves are of complex dimension 1 and the Reeb component is given by the Hopf construction. After the computation, if the situation allows us, we discuss further on the realization of such Reeb components in a Levi flat hypersurface in a complex surface and also on the extendability of automorphisms to those of ambient complex surfaces.

For higher dimensional ones, basically we can compute them to a large degree, once we know the automorphism group of the boundary leaf. *e.g.*, Horiuchi computed them for complex leaf dimension 2. (He completed the computation when the holonomy is infinitely tangent to the identity, while in other cases the description becomes quite complicated,but it is possible.) In this talk the boundary leaf is an elliptic curve, so that we know its automorpshim group well.

Here an *automorpshism* means a smooth, foliation preserving diffeomorphism which is holomorphic between leaves.

The result shows different features depending on the character of the holonomy of the boundary leaf. We assume that the holonomy diffeomorphism $\varphi \in \mathit{Diff}^{\infty}([0,\infty))$ of the boundary leaf (*i.e.*, a generator of the holonomy group) is expanding at $x = 0$. We have the following three cases.

Case (1) : The linear part of the holonomy is non-trivial.

Case (2) : The linear part is trivial but the infinite jet is non-trivial.

Case (3) : The holonomy is expanding but is flat to the identity.

THEOREM 1 The automorphism group $AutR$ of a Reeb component R with complex leaves is

Case (1) : a 3 dimensional or 5 dimensional solvable Lie group,

Case (2) : an ∞-dimensional solvable Lie group,

Case (3) : an ∞-dimensional solvable Lie group or slightly more complicated depending on the centralizer $Z_\varphi = \{\exp(tX) \,|\, t \in \mathbb{R}\}$ in $\mathit{Diff}^{\infty}([0,\infty))$ for some smooth vector field X or not.

1 Structure of the group of automorphisms

Let $H = H_\lambda = \mathbb{C}^*/\lambda^{\mathbb{Z}}$ ($\lambda \in \mathbb{C}^*$, $|\lambda| > 1$) denote the boundary elliptic curve of the Reeb component R. We easily see the following.

PROPOSITION 2 $AutR$ admits the following decompositions into extensions.

$$0 \to Aut(R, H) \to AutR \to Aut_0 H \to 0$$

and

$$0 \to \mathcal{K}_{\lambda,\varphi} \to Aut_0 H \to Z_\varphi \to 0$$

where $Aut_0 H$ denotes the identity component $\cong H \cong T^2$.

PROPOSITION 3 In Case (1), thanks to Sternberg's linearization, and in Case (2), thanks to Takens' normal form, the Szekeres vector field for φ is smooth, and the centralizer Z_φ coincides with $\{\exp(tX) \,|\, t \in \mathbb{R}\} \cong \mathbb{R}$.

In Case (3) in general we only have $\varphi^{\mathbb{Z}} \subset Z_\varphi \subset \{\exp(tX) \,|\, t \in \mathbb{R}\}$ where X is the Szekerez vector field which is guranteed only of C^1 at the origin.

In the first sequence $AutR \to Aut_0 H$ is obtained by the restriction to the boundary and $Aut(R, H)$ is defined to be the kernel. Then $Aut_0 H \to Z_\varphi \subset \mathit{Diff}^\infty([0, \infty))$ is looking at the action on the transverse space. The kernel $\mathcal{K}_{\lambda,\varphi}$ is a translation inside each leaf in the interior, where all leaves are biholomorphic to the cmplex plane.

As a consequence the problem of determining the automorphism group is reduced to the computation of $\mathcal{K}_{\lambda,\varphi}$.

2 Schröder's equation on $[0, \infty)$

The computation of the kernel $\mathcal{K}_{\lambda,\varphi}$ is nothing but solving the following Schröder type functional equation on the half line $[0, \infty)$.

$$\text{(I)} \qquad \beta \circ \varphi = \lambda\beta \qquad \beta \in C^\infty([0, \infty); \mathbb{C}).$$

If we consider this equation on the open half line $(0, \infty)$, we easily see that the solution space $\mathcal{Z}_{\lambda,\varphi} \subset C^\infty((0, \infty); \mathbb{C})$ is isomorphic to $C^\infty(S^1; \mathbb{C})$.

Main Theorem

The space $\mathcal{K}_{\lambda,\varphi}$ of solutions to the equation (I) is as follows.

Case (1) : $\mathcal{K}_{\lambda,\varphi} = \{cx^p \,;\, c \in \mathbb{C}\} \cong \mathbb{C}$ if $\lambda = \mu^p$ and $p \in \mathbb{N}$ where $\mu = \varphi'(0)$,
and otherwise $\mathcal{K}_{\lambda,\varphi} = 0$.

Case (2) and (3) : $\mathcal{K}_{\lambda,\varphi} \cong \mathcal{Z}_{\lambda,\varphi}$, namely any $\beta \in \mathcal{Z}_{\lambda,\varphi}$ extends to $[0, \infty)$ by $\beta(0) = 0$ as a smooth function and is flat at $x = 0$.

About the proof of Main Theorem:

For Case (I), by Sternberg's linearization [4], it is nothing but to look for weighted homogeneous functions and the resuts is well-known.

For Case (2) and (3) two ways are possible. One allows us to prove two cases together. This unified treatment relies on the center manifold theorem or C^r-section theorem (cf. [3]).

Case (2) is also proven by relying on Takens' normal form [5] and Fourier expansion/series. In this prove, after taking the normal form we compute the solution quite explicitely for natural ODE's related to the functional equation (I). A functional equation is decomposed into a family of ODE's and then the the solutions are brought together into those of (I) by Fourier series.

This method does not work for Case (3), to which we can give antoher proof, which is not applicable to Case (2) in turn. We take higher order derivatives of the equation (I) and estimate the results in somewhat smart way. This enables us to verify the convergence of any higher order derivatives of $\beta \in \mathcal{Z}_{\lambda,\varphi}$ to 0 when $x \to 0+0$.

3 Applications and discussions

Pasting two Reeb components of Case (3) a Reeb foliation on S^3 is constructed. Of course this method is generalized to construct foliations with complex leaves on lens spaces. In thses cases the boundary is common to two Reeb components R_1 and R_2.

THEOREM 4 In the above cases, the automorphism group of the resultant foliation on a lens space or S^3 is the fibre product of $AutR_1$ and $AutR_2$ over Aut_0H.

If we start from a codimension one foliation with complex leaves, we can perform a usual *turbulization* and get a new Reeb component R. For this modification, any of Case (1), (2), or (3) is possible for R.

THEOREM 5 For a tubulization wiht the new Reeb component R of Case (2) or (3), any of manifold as the identity outside R, namely, the automorphism group of the resultant foliation includes $AutR$.

If we construct a *Hopf surface* W by $(\mathbb{C}^2 \setminus \{O\})/T^{\mathbb{Z}}$ where $T(z,w) = (\lambda \cdot z, \mu \cdot w)$ with $\lambda \in \mathbb{C}^*$ and $|\lambda|, \mu > 1$, the real hypersurface $M^3 = (\mathbb{C} \times \mathbb{R} \setminus \{O\})/T^{\mathbb{Z}}$ is *Levi-flat* and composed of two Reeb components $R_{\mathcal{P}}(M)$ with Levi-foliations.

THEOREM 6 Any element of $AutR_+$ or of $AutR_-$ extends to the ambient Hopf surface W as a holomorphic automorphism.

Discussions

Theorem 1 and Theorem 6 shows that Reeb components of Case (1) exhibits a character simlar to compact complex manifolds.

Still it should be confirmed whether if the automorphism extends to the ambient surface in the case where the Reeb component appears as a part of a Levi-flat hypersurface which bounds a Stein surface.

On the other hand, some recent works relying on the Ueda theory suggests that Reeb components of Case (3) can not be realized in Levi-flat real hypersurfaces.

Our results mildly seduces us to imagine that the same might apply to Case (2).

Similar results on 5-dimensional Reeb components with complex 2-dimensional leaves are obtained by T. Horiuchi in [2]. There a complete computaion of the automorphism groups of all Hopf surfaces is done base on Kodaira's classification. Combined with a slight extension of Main Theorem, up to dimension 5 we can compute the automorphism groups of Reeb components.

A detailed exposition of this talk is found in [1]. The author was partially supported by Grant-in-Aid for Scientific Research (B) No. 22340015. This is a report on a joint work with Horiuchi Tomohiro.

1. T. Horiuchi & Y. Mitsumatsu, *Reeb components with complex leaves and their symmetries I. The automorphism groups and Schröder's equation on the half line.* Preprint, arXiv:1605.08977.
2. T. Horiuchi, *Reeb components of leafwise complex foliations and their symmetries II.* arXiv:1511.03424, to appear in Hokkaido Math. Journal.
3. M. Shub, *Global stability of dynamical systems.* Springer-Verlag, New York, 1987.
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5. F. Takens, *Normal forms for certain singularities of vectorfields.* Ann. Inst. Fourier, **23**-2, (1973), 163 – 195.

Frame bundle approach to generalized minimal submanifolds

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We extend the notion of r–minimality of a submanifold in arbitrary codimension to u–minimality for a multi–index $u \in \mathbb{N}^q$, where q is the codimension. This approach is based on the analysis on the frame bundle of orthonormal frames of the normal bundle to a submanifold and vector bundles associated with this bundle. The notion of u–minimality comes from the variation of σ_u–symmetric function obtained from the family of shape operators corresponding to all possible bases of the normal bundle. We obtain the variation field, which gives alternative definition of u–minimality. Finally, we give some examples of u–minimal submanifolds for some choices of u and state some relations between generalized symmetric functions σ_u.

More precisely, let M be a Riemannian manifold and L a codimension q submanifold. For a fixed orthonormal basis $e = (e_1, \ldots, e_q)$ in the normal bundle to L in M let $\mathbf{A}(e) = (A_1, \ldots, A_q)$ be family of shape operators, $A_\alpha = A^{e_\alpha}$. We may associate with the family $\mathbf{A}(e)$ symmetric functions σ_u, where $u \in \mathbb{N}^q$, as follows

$$\det(I + t_1 A_1 + \ldots, t_q A_q) = \sum_u \sigma_u t_1^{u_1} \ldots t_q^{u_q}.$$

Here σ_u depends on the choice of the basis e. Integrating over all such bases (with respect to the natural measure on the orthonormal frame bundle $O(T^\perp L)$) we get, so called, generalized symmetric functions $\hat{\sigma_u} : L \to \mathbb{R}$. Critical points of the variation of $\hat{\sigma}_u$ are called u–minimal submanifolds.

We derive the formula for the variation field, which gives alternative definition of u–minimality. For $u = (0, \ldots, 0)$ we get the notion of classical minimality. We show existence of u–minimal submanifolds for $u = (0, \ldots, 0, 2, 0, \ldots, 0)$.

The talk is based on the article [**2**] and we heavily rely on the results concerning generalized Newton transformation, obtained in the article [**1**].

1. K. Andrzejewski, W. Kozłowski, K. Niedziałomski, *Generalized Newton transformation and its applications to extrinsic geometry*, Asian J. Math. 20 (2016), No. 2, 293–322.
2. K. Niedziałomski, *Frame bundle appriach to generalized minimal submanifolds*, arXiv, http://arxiv.org/abs/1601.02248

Independent variation of secondary characteristic classes of Riemannian foliations

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Secondary characteristic classes of Riemannian foliations with framed normal bundle were introduced by Lazarov-Pasternack [4]. It is a generalization of Chern-Simons invariants of framed Riemannian manifolds. Hurder [2] showed that all variable classes of Lazarov-Pasternack vary independently based on a partial result due to Lazarov-Pasternack [5]. Independent variation implies that the integral homology of the classifying space $FR\Gamma_q$ of codimension q Riemannian foliations with framed normal bundle surjects onto certain real vector space. Later Morita [7] discovered new classes in terms of canonical Cartan connections, and showed that these new classes vary independently. The main result of this talk is the following.

THEOREM 1. *All derivable classes of Lazarov-Pasternack and Morita are independently derivable.*

This generalizes results of Hurder and Morita mentioned above. We also show that $\pi_{q+1}(FR\Gamma_q)$ surjects to a real vector space and the universal homomorphism to $H^\bullet(FR\Gamma_q;\mathbb{R})$ is injective (Theorems 2 and 3) below are transversely Kähler analog of these results).

We prove the above results on Riemannian foliations by its transversely Kähler analog. Secondary characteristic classes of transversely Kähler foliations were introduced by Matsuoka-Morita [6]. Consider a differential graded algebra

$$KW_n = \bigwedge(u_1,\dots,u_n)\otimes\left(\mathbb{R}[s_1,\dots,s_n,\Phi]/\operatorname{Span}\{s_J\Phi^k \mid \deg J + k > n\}\right),$$

where

- $\deg\Phi = 2$, $\deg s_i = 2i$, $\deg u_i = 2i-1$, $d\Phi = 0$, $ds_i = 0$ and $du_i = s_i$.
- Φ corresponds to the basic Kähler class, s_i corresponds to the trace of the i-th power of the curvature of the normal bundle and u_i is the transgression of s_i.

For a manifold M with complex codimension n transversely Kähler foliation $\mathcal{F}$ with framed normal bundle, Matsuoka-Morita's construction yields a characteristic homomorphism

$$\Delta_{\mathcal{F}} : H^\bullet(KW_n) \longrightarrow H^\bullet(M;\mathbb{R}).$$

A class $\alpha \in H^\bullet(KW_n)$ is called ***rigid*** if for any manifold M with a one parameter family $\{\mathcal{F}_t\}$ of transversely Kähler foliations, the class $\Delta_{\mathcal{F}_t}(\alpha)$ is constant with respect to t. Otherwise, α is called *derivable*. For $k > 0$, it is easy to see that a class of the form $[u_I s_J \Phi^k]$ is derivable by the dilatation of the Kähler form. By Heitsch's formula, Morita-Matsuoka proved that $[u_I s_J]$ is rigid if $\min I + \deg J > n + 1$. Then the space of secondary classes which are potentially derivable are spanned by the following

1. $[u_I s_J \Phi^k]$ such that $\min I + \deg J + k = n + 1$.
2. $[u_I s_J \Phi^k]$ such that $\min I + \deg J \geqslant n + 1$ and $k > 0$.

The independent variation of the classes of the form (2) is proved by computing the characteristic classes of the simple foliation on the unitary frame bundle over certain union of products of complex tori and projective spaces considered by Hurder [3]. We show the independent variation of the classes of the form (1) by computing the characteristic classes of the pull back to the unitary normal frame bundle of linear deformations of the Hopf fibration $S^{2n+1} \to \mathbb{C}P^n$ by using the X-connections of Baum-Bott [1]. We show the independent variation of all derivable classes in KW_n by combining these two computations. Let $FK\Gamma_n$ be the the classifying space of complex codimension n transversely Kähler foliations with framed normal bundle. As a consequence of this computation, we have that

THEOREM 2. *There exists a surjective homomorphism*

$$\pi_{2n+1}(FK\Gamma_n) \longrightarrow \mathbb{R}^{v(n)},$$

where $v(n)$ is the dimension of the vector space generated by derivable classes of KW_n of degree $2n+1$.

THEOREM 3. *The canonical characteristic homomorphism $H^\bullet(KW_n) \longrightarrow H^\bullet(FK\Gamma_n; \mathbb{R})$ is injective.*

1. P. BAUM & R. BOTT, *Singularities of holomorphic foliations*, J. Differential Geom. **7**, no. 3-4 (1972), 279–342.
2. S. HURDER, *On the secondary classes of foliations with trivial normal bundles*, Comment. Math. Helv. **56** (1981), no. 2, 307–326.
3. S. HURDER, *Characteristic classes for Riemannian foliations*, Differential geometry, pp. 11–35, World Sci. Publ., Hackensack, NJ, 2009
4. C. LAZAROV & J. PASTERNACK, *Secondary characteristic classes for Riemannian foliations*, J. Differential Geom. **11** (1976), 365–385.
5. C. LAZAROV & J. PASTERNACK, *Residues and characteristic classes for Riemannian foliations*, J. Differential Geom. **11** (1976), 599–612.
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7. S. MORITA, *On characteristic classes of Riemannian foliations*, Osaka J. Math. *16*, no. 1 (1979), 161–172.

On $\mathcal{A}$-focal entropy points

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In my talk I will consider points focusing entropy and such that this fact is influenced exclusively by the behaviour of the function around these points (i.e. it is independent from the form of the function at any distance from these points).

Let $X = I^m$ ($I = [0,1]$ and $m = 1, 2, \ldots$) and let $\mathcal{A}$ be the family of all arcs in X. By $\vartheta_{\mathcal{A}}^{Y}$ we will denote the family of all finite sequences of pairwise disjoint arcs contained in $Y \subset X$. For simplicity of notation, let $\vartheta_{\mathcal{A}}$ stand for $\vartheta_{\mathcal{A}}^{X}$. Moreover, $\mathcal{A}|Y = \{K \cap Y : K \in \mathcal{A}\}$.

If $F = (A_1, \ldots, A_m) \in \vartheta_{\mathcal{A}}$ and $f : X \to X$ is a function then we define so called structural matrix $\mathcal{M}_f = [a_{ij}]_{i,j=1}^{m}$ in the following way: $a_{ij} = 1$ if $A_i \underset{f}{\to} A_j$ and $a_{ij} = 0$ otherwise.

A generalized entropy of a function f (not necessarily continuous) with respect to the sequence $F \in \vartheta_{\mathcal{A}}$ is the number $H_f(F) = \log \sigma(\mathcal{M}_f)$ if $\sigma(\mathcal{M}_f) > 0$ and $H_f(F) = 0$ if $\sigma(\mathcal{M}_f) = 0$, where
$\sigma(\mathcal{M}_f) = \limsup_{n\to\infty} \sqrt[n]{\mathrm{tr}(\mathcal{M}_f^n)}$.

Let $Y \subset X$ be a nonempty open set. An entropy of f on Y with respect to the family $\mathcal{A}$ is the number

$$H_f(Y) = \sup\left\{\frac{1}{n} H_{f^n}(F) : F \in \vartheta_{\mathcal{A}}^{Y}\right\}.$$

Now, let us introduce the following notation

$$d(f,Y) = \begin{cases} \frac{H_f(Y)}{h(f)} & \text{if } h(f) \in (0,\infty), \\ 1 & \text{if } H_f(Y) = \infty \text{ or } h(f) = 0, \\ 0 & \text{if } H_f(Y) \in [0,\infty) \text{ and } h(f) = \infty. \end{cases}$$

A density of entropy of f at a point x_0 is the number

$$E_f(x_0) = \inf\{d(f,V) : V \in O(x_0)\}.$$

We say that $x_0 \in X$ is an *$\mathcal{A}$-focal entropy point of* f (or briefly: focal entropy point) if $E_f(x_0) = 1$.

The first results will be connected with the fact that each continuous function mapping the unit interval into itself has such kind of points. Moreover, we will discuss the basic properties of the set of all focal entropy points and the possibility of improving functions $f \colon I^m \to I^m$ so that any fixed point of the function becomes its focal entropy point.

Helicity is the only integral invariant of volume-preserving diffeomorphisms

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Let M be a compact 3-dimensional manifold without boundary, endowed with a Riemannian metric. We denote by $\mathfrak{X}^1_{\rm ex}$ the vector space of exact divergence-free vector fields on M of class C^1, endowed with its natural C^1 norm. We recall that a divergence-free vector field w is *exact* if the 2-form $i_w\mu$ is exact, where μ is the Riemannian volume form.

On exact fields, the curl operator has a well defined inverse $\mathrm{curl}^{-1}:\mathfrak{X}^1_{\rm ex}\to\mathfrak{X}^1_{\rm ex}$. The inverse of curl is a generalization to compact 3-manifolds of the Biot–Savart operator, and can also be written in terms of a (matrix-valued) integral kernel $k(x,y)$ as

$$\mathrm{curl}^{-1}w(x)=\int_M k(x,y)\,w(y)\,dy\,, \tag{2}$$

where dy stands for the Riemannian volume measure. Using this integral operator, one can define the helicity of a vector field w on M as

$$\mathcal{H}(w):=\int_M w\cdot\mathrm{curl}^{-1}w\;dx\,.$$

Here the dot denotes the scalar product of two vector fields defined by the Riemannian metric on M. It is well known that the helicity is invariant under volume-preserving diffeomorphisms, that is, $\mathcal{H}(w)=\mathcal{H}(\Phi_*w)$ for any diffeomorphism Φ of M that preserves volume (and orientation).

In view of the expression (2) for the inverse of the curl operator, it is clear that the helicity is an *integral invariant*, meaning that it is given by the integral of a density of the form

$$\mathcal{H}(w)=\int G(x,y,w(x),w(y))\,dx\,dy\,.$$

Our objective in this talk is to show, under some natural regularity assumptions, that the helicity is the only integral invariant under volume-preserving diffeomorphisms. To this end, let us define a regular integral invariant as follows:

Definition *Let $\mathcal{I}:\mathfrak{X}^1_{\rm ex}\to\mathbb{R}$ be a C^1 functional. We say that $\mathcal{I}$ is a* regular integral invariant *if:*

1. *It is invariant under volume-preserving transformations, i.e., $\mathcal{I}(w)=\mathcal{I}(\Phi_*w)$ for any diffeomorphism Φ of M that preserves volume (and orientation).*

2. *At any point $w \in \mathfrak{X}^1_{\mathrm{ex}}$, the (Fréchet) derivative of $\mathcal{I}$ is an integral operator with continuous kernel, that is,*

$$(D\mathcal{I})_w(u) = \int_M K(w) \cdot u \,,$$

for any $u \in \mathfrak{X}^1_{\mathrm{ex}}$, where $K : \mathfrak{X}^1_{\mathrm{ex}} \to \mathfrak{X}^1_{\mathrm{ex}}$ is a continuous map.

The following theorem shows that the helicity is essentially the only regular integral invariant in the above sense:

Theorem *Let $\mathcal{I}$ be a regular integral invariant. Then $\mathcal{I}$ is a function of the helicity, i.e., there exists a C^1 function $f : \mathbb{R} \to \mathbb{R}$ such that $\mathcal{I} = f(\mathcal{H})$.*

The idea of the proof is that the invariance of the functional $\mathcal{I}$ under volume-preserving diffeomorphisms implies the existence of a continuous first integral for each exact divergence-free vector field. Because a generic vector field in $\mathfrak{X}^1_{\mathrm{ex}}$ is not integrable, we conclude that the aforementioned first integral is a constant (that depends on the field), which in turn implies that $\mathcal{I}$ has the same value for all vector fields in a connected component of the level sets of the helicity. Because these level sets are path connected, the theorem follows.

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Minimal laminations in $\mathbb{R}^3$ and the Hoffman-Meeks conjecture

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The Hoffman–Meeks conjecture is one of the basic open problems in classical minimal surface theory, and states that if M is a minimal surface with finite total curvature in $\mathbb{R}^3$ with genus g and k ends, then $k \leqslant g + 2$. This open problem motivates the study of the possible limits of a sequence of embedded minimal surfaces $M_n \subset \mathbb{R}^3$ with fixed genus. Typically, minimal laminations with singularities appear as such limits. By using Colding–Minicozzi theory, we will give a convergence result for (a subsequence of) the M_n if we assume a uniform bound for the injectivity radius of the M_n outside a closed countable set of $\mathbb{R}^3$. We will also show how one can use this convergence result to obtain a (non-explicit) bound $k \leqslant C(g)$ only depending on the genus, for the Hoffman–Meeks conjecture.

A short story on the ellipticity of the Stein-Weiss gradients

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The main thesis of some three publications will be presented in a historical context.

1. J. Kalina, A. Pierzchalski & P. Walczak, *Only one of the generalized gradients can be elliptic* Ann. Polon. Math. **67** (1997) 111–120.
2. J. Kalina, B. Ørsted, A. Pierzchalski, P.Walczak & G. Zhang, *Elliptic gradients and highest weights*, Bull. Polon. Acad. Sci. Ser. Math. **44** (1996), 511–519.
3. T. Branson, *Stein Weiss operators and ellipticity* J. Funct. Anal. **151** (1997), 334–383.

Integral formulae for codimension-one foliated Finsler manifolds

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The talk is based on our joint with P. Walczak works [**2,3**] about a codimension-one foliated Finsler space (M, F), in particular, a Randers space (i.e., $F = \alpha+\beta$, α being the norm of a Riemannian structure on M and β a 1-form of α-norm smaller than 1 on M). Using a unit vector field orthogonal (in the Finsler sense) to the leaves we define a new Riemannian metric g on M. For that g we calculate several geometric invariants of F, express them in terms of invariants arising from α and some quantities related to β, and then, using the approach of [**1**], we obtain the integral formulae for closed (M, F) and $(M, \alpha + \beta)$. On this way, we generalize Reeb's formula (that the total mean curvature of the leaves is zero) and its companion (that twice total second mean curvature of the leaves equals to the total Ricci curvature in the normal direction). We also extend result by Brito-Langevin-Rosenberg (1981) (that total mean curvatures of arbitrary order for a codimension-one foliated Riemannian manifold of constant curvature don't depend on a foliation).

1. V. R. & P. Walczak, *Integral formulae on foliated symmetric spaces*, Math. Ann. **352**, (2012) 223–237.
2. V. R. & P. Walczak, *Integral formulae for codimension-one foliated Finsler spaces*, Balkan J. Geom. & Appl. 2016 (see ArXiv:1602.00610).
3. V. R. & P. Walczak, *Integral formulae for codimension-one foliated Randers spaces*, preprint, ArXiv:1604.04069.

Exotic open 4-manifolds which are non-leaves

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We study the possibility of realizing exotic smooth structures on punctured simply connected 4-manifolds as leaves of a codimension one foliation on a smooth compact manifold. In particular, we show the existence of an uncountable set of smooth open 4-manifolds which are not diffeomorphic to any leaf of a codimension one transversely C^2 foliation on a compact manifold. These examples include some exotic $\mathbb{R}^4$'s and exotic cylinders $S^3 \times \mathbb{R}$. See [**10**] for the complete paper.

Our results involve a set $\mathcal{Y}$ of smooth open 4-manifolds which we define below.

Theorem 1. *If $Y \in \mathcal{Y}$ is a leaf in a $C^{1,0}$ codimension one foliation of a closed 5-manifold, then it is a proper leaf and each connected component of the union of the leaves diffeomorphic to Y fibers over the circle with the leaves as fibers.*

Theorem 2. *For any manifold $Y \in \mathcal{Y}$ there exists an uncountable subset $\mathcal{Y}_Y \subset \mathcal{Y}$ of manifolds homeomorphic to Y that are not diffeomorphic to any leaf of a C^2 codimension one foliation of a compact manifold.*

The following result of independent interest, which uses the theory of levels and depth, will be used in the proof of Theorem 2.

Theorem 3. *The set of diffeomorphism classes of smooth manifolds of arbitrary dimension which are diffeomorphic to leaves of finite depth in C^2 codimension one foliations of compact manifolds is countable.*

Let us recall some important steps in the history of leaves and non-leaves. Cantwell and Conlon [**3**] showed that every open surface is diffeomorphic to a leaf of a foliation on every closed 3-manifold. The first examples of topological non-leaves were due to Ghys [**5**] and Inaba, Nishimori, Takamura, and Tsuchiya [**7**]. Later on, Attie and Hurder [**1**] constructed simply connected 6-dimensional non-leaves, among other results.

To define the set $\mathcal{Y}$ we need the concept of "end sum". Given two open smooth oriented 4-manifolds M and N with proper smooth embedded paths $c_1 : [0,\infty) \to M$ and $c_2 : [0,\infty) \to N$ defining ends of M and N, let V_1 and V_2 be tubular neighborhoods of $c_1([0,\infty))$ and $c_2([0,\infty))$. Then the end sum is $M \natural N = (M \setminus V_1) \bigcup_\partial (N \setminus V_2)$, where the boundaries, both diffeomorphic to $\mathbb{R}^3$, are identified so as to preserve the orientation. If N is homeomorphic to $\mathbb{R}^4$ then $M \natural N$ is homeomorphic to M. The end sum, up to diffeomorphism, depends only on the smooth proper isotopy classes of the curves. End sum was the first technique which made it possible to find infinitely many exotic structures on $\mathbb{R}^4$ [6] and it is an important tool for dealing with the problem of generating infinitely many smooth structures on open 4-manifolds [2,4].

An end of a smooth 4-manifold is *smoothly periodic* if there exists an unbounded domain $V \subset M$ homeomorphic to $S^3 \times (0,\infty)$ and a diffeomorphism $h : V \to V$ such that $h^n(V)$ defines the given end (i.e., $\{h^n(V)\}$ is a neighborhood base for the end).

It is well known that by removing a closed set carrying the 2-homology of the Kummer complex surface K3 it is possible to obtain a smooth 4-manifold $\mathbf{R}$ homeomorphic to $\mathbb{R}^4$ with an exotic end. For a homeomorphism $\psi : \mathbb{R}^4 \to \mathbf{R}$, we let $\mathbf{K}_t$ denote $\psi(D(t))$, where $D(t)$ is the standard closed 4-disk of radius t centered at the origin, so that its interior $\mathring{\mathbf{K}}_t$ has a smooth structure induced from $\mathbf{R}$ by ψ. Let $\natural\mathbf{R}_\infty = \natural_{i=1}^{\infty}\mathbf{R}$ be the infinite end sum. Then we have the following special case of Theorem 1.4 of Taubes [10].

Theorem (Taubes, [10]) *Let M be an open smooth simply connected 4-manifold with definite intersection form and exactly one end. If the end of M is homeomorphic to $S^3 \times (0,\infty)$ and smoothly periodic, then the intersection form is isomorphic to a diagonal form. As a consequence, for any homeomorphism $\psi : \mathbb{R}^4 \to \mathbf{R}$, there exists $r_0 > 0$ such that, for any $t, s > r_0$, $t \neq s$, $\mathring{\mathbf{K}}_t$ is not diffeomorphic to $\mathring{\mathbf{K}}_s$.*

Now we define $\mathcal{Y}$ to be the set of smooth manifolds Y (up to diffeomorphism) that are homeomorphic to simply connected compact 4-manifolds with finitely many punctures satisfying the following conditions:

1. Y has an end diffeomorphic to the end of a non-trivial finite end sum $\natural_{i=1}^{k}\mathbf{R}$, to $\natural_{i=1}^{k}\mathring{\mathbf{K}}_t$ or to $\mathring{\mathbf{K}}_t\natural\mathbf{R}_\infty$ with $t > r_0$, and
2. if $H_2(Y) = 0$, then Y has only one exotic end and the other ends (if there are any) are standard.
3. In the particular case where Y is homeomorphic to $\mathbb{R}^4$ we only consider smooth structures with finite Taylor-index (See [11].)

This is joint work with Carlos Meniño Cotón (Universidade Federal do Rio de Janeiro).

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2. Ž. Bižaca, J. Etnyre, *Smooth structures on collarable ends of 4-manifolds*. Topology **37-3**, 461–467 (1998).
3. J. Cantwell & L. Conlon, *Every surface is a leaf*. Topology **25-3**, 265–285 (1987).
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5. É. Ghys, *Une variete qui n'est pas une feuille*. Topology **24-1**, 67–73 (1985).

6. R.E. Gompf, *An exotic menagerie*. J. Diff. Geom. **37**, 199–223 (1993).
7. T. Inaba, T. Nishimori, M. Takamura & N. Tsuchiya, *Open manifolds which are non-realizable as leaves*. Kodai Math. J., 8, **112–119** (1985).
8. C. Meniño Cotón, P.A. Schweitzer, S.J., *Exotic open 4-manifolds which are non-leaves* Arxiv:1410.8182.
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10. C.H. Taubes, *Gauge theory on asymptotically periodic 4-manifolds*. J. Diff. Geom. **25**, 363–430 (1987).
11. L. Taylor, *An invariant of smooth 4-manifolds*. Geom. Topol. **1**, 71–89, (1997).

Vaisman manifolds, canonical foliations and the associated spectral sequence

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Vaisman manifolds are classical examples of locally conformally Kähler manifolds. These spaces are known to admit local Kähler structures that cannot be extended globally. Any Vaisman manifold induces a canonical foliated structure that turn the manifold into a Riemannian foliation. For any foliation of this type it is possible to construct a spectral sequence such that the terms of this cohomological object stand as topological invariants. We investigate the terms of the spectral sequence using a Hodge approach. In the attempt to associated a Vaisman structure to an arbitrary foliation we show that these invariants offer us topological obstructions. Several examples are presented.

Smale-Barden manifolds with K-contact and Sasakian structures

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We show that under some mild restrictions, there exists a closed 5-dimensional manifold with vanishing first integral homology which carries K-contact structures but does not carry any Sasakian structure. This yields a partial answer to a question posed by Boyer and Galicki. In the talk, I present an overview of this result together with a more general research program. This is a joint work with Vicente Munoz and Juan Angel Rojo.

Geometric structures on foliated manifolds. Tangentially $\mathfrak{g}$-foliations revisited

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A tangentially g-foliation is a regular foliation whose tangent subbundle is trivial and the trivialisation is given by a set global vector fields forming a Lie algebra isomorphic to $\mathfrak{g}$. Such foliations appear in many a geometrical context. Let us just mention Sasakian manifolds, 3-Sasakian manifolds, or totally geodesic foliations. The topics investigated include the existence of such foliations with rich tranverse structures à Sasaki and the fatness condition.

The lecture is based on recent joint research with A. Tralle, M. Bocheński, and M. Sroka.

Variations of total mixed scalar curvature

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In [1] authors considered variations of total mixed scalar curvature (of the Levi-Civita connection) as a functional on the space of pseudoriemannian metrics preserving a fixed almost product structure. The Euler-Lagrange equations were obtained and some examples of their solutions were given.

These results will be presented and generalised – by considering the total mixed scalar curvature of a fixed distribution and varying it with respect to all pseudoriemannian metrics, as well as connections.

1. V. Rovenski, T. Zawadzki, *The Einstein-Hilbert type action on foliated pseudo-Riemannian manifolds*, ArXiv:1604.00985

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USEFUL INFORMATION

Registration desk

Registration desk is located at the main lecture hall. It is open on Monday (July 11) at 5-7 pm and on Tuesday (July 13) at 8:30 – 11 am. Anyway, the organizing committee memmbers help you in any case.

Meeting Venue

The conference takes place in the main lecture hall equipped with a computer, projectors and blackboards. You can also use a small lecture room and the common room for some individual meetings. Będlewo Palace provides full-board accommodation for participants of the conference. Hotel rooms are in the same building as lecture halls and the restaurant is located in the neighbouring palace.

Social events

The conference banquet starts on Wednesday (July 13) at 8 pm. It is included in the registration fee.

On Thursday (July 14) afternoon we plan (depending on weather) an excursion around Wielkopolski National Park in two ways : draisines (less effort) or bikes (more effort).

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Index

Notes

Notes

Notes

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