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THE CHARACTERISTIC OF THEORETICAL INCOME DISTRIBUTIONS AND THEIR APPLICATION TO THE ANALYSIS OF WAGE DISTRIBUTIONS IN POLAND BY REGIONS

Abstract. In the paper we compare the properties of theoretical income distributions from the point of view of their application to the analysis of wage distributions in Poland. Among them the lognormal and Dagum distributions were taken into consideration. On the basis of the theoretical density curves well fitted to the empirical ones income inequality measures were calculated. The estimation was conducted for the wages distributions in Poland in different divisions: by gender, by economic sector and by regions.

Key words: income distribution, inequality, goodness-of-fit.

1. INTRODUCTION

In income distribution comparisons in time and in space various synthetic inequality measures based on theoretical distribution parameters can be helpful. Since Pareto (1896) proposed his first income distribution model, many economists and mathematicians tried to describe empirical distributions by simple mathematical formulas with a small number of parameters. These formulas can be useful for many reasons. Firstly, applying the theoretical model simplifies the analysis, because different distribution characteristics can be performed by the same parameters. Secondly, the theoretical model well fitted to the data can be used to the prediction of wage and income distributions in different divisions. Moreover, the approximation of the empirical wage and income distributions by means of the theoretical curves can smooth the irregularities coming from the method of data collecting.

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2. INCOME DISTRIBUTION MODEL

Many authors who consider income distributions propose a set of economic, econometric, stochastic and mathematical properties to motivate the identification of a particular mathematical model of income distributions. The final choice depends on the model capacity to account well to these properties.

Aitchison and Brown (1957) stated four fundamental features as a guide to identify the most representative model of the unknown stochastic process that generates an income distribution. Other authors as Metcalf (1972) and Dagum (1977) considered the same problem. Finally, the set of properties, necessary for a good income distribution model, can be specified:

1. Empirical and theoretical foundations of the model.
2. Convergence to the Pareto law for high income groups.
3. Existence of only a small number of finite moments of a distributions (heavy tails).
4. Goodness of fit for a whole range of a distribution.
5. Economic interpretation of parameters.
6. Model flexibility to describe wage, income and wealth distributions in different periods of time. It should be possible by changes in parameter values.
7. The good model should be able to account for negative and null income through changes in the values of its parameters.
8. The model should be useful for strictly positive income without truncating the distribution.
9. Simple and efficient methods of parameters estimation.
10. Possibility of derivation of the explicit mathematical forms of the Lorenz curve and the Gini concentration index. Simplicity (or small number of parameters).

In the paper we compare the lognormal distribution with the Dagum one from the point of view of their application to the analysis of wage distributions in Poland.

The lognormal distribution is a theoretical model most often used to the approximation and estimation of wage and income distributions in different divisions. The lognormal density function is the following:

$$f(x) = \begin{bmatrix} \frac{1}{y\sigma\sqrt{2\pi}} \exp\left[\frac{-1}{2\sigma^2}(\ln y - \mu)^2\right], y > 0 \\ 0 \qquad y \leqslant 0, \end{bmatrix} \tag{1}$$

where:

μ – the expected value of the logarithms of a random variable X,

σ – the standard deviation of the logarithms of a random variable X.

The consistency of the lognormal curve with the empirical evidence was quite good especially in the centrally-planned economy (Aitchison, Brown 1957, Vielrose 1960, Kordos, Stroińska 1971, Kordos 1976, Domański 1975). The advantage of this model is its simplicity, because the lognormal density function is characterized by only two parameters. On the other hand this simplicity limits the opportunities of perfect fitting to the data especially in the right tail of the distribution. Moreover, the lognormal curve cannot be used to the analysis of wealth distributions because it is always unimodal while wealth distributions are strictly decreasing.

The characteristics of the lognormal distribution mentioned above induced many scholars to further investigations on the mathematical model describing income distributions. The empirical observations conducted by Dagum showed, that the behavior of income elasticity of the cumulative distribution function of income ($F(x)$) can be presented by means of the following differential equation:

$$\varepsilon(y, F) = \frac{d \ln F(y)}{d \ln y} = \beta_1[1 - [F(y)^{\beta_2}], \tag{2}$$

$$\text{for} \quad y \geqslant 0; \quad \beta_1 \beta_2 > 0.$$

Solving the equation Dagum obtained the cumulative distribution function of the form:

$$F(y) = \begin{bmatrix} (1 + \lambda y^{-\delta})^{-\beta}, & y > 0 \\ 0 & y \leqslant 0, \end{bmatrix} \tag{3}$$

$$\text{for} \quad \beta, \lambda, \delta > 0,$$

where:

$\beta = 1/\beta_2$,

$\delta = \beta_1 \beta_2$

$\lambda = \exp$,

c – constant of integration coming from the solution of (2).

The density function corresponding to the cumulative distribution function given in (3) is the following:

$$f(y) = \begin{bmatrix} \beta\ \lambda\ \delta\ y^{-\delta-1}(1+\lambda y^{-\delta})^{-\beta-1}, & y>0 \\ 0 & y \leqslant 0. \end{bmatrix} \tag{4}$$

The distribution moments about the origin can be written in the form:

$$\mu_r = \beta \lambda^{r/\delta} B(1 - r/\delta, \beta + r/\delta), \quad \text{for} \quad r < \delta, \tag{5}$$

where: $B(1 - r/\delta, \beta + r/\delta$ – the beta functions with parameters $(1 - r/\delta, \beta + r/\delta)$.

It follows from the equation (5) that the distribution under consideration possesses finite moments of order $r < \delta$. Hence the moments of orders $r \geqslant \delta$ tend to infinity.

The theoretical distribution obtained by Dagum on the basis of empirical evidence, has many interesting properties from the point of view of its application to the analysis of wage, income and wealth distributions. The investigations conducted in many countries (cf. Dagum, Lemmi 1987, Jędrzejczak 1991) showed, that the estimates of δ are usually in the neighbourhood of four. So the number of finite moments of the distribution is also small – usually three or four. Hence the model given by (3) presents a "heavy tail", what is desirable in the analysis of income distributions. As the consequence, the Dagum distribution tends to the Pareto curve for high income groups (convergence to the Pareto law).

The distribution (2) is described by three parameters β, λ and δ; β and δ are inequality parameters while λ is a parameter of scale. Changes in a monetary unit will change λ and will leave invariant the remaining two parameters. The "new" value of λ can be obtained by means of the following relation:

$$\lambda^* = \lambda k^{-\delta}, \tag{6}$$

where: k – a rate of a new monetary unit.

Parameters β and δ are the shape parameters of the Dagum density function. Inequality measures (as the Gini ratio) are increasing functions of β and decreasing functions of δ. The Lorenz curve corresponding to the model (3) can be written in the form:

$$L(p) = B[p^{1/\beta};\ \beta + 1/\delta,\ 1 - 1/\delta], \quad \text{for} \quad \delta > 1,\ 0 \leqslant p \leqslant 1, \tag{7}$$

where: $B[p^{1/\delta};\ \beta + 1/\delta,\ 1 - 1/\delta]$ – incomplete beta function with parameters $(\beta + 1)/\delta$, $B[\beta + 1/\delta,\ 1 - 1/\delta]$.

The Gini coefficient based on the Lorenz curve given by (7) is the following:

$$G = -1 + B(\beta, \beta)/B(\beta, \beta + 1/\delta), \tag{8}$$

The Dagum model possesses another important property, that its density function can be unimodal or non-modal depending on the parameters. When $\beta\delta > 1$ the distribution is unimodal, while for $0 < \beta\delta < 1$ the density function is strictly decreasing (non-modal). The latter situation can occur in the upper past of the distribution (the Pareto case) or for the actual income distribution in a poor and overpopulated country.

It is worth mentioning that also a four-parameter distribution has been proposed (Dagum 1977). The fourth parameter α can be used to fit the model to strictly positive or negative incomes.

In Section 3 we compare the consistency of the lognormal and the Dagum distributions with wage distributions in Poland by regions. The parameters of both the density functions were estimated by means of the maximum likelihood methods. The basis for the estimation were grouped data concerning wage distributions in Poland in 1999 and 2001. The general form of the logarithm of the likelihood function is the following:

$$\ln L = \sum_{i=1}^{k} y_i \ln[F(y_i) - F(y_{i-1})], \tag{9}$$

where:

y_i – empirical frequencies in intervals,

$F(y)$ – cumulative distribution function of income.

For the lognormal distribution (9) is a function of two variables μ and σ while for the Dagum model it is the function of λ, β and δ. To find its maximum an individual numerical procedure has been applied.

The results of the calculations are presented in Tables 1–6. The coefficient of distributions similarity (Vielrose 1960) and the standard deviation of differences between relative frequencies were used as goodness-of-fit measures.

Table 1. Goodness-of-fit measures for the total wage distributions in Poland by regions (1999 and 2001)

Voivodeship		Lognormal distribution		Dagum distribution	
		W_p	S_d	W_p	S_d
Dolnośląskie	1999	0.9199	0.0163	0.9456	0.0133
	2001	0.9249	0.0147	0.9371	0.0119
Kujawsko-pomorskie	1999	0.9235	0.0151	0.9456	0.0143
	2001	0.9241	0.0261	0.9247	0.0155
Lubelskie	1999	0.9110	0.0187	0.9364	0.0170
	2001	0.9288	0.0138	0.9330	0.0135
Lubuskie	1999	0.9175	0.0167	0.9394	0.0158
	2001	0.9394	0.0107	0.9389	0.0111
Łódzkie	1999	0.9148	0.0167	0.9332	0.0148
	2001	0.9308	0.0137	0.9355	0.0128
Małopolskie	1999	0.9200	0.0167	0.9412	0.0146
	2001	0.9349	0.0125	0.9431	0.0114
Mazowieckie	1999	0.9117	0.0164	0.9456	0.0107
	2001	0.9078	0.0153	0.9468	0.0093
Opolskie	1999	0.9159	0.0180	0.9363	0.0165
	2001	0.9304	0.0135	0.9291	0.0144
Podkarpackie	1999	0.9125	0.0184	0.9396	0.0167
	2001	0.9376	0.0119	0.9322	0.0132
Podlaskie	1999	0.9220	0.0158	0.9473	0.0141
	2001	0.9185	0.0163	0.9071	0.0195
Pomorskie	1999	0.9325	0.0137	0.9463	0.0125
	2001	0.9251	0.0136	0.9275	0.0146
Śląskie	1999	0.9410	0.0126	0.9353	0.0128
	2001	0.9373	0.0120	0.9289	0.0120
Świętokrzyskie	1999	0.9230	0.0175	0.9371	0.0157
	2001	0.9311	0.0130	0.9456	0.0177
Warmińsko-mazurskie	1999	0.9174	0.0173	0.9366	0.0158
	2001	0.9407	0.0114	0.9477	0.0105
Wielkopolskie	1999	0.9183	0.0167	0.9414	0.0141
	2001	0.9348	0.0124	0.9385	0.0123
Zachodniopomorskie	1999	0.9215	0.0156	0.9389	0.0135
	2001	0.9184	0.0155	0.9190	0.0153

Table 2. Goodness-of-fit measures for the wage distributions of men by regions (1999 and 2001)

Voivodeship		Lognormal distribution		Dagum distribution	
		W_p	S_d	W_p	S_d
Dolnośląskie	1999	0.9311	0.0160	0.9420	0.0125
	2001	0.9269	0.0149	0.9346	0.0118
Kujawsko-pomorskie	1999	0.9280	0.0146	0.9493	0.0125
	2001	0.9191	0.0155	0.9350	0.0132
Lubelskie	1999	0.9113	0.0206	0.9293	0.0172
	2001	0.9262	0.0143	0.9371	0.0117
Lubuskie	1999	0.9045	0.0188	0.9370	0.0155
	2001	0.9285	0.0134	0.9363	0.0115
Łódzkie	1999	0.9241	0.0154	0.9354	0.0140
	2001	0.9257	0.0146	0.9310	0.0131
Małopolskie	1999	0.9208	0.0192	0.9317	0.0151
	2001	0.9229	0.0162	0.9382	0.0127
Mazowieckie	1999	0.9152	0.0152	0.9472	0.0104
	2001	0.9024	0.0162	0.9435	0.0098
Opolskie	1999	0.9114	0.0201	0.9383	0.0148
	2001	0.9312	0.0149	0.9393	0.0118
Podkarpackie	1999	0.9123	0.0201	0.9337	0.0172
	2001	0.9345	0.0134	0.9424	0.0116
Podlaskie	1999	0.9309	0.0155	0.9388	0.0143
	2001	0.9066	0.0184	0.9094	0.0191
Pomorskie	1999	0.9356	0.0131	0.9513	0.0108
	2001	0.9262	0.0132	0.9343	0.0121
Śląskie	1999	0.9237	0.0129	0.9274	0.0140
	2001	0.9324	0.0124	0.9240	0.0149
Świętokrzyskie	1999	0.9238	0.0196	0.9318	0.0157
	2001	0.9342	0.0131	0.9518	0.0098
Warmińsko-mazurskie	1999	0.9071	0.0202	0.9302	0.0172
	2001	0.9316	0.0124	0.9622	0.0067
Wielkopolskie	1999	0.9244	0.0153	0.9464	0.0126
	2001	0.9348	0.0115	0.9422	0.0107
Zachodniopomorskie	1999	0.9232	0.0166	0.9389	0.0133
	2001	0.9102	0.0179	0.9150	0.0163

Table 3. Goodness-of-fit measures for the wage distributions of women by regions (1999 and 2001)

Voivodeship		Lognormal distribution		Dagum distribution	
		W_p	S_d	W_p	S_d
Dolnośląskie	1999	0.9226	0.0160	0.9467	0.0148
	2001	0.9377	0.0169	0.9367	0.0130
Kujawsko-pomorskie	1999	0.9264	0.0156	0.9367	0.0170
	2001	0.9243	0.0181	0.9072	0.0188
Lubelskie	1999	0.9153	0.0227	0.9400	0.0175
	2001	0.9287	0.0146	0.9795	0.0167
Lubuskie	1999	0.9346	0.0157	0.9319	0.0173
	2001	0.9564	0.0091	0.9385	0.0120
Łódzkie	1999	0.9084	0.0176	0.9333	0.0160
	2001	0.9412	0.0123	0.9369	0.0132
Małopolskie	1999	0.9152	0.0183	0.9370	0.0150
	2001	0.9365	0.0118	0.9426	0.0112
Mazowieckie	1999	0.9045	0.0179	0.9424	0.0113
	2001	0.9145	0.0142	0.9490	0.0093
Opolskie	1999	0.9141	0.0173	0.9241	0.0193
	2001	0.9281	0.0150	0.9075	0.0181
Podkarpackie	1999	0.9147	0.0172	0.9376	0.0167
	2001	0.9415	0.0122	0.9196	0.0161
Podlaskie	1999	0.9210	0.0163	0.9469	0.0146
	2001	0.9296	0.0155	0.9052	0.0198
Pomorskie	1999	0.9319	0.0142	0.9406	0.0144
	2001	0.9255	0.0152	0.9079	0.0182
Śląskie	1999	0.9293	0.0152	0.9330	0.0160
	2001	0.9333	0.0134	0.9194	0.0162
Świętokrzyskie	1999	0.9281	0.0158	0.9333	0.0176
	2001	0.9323	0.0138	0.9373	0.0146
Warmińsko-mazurskie	1999	0.9320	0.0152	0.9422	0.0150
	2001	0.9420	0.0125	0.9217	0.0160
Wielkopolskie	1999	0.9131	0.0177	0.9361	0.0162
	2001	0.9309	0.0134	0.9320	0.0148
Zachodniopomorskie	1999	0.9240	0.0170	0.9372	0.0143
	2001	0.9298	0.0147	0.9155	0.0170

Table 4. Goodness-of-fit measures for the total wage distributions in Poland by regions (2002)

Voivodeship	Lognormal distribution		Dagum distribution	
	W_p	S_d	W_p	S_d
Ogółem	0.9547	0.0084	0.9752	0.0052
Dolnośląskie	0.9348	0.0133	0.9641	0.0076
Kujawsko-pomorskie	0.9600	0.0075	0.9758	0.0051
Lubelskie	0.9628	0.0081	0.9590	0.0103
Lubuskie	0.9573	0.0097	0.9525	0.0117
Łódzkie	0.9584	0.0078	0.9734	0.0057
Małopolskie	0.9614	0.0073	0.9679	0.0068
Mazowieckie	0.9285	0.0123	0.9806	0.038
Opolskie	0.9546	0.0082	0.9516	0.0106
Podkarpackie	0.9686	0.0059	0.9775	0.0053
Podlaskie	0.9519	0.0100	0.9855	0.0028
Pomorskie	0.9659	0.0060	0.9639	0.0074
Śląskie	0.9668	0.0066	0.9525	0.0091
Świętokrzyskie	0.9562	0.0081	0.9663	0.0087
Warmińsko-mazurskie	0.9727	0.0051	0.9649	0.0074
Wielkopolskie	0.9665	0.0060	0.9671	0.0065
Zachodniopomorskie	0.9676	0.0054	0.9775	0.0045

Table 5. Goodness-of-fit measures for the wage distributions of women by regions (2002)

Voivodeship	Lognormal distribution		Dagum distribution	
	W_p	S_d	W_p	S_d
Ogółem	0.9563	0.0084	0.9600	0.0086
Dolnośląskie	0.9581	0.0088	0.9581	0.0088
Kujawsko-pomorskie	0.9598	0.0194	0.9560	0.0092
Lubelskie	0.9406	0.0139	0.9286	0.0171
Lubuskie	0.9519	0.0122	0.9334	0.0159
Łódzkie	0.9658	0.0069	0.9655	0.0077
Małopolskie	0.9529	0.0101	0.09536	0.0102
Mazowieckie	0.9454	0.0115	0.9705	0.0069
Opolskie	0.9385	0.0136	0.9181	0.0180
Podkarpackie	0.9572	0.0088	0.9493	0.0116
Podlaskie	0.9538	0.0101	0.9801	0.0051
Pomorskie	0.9619	0.0073	0.9502	0.0102
Śląskie	0.9640	0.0080	0.9474	0.0114
Świętokrzyskie	0.9515	0.0101	0.9372	0.0140
Warmińsko-mazurskie	0.9532	0.0100	0.9399	0.0130
Wielkopolskie	0.9705	0.0063	0.9618	0.0085
Zachodniopomorskie	0.9715	0.0056	0.9573	0.0094

Table 6. Goodness-of-fit measures for the wage distributions of men by regions (2002)

Voivodeship	Lognormal distribution		Dagum distribution	
	W_p	S_d	W_p	S_d
Ogółem	0.9505	0.0080	0.9833	0.0036
Dolnośląskie	0.9331	0.0144	0.9545	0.0094
Kujawsko-pomorskie	0.9501	0.0085	0.9804	0.0041
Lubelskie	0.9523	0.0085	0.9741	0.0055
Lubuskie	0.9495	0.0102	0.9640	0.0085
Łódzkie	0.9511	0.0093	0.9718	0.0057
Małopolskie	0.9555	0.0084	0.9789	0.0042
Mazowieckie	0.9190	0.0135	0.9785	0.0041
Opolskie	0.9416	0.0137	0.9587	0.0091
Podkarpackie	0.9472	0.0093	0.9822	0.0032
Podlaskie	0.9332	0.0132	0.9678	0.0079
Pomorskie	0.9650	0.0059	0.9757	0.0048
Śląskie	0.9441	0.0085	0.9360	0.0117
Świętokrzyskie	0.9450	0.0113	0.9646	0.0088
Warmińsko-mazurskie	0.9614	0.0065	0.9852	0.0028
Wielkopolskie	0.9636	0.0060	0.9691	0.0059
Zachodniopomorskie	0.9463	0.0104	0.9738	0.0053

Analyzing the results of conducted approximations, one can easily notice that the Dagum curve is better fitted to the data than the lognormal one. For the lognormal model the consistency with the empirical distributions is often poor (especially in 1999). In 2001 the lognormal curve seemed to be relatively well fitted to the data while for the Dagum model the consistency was often unsatisfactory. In 2002 the consistency measures for both the distributions indicated better fitting which, for the Dagum model, was very good ($S_r < 0.01$). It can be explained by growing concentration of wage and income distributions in Poland.

3. THE ANALYSIS OF DISTRIBUTION INEQUALITY ON THE BASIS OF A CHOSEN THEORETICAL MODEL

Most often used inequality measures were based on the Lorenz curve. They differ from each other in the method of calculating the concentration area that is the area between the line of equal shares and the Lorenz curve which, for the continuous distribution, can be written as follows:

$$L(p) = \mu^{-1}\int_0^p F^{-1}(t)dt, \tag{10}$$

where:

$F^{-1}(p)$ – the distribution p-th quantile,

μ – expected value.

The literature on income distribution had agreed, that the Gini ratio is the most useful, and certainly the most widely used, measure of changes in inequality. The Gini ratio is defined as follows:

$$G = 2\int_0^1 (p - L(p))dp \tag{11}$$

The concentration coefficient given by (11) can also be expressed by means of the following formula:

$$G = \frac{\Delta}{2\mu}. \tag{12}$$

where: Δ – the Gini mean difference given by the following equation:

$$\Delta = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |x - y| dF(x)dF(y). \tag{13}$$

Comparing two populations of economic units we can also assess the level of domination of one of them with respect to another. In order to do it we can apply a measure of distance between distributions. Interesting distance measures based on the Gini mean difference were proposed by Dagum and called economic distances.

Economic distance d_0 between an income distribution $f_1(x)$ with cumulative distribution function $F_1(x)$ and a distribution $f_2(y)$ with cumulative distribution function $F_2(y)$ can be defined as a probability, that an income variable Y is greater than X given that $E(Y) > E(X)$:

$$d_0 = P\{Y > X | E(Y) > E(X)\} = \int_0^{\infty} \int_0^{y} dF_1(x) dF_2(y), \tag{14}$$

where: $E(X), E(Y)$ – expected values of income variables X and Y.

Economic distance d_1 between income distributions $f_1(x)$ and $f_2(x)$ is defined as the weighted arithmetic mean of the differences $Y - X$ (for $y > x$) given that $E(Y) > E(X)$. The weighting factor is the joint density function $f_1(x) f_2(y)$.

$$d_1 = \int_0^{\infty} \int_0^{y} (y - x) dF_1(x) dF_2(y). \tag{15}$$

The economic distance d_0 takes values between 0 and 1/2, while the economic distance d_1 takes values between $\frac{\Delta}{2}$ and Δ, where Δ is the Gini mean difference modified for two income variables X and Y. The minimum values are attained when the compared random variables are independent and identically distributed. The maximum value of d_1 depends on the unit of income and is equal to the unconditional weighted arithmetic mean of all possible absolute differences between X and Y. The normalized forms of d_0 and d_1 called the economic distance ratios are the following:

$$\begin{gathered} D_0 = 2d_0 - 1, \\ D_1 = [E(Y) - E(X)]/[2d_1 - E(Y) + E(X)]. \end{gathered} \tag{16}$$

D_0 and D_1 are dimensionless and take values in the unit interval. They measure the proportion by which the more affluent population is "better" off than the other, taking into account the level of income. D_1 measures not only the frequency but also the amount by which one population dominates the other one, taking into account the mean, dispersion and asymmetry of the compared distributions.

The results of the calculations concerning the level of distribution inequality are presented in Tables 7 and 8. The Gini coefficients and economic distance ratios were calculated on the basis of the Dagum distribution parameters. It can be easily noticed that the level of concentration increased significantly in the period 1999–2002. The Gini coefficient is higher for men – the highest value was observed for the voivodeship "mazowieckie" (0.35).

Table 7. Gini coefficients and economic distance ratios between men and women by regions in 1999 and 2001

Voivodeship		Gini ratio			Economic distance ratio	
		total	men	women	D_0	D_1
Dolnośląskie	1999 2001	0.29 0.30	0.30 0.32	0.25 0.27	0.27 0.18	0.45 0.36
Kujawsko-pomorskie	1999 2001	0.28 0.29	0.29 0.30	0.26 0.28	0.20 0.13	0.34 0.22
Lubelskie	1999 2001	0.25 0.28	0.27 0.29	0.23 0.26	0.20 0.17	0.35 0.30
Lubuskie	1999 2001	0.27 0.28	0.28 0.29	0.25 0.26	0.13 0.11	0.23 0.21
Łódzkie	1999 2001	0.29 0.30	0.31 0.31	0.26 0.27	0.16 0.15	0.32 0.31
Małopolskie	1999 2001	0.28 0.28	0.28 0.29	0.26 0.27	0.22 0.18	0.34 0.28
Mazowieckie	1999 2001	0.38 0.40	0.39 0.43	0.35 0.37	0.14 0.11	0.28 0.28
Opolskie	1999 2001	0.26 0.26	0.27 0.26	0.25 0.26	0.22 0.18	0.34 0.26
Podkarpackie	1999 2001	0.24 0.25	0.25 0.25	0.23 0.24	0.17 0.13	0.30 0.22
Podlaskie	1999 2001	0.26 0.25	0.28 0.27	0.23 0.23	0.13 0.17	0.27 0.31
Pomorskie	1999 2001	0.30 0.30	0.31 0.30	0.28 0.28	0.17 0.19	0.32 0.31
Śląskie	1999 2001	0.30 0.30	0.28 0.29	0.25 0.26	0.40 0.36	0.58 0.53
Świętokrzyskie	1999 2001	0.26 0.27	0.27 0.28	0.24 0.26	0.21 0.18	0.35 0.30
Warmińsko-mazurskie	1999 2001	0.28 0.28	0.29 0.29	0.26 0.25	0.20 0.14	0.34 0.28
Wielkopolskie	1999 2001	0.29 0.29	0.30 0.30	0.27 0.27	0.18 0.16	0.33 0.29
Zachodniopomorskie	1999 2001	0.28 0.29	0.29 0.30	0.26 0.26	0.19 0.19	0.30 0.32

Table 8. Gini coefficients and economic distance ratios between men and women by regions in 2002

Voivodeship	Gini ratio		Economic distance ratio	
	men	women	D_0	D_1
Dolnośląskie	0.35	0.29	0.06	0.22
Kujawsko-pomorskie	0.32	0.28	0.08	0.19
Lubelskie	0.31	0.27	0.13	0.26
Lubuskie	0.31	0.27	0.10	0.21
Łódzkie	0.33	0.28	0.08	0.23
Małopolskie	0.30	0.28	0.17	0.28
Mazowieckie	0.43	0.35	0.09	0.28
Opolskie	0.29	0.28	0.16	0.26
Podkarpackie	0.28	0.26	0.11	0.19
Podlaskie	0.30	0.26	0.10	0.22
Pomorskie	0.32	0.29	0.16	0.28
Śląskie	0.32	0.28	0.29	0.47
Świętokrzyskie	0.31	0.27	0.12	0.25
Warmińsko-mazurskie	0.31	0.28	0.10	0.23
Wielkopolskie	0.33	0.29	0.15	0.28
Zachodniopomorskie	0.31	0.28	0.10	0.21

Simultaneously the economic distance between men and women decreased for almost all distributions under consideration. What is important, the same conclusions can be made analysing the economic distance ratios D_0 and D_1. Hence, the diminishing economic distance between men and women is connected not only with greater probability of gaining higher income for women but also with decreasing discrepancy between average income levels.

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CHARAKTERYSTYKA TEORETYCZNYCH I EMPIRYCZNYCH ROZKŁADÓW PŁAC I DOCHODÓW

(Streszczenie)

W artykule porównywano własności teoretycznych i empirycznych rozkładów płac i dochodów z punktu widzenia możliwości ich zastosowania do analizy rozkładów płac w Polsce. W szczególności rozważano takie rozkłady, jak logarytmiczno-normalny czy Daguma. Rozkłady teoretyczne, wykazujące wysoką zgodność z empirycznymi, zostały następnie wykorzystane do estymacji miar nierównomierności. Estymację przeprowadzono na podstawie rozkładów płac w Polsce w różnych przekrojach – według płci, sektora gospodarki oraz regionów.