ACTA UNIVERSITATIS LODZIENSIS
FOLIA OECONOMICA 216, 2008

*Alicja Ganczarek*[*]

# *VaR* IN RISK ANALYSIS ON DAM AND MODELS OF VOLATILITY OF VARIANCE

**ABSTRACT.** The aim of this paper is to describe and measure risk on the Day Ahead Marked (DAM) of the Polish Power Exchange. In this paper downside risk measures such as *Value-at-Risk (VaR)* and *Conditional Value–at–Risk (CVaR)* are presented. These measures were estimated on the basis of the *Generalized Autoregressive Conditional Heteroscedasticity (GARCH)*. They are applied to time series of the logarithmic rate of return of prices from the DAM from March to October 2003. The Kupiec test was used to choose an appropriate heteroscedasticity model to compute *VaR* and *CVaR* and to describe and measure risk on the DAM.

**Key words:** *Generalized Autoregressive Conditional Heteroscedasticity* (GARCH), Generalized Error Distribution (GED), normal distribution, t-Student's distribution, Value – at –Risk (VAR) , Conditional Value – at –Risk (CVaR), failure test.

## I. INTRODUCTION

The Day Ahead Market (DAM) was the first market which was established on the Polish Power Exchange. This whole-day market consists of the twenty-four separate, independent markets. A separate price is established for each hour of the day, one day before the delivery. A price for each hour balances the aggregate supply and demand for this hour.

The advantage of the exchange is that all participants of the market can buy and sell electric energy, irrespective of whether they are producers or receivers.

The empirical results show that the time series on DAM rates of return are not dependent only at the first moment of the data: the volatility of rates of return is characterized with volatility clustering, the rates of return have the leptokurtic distribution and fat-tails, the volatility of rates of return is in inverse correlation with their volatility and the long memory processes in the series of variance, the squares returns data are characterized with the significant autocorrelation coefficients. Moreover, downside risk measures are more effective than the

[*] Ph. D., Department of Statistics, Karol Adamiecki University of Economics, Katowice.

measures of volatility to estimate risk on the electric energy market, where the changes in prices and demand are quick and considerable. Therefore, in order to estimate risk on DAM, we used the *Value-at-Risk (VaR)* and *the Conditional Value–at–Risk (CVaR),* which were calculated on the basis of the *Generalized Autoregressive Conditional Heteroscedasticity (GARCH).*

## II. METHODOLOGY

Engel (1982) introduced **the Autoregressive Conditional Heteroscedasticity (ARCH)** model, which incorporated some of the stylized characteristics common to the second of moment of financial basset price information into the variance equation. A more generalized version of ARCH, **the Generalized Autoregressive Conditional Heteroscedasticity (GARCH)**, was formulated by Bollerslev (1986):

$$Z_t = \mu + \sqrt{h_t}\,\varepsilon_t, \tag{1}$$

$$h_t = c_0 + \sum_{i=1}^{q} c_i Z_{t-i}^2 + \sum_{i=1}^{p} b_i h_{t-1}, \tag{2}$$

where:

$\mu$ – mean of rates of return,

$\varepsilon_t$ – noise

$Z_t = \ln(\frac{X_t}{X_{t-1}})$ – logarithmic rates of return

$h_t$ – conditional variance.

$c_0, c_q, b_p > 0$, if $\sum_{i=0}^{q} c_i + \sum_{i=1}^{p} b_i < 1$, then the time series $Z_t$ – is strictly stationary.

An effective method used to estimate the coefficients in GARCH(p, q) models is the maximum likelihood method (ML).

**The *Value-at-Risk (VaR)*** is such a loss of value which is not exceeded with the given probability over a predefined time period (Jajuga (2000)). *VaR* is a number that represents an estimate of how much value may be lost due to market movements for a particular time horizon and for a given confidence level. If we used volatility of variance (2) to calculate *VaR,* we can write:

$$VaR_{\alpha} = (e^{F^{-1}(\alpha)\sqrt{h_t}+\mu} - 1)P_0, \tag{3}$$

where:

$F^{-1}(\alpha)$ - is $\alpha$ - quintile of $\varepsilon_t$ distribution of equation (1),

$P_0$ – is a present price (price of 1MWh electric energy).

The *VaR* quantity represents the maximum possible loss which is not exceeded with the given probability. **The *Conditional Value–at–Risk (CVaR)*** quantity is the conditional expected loss given the loss strictly exceeds its *VaR* (Rockafellar and Uryasev (2000)):

$$CVaR_{\alpha}(X) = \mathrm{ES}_{\alpha}(X) = E\{X \mid X \leq VaR_{\alpha}\}, \tag{4}$$

where:

$X$ – random variable,

To estimate the effectiveness of *VaR* we used **the failure test**, which was proposed by Kupiec (1995). We are testing the hypothesis:

$$H_0: \quad \omega = \alpha$$
$$H_1: \quad \omega \neq \alpha$$

where $\omega$ is a proportion of the number of results exceeding $VaR_{\alpha}$ to the number of all results.

The number of the excesses of $VaR_{\alpha}$ has binomial distribution for a given size of the theoretical sample. Consider the test statistic (Kupiec (1995)):

$$LR_{uc} = -2ln[(1-\alpha)^{T-N}\alpha^{N}] + 2ln\left\{\left[1-\left(\frac{N}{T}\right)\right]^{T-N}\left(\frac{N}{T}\right)^{N}\right\}, \tag{5}$$

where:

$N$ – is a number of excesses,

$T$ – is a length of time series,

$\alpha$ – is the given probability of the loss of value not exceeding *VaR*.

Assuming that the null hypothesis is true, the statistic above has an asymptotic $\chi^2$ – distribution with 1 degree of freedom.

## III. EMPIRICAL ANALYSIS

The electric energy volumes and prices feature daily, weekly and yearly seasonal peaks and lows. To eliminate the daily periodicity, these twenty four separate markets were grouped into three clusters, respectively associated with hours of the day: {1–6}, {7–19}, {20–24}. The division is based on classification results presented in Ganczarek (2003). Consequently, three series of logarithmic rates of return were identified, each associated with the corresponding group. Further analysis is carried out for these three time series. Next, the time series of rates of return of electric energy prices noted in groups of hours {1–6}, {7–19}, {20–24} were described by Generalized Autoregressive Conditional Heteroscedasticity GARCH(1,1) models. We considered GARCH(1,1) models with the distributions of residuals: normal, t-Student and GED (Ganczarek (2006)). Next we estimated the VaR using GARCH(1,1) models based on equation (3) in all three groups of hours (table 1– table 3). Already in the initial analysis in tables 1–3 we see that the biggest losses of all presented distributions are obtained based on the results of *VaR* using GARCH with t-Student distribution (because t-Student distribution is the fattest tailed of all presented distribution).

Table 1

Values of *VaR* estimated using GARCH(1,1) models on DAM for hours {1–6} from 30.03.03 to 25.10.03

| Residual distribution | $\alpha$ | 0.01 | 0.05 | 0.95 | 0.99 |
|---|---|---|---|---|---|
| Normal | $F_N^{-1}(\alpha)$ | –2.33 | –1.64 | 1.64 | 2.33 |
| | $VaR_\alpha$ | –6.43 | –4.53 | 4.59 | 6.49 |
| t–Student | $F_S^{-1}(\alpha)$ | –3.75 | –2.13 | 2.13 | 3.75 |
| | $VaR_\alpha$ | –9.00 | –5.11 | 5.17 | 9.06 |
| GED | $F_G^{-1}(\alpha)$ | –2.84 | –1.77 | 1.77 | 2.84 |
| | $VaR_\alpha$ | –7.33 | –4.56 | 4.62 | 7.39 |

Source: own elaboration.

Table 2

Values of *VaR* estimated using GARCH(1,1) models on DAM for hours {7–19} from 30.03.03 to 25.10.03

| Residual distribution | $\alpha$ | 0.01 | 0.05 | 0.95 | 0.99 |
|---|---|---|---|---|---|
| Normal | $F_N^{-1}(\alpha)$ | –2.33 | –1.64 | 1.64 | 2.33 |
| | $VaR_\alpha$ | –20.62 | –14.54 | 14.69 | 20.77 |
| t–Student | $F_S^{-1}(\alpha)$ | –3.37 | –2.02 | 2.02 | 3.37 |
| | $VaR_\alpha$ | –27.02 | –16.15 | 16.30 | 27.17 |
| GED | $F_G^{-1}(\alpha)$ | –3.06 | –1.90 | 1.90 | 3.06 |
| | $VaR_\alpha$ | –25.04 | –15.49 | 15.64 | 25.19 |

Source: own elaboration.

Table 3

Values of *VaR* estimated using GARCH(1,1) models on DAM for hours {20–24} from 30.03.03 to 25.10.03

| Residual distribution | $\alpha$ | 0.01 | 0.05 | 0.95 | 0.99 |
|---|---|---|---|---|---|
| Normal | $F_N^{-1}(\alpha)$ | –2.33 | –1.64 | 1.64 | 2.33 |
| | $VaR_\alpha$ | –11.48 | –8.09 | 8.18 | 11.57 |
| t–Student | $F_S^{-1}(\alpha)$ | –4.54 | –2.35 | 2.35 | 4.54 |
| | $VaR_\alpha$ | –23.50 | –12.16 | 12.25 | 23.59 |
| GED | $F_G^{-1}(\alpha)$ | –2.83 | –1.68 | 1.68 | 2.83 |
| | $VaR_\alpha$ | –14.28 | –8.45 | 8.54 | 14.38 |

Source: own elaboration.

For example when we look at *VaR* estimated using GARCH(1,1) models on DAM for hours {20–24} and with t-Student distribution (table 3) we can say, that if we take short position with the probability of 0.99, on the next day we will not lose more than 23.50 PLN/MWh. The results obtained for $CVaR_{99\%}$ inform about the average of 1% of the biggest loss. For example $CVaR_{99\%}$= –32.65 PLN/MWh (table 4) means, that the average of 1% of the worst losses equals –32.65 PLN/MWh.

Table 4

Values of $CVaR$, estimated using GARCH(1,1) models on DAM from 30.03.03 to 25.10.03

| Hour | Distribution | $CVaR_{0.01}$ | $CVaR_{0.05}$ | $CVaR_{0.95}$ | $CVaR_{0.99}$ |
|---|---|---|---|---|---|
| {1 –6} | Normal | –13.43 | –9.47 | 9.53 | 13.49 |
| | t–Student | –20.66 | –11.74 | 11.80 | 20.72 |
| | GED | –16.09 | –10.03 | 10.09 | 16.15 |
| {7 –19} | Normal | –22.03 | –15.53 | 15.68 | 22.18 |
| | t–Student | –38.02 | –22.74 | 22.88 | 38.17 |
| | GED | –33.81 | –20.91 | 21.06 | 33.95 |
| {20 –24} | Normal | –12.19 | –8.59 | 8.69 | 12.28 |
| | t–Student | –32.65 | –16.90 | 16.99 | 32.74 |
| | GED | –19.51 | –11.54 | 11.64 | 19.60 |

Source: own elaboration.

The results of the Kupiec test (5) for $VaR$, which have been estimated on the basis of GARCH(1,1) models with the distributions of residuals: normal, t-Student and GED, are presented in tables 5–7.

Table 5

The results of the Kupiec test $LR_{uc}$ for $VaR$, estimated using GARCH(1,1) models on DAM for hours {1–6} from 30.03.03 to 25.10.03

| $T$ = 1252 | $VaR_{0.01}$ | $VaR_{0.05}$ | $VaR_{0.95}$ | $VaR_{0.99}$ |
|---|---|---|---|---|
| The number of the excesses of $VaR_{\alpha}$ – $N$ | | | | |
| Normal | 27 | 61 | 57 | 30 |
| t-Student | 9 | 31 | 38 | 11 |
| GED | 15 | 52 | 47 | 19 |
| The proportion of the number of results exceeding $VaR_{\alpha}$ to the number of all results – $N/T$ | | | | |
| Normal | 0.02 | 0.05 | 0.05 | 0.02 |
| t-Student | 0.01 | 0.02 | 0.03 | 0.01 |
| GED | 0.01 | 0.04 | 0.04 | 0.02 |
| The value of statistic $LR_{uc}$ | | | | |
| Normal | 12.709 | 0.043** | 0.543** | 17.720 |
| t-Student | 1.108** | 20.460 | 11.768 | 0.194** |
| GED | 0.467** | 2.000** | 4.462* | 2.924** |

Source: own elaboration.

Table 6

The results of the Kupiec test $LR_{uc}$ for $VaR$, estimated using GARCH(1,1) models on DAM for hours {7–19} from 30.03.03 to 25.10.03

| $T = 2715$ | $VaR_{0.01}$ | $VaR_{0.05}$ | $VaR_{0.95}$ | $VaR_{0.99}$ |
|---|---|---|---|---|
| The number of the excesses of $VaR_{\alpha}$ – $N$ | | | | |
| Normal | 26 | 103 | 163 | 52 |
| t-Student | 2 | 27 | 36 | 4 |
| GED | 3 | 32 | 49 | 7 |
| The proportion of the number of results exceeding $VaR_{\alpha}$ to the number of all results – $N/T$ | | | | |
| Normal | 0.01 | 0.04 | 0.06 | 0.02 |
| t-Student | 0.00 | 0.01 | 0.01 | 0.00 |
| GED | 0.00 | 0.01 | 0.02 | 0.00 |
| The value of statistic $LR_{uc}$ | | | | |
| Normal | 0.05** | 9.04 | 5.43* | 18.12 |
| t-Student | 40.10 | 134.81 | 107.74 | 31.18 |
| GED | 35.30 | 119.13 | 76.52 | 21.47 |

Source: own elaboration.

Table 7

The results of the Kupiec test $LR_{uc}$ for $VaR$, estimated using GARCH(1,1) models on DAM for hours {20–24} from 30.03.03 to 25.10.03

| $T = 1043$ | $VaR_{0.01}$ | $VaR_{0.05}$ | $VaR_{0.95}$ | $VaR_{0.99}$ |
|---|---|---|---|---|
| The number of the excesses of $VaR_{\alpha}$ – $N$ | | | | |
| Normal | 15 | 39 | 50 | 19 |
| t-Student | 3 | 13 | 19 | 2 |
| GED | 8 | 34 | 47 | 10 |
| The proportion of the number of results exceeding $VaR_{\alpha}$ to the number of all results – $N/T$ | | | | |
| Normal | 0.01 | 0.04 | 0.05 | 0.02 |
| t-Student | 0.00 | 0.01 | 0.02 | 0.00 |
| GED | 0.01 | 0.03 | 0.05 | 0.01 |
| The value of statistic $LR_{uc}$ | | | | |
| Normal | 1.781** | 3.810** | 0.095** | 5.722* |
| t-Student | 7.437 | 43.708 | 29.029 | 10.322 |
| GED | 0.622** | 7.543 | 0.553** | 0.018** |

[*means, that on significance level 0.01 we do not reject the null hypothesis
**means, that on significance level 0.05 we do not reject the null hypothesis]

Source: own elaboration.

## IV. CONCLUSION

Based on the results from Ganczarek (2006), we can say that time series of rates of return of electric energy prices are described most accurately by the t-Student distribution (especially in tail of distributions). But *VaR*, which were estimated based on GARCH models with t-Student distribution of residuals, are overestimated. Surprisingly, the ratio of the number of results exceeding $VaR_a$ to the number of all results is very small.

To calculate *VaR* based on GARCH models, the models with normal distribution of residuals were the most useful (with GED distribution of residuals being the second best).

The t – Student distribution, which has the fattest tails should be rejected, because *VaR* calculated by GARCH models using this distribution are overestimated.

## REFERENCES

Bollerslev T. (1986), Generalized autoregressive conditional heteroscedasticy, J. Econometrics, 31, 307–328.

Engel R. F. (1982), Autoregressive conditional heteroscedasticy with estimates of the variance of United Kingdom inflation, Econometrica, 50, 987–1007.

Ganczarek A, (2003), Klasyfikacja polskiego rynku energii, Inżynieria Ekonomiczna w Badaniach Społeczno-Gospodarczych (red. M. J. Król), PRz, Rzeszów, 51–66.

Ganczarek A. (2006), Modele autoregresyjne na Rynku Dnia Następnego Towarowej Giełdy Energii S.A., XLII[th] Conference of SEMPP, AE Wrocław, (in press).

Jajuga K. (red.) (2000), Metody ekonometryczne i statystyczne w analizie rynku kapitałowego, AE, Wrocław.

Kupiec P. (1995), Techniques for verifying the accuracy of risk management models, Journal of Derivatives, 2, 173–184.

Rockafellar R. T. and Uryasev S. (2000), Optimization of Conditional Value-at-Risk. Journal of Risk, 2, 21–41.

*Alicja Ganczarek*

## *VaR* W ANALIZIE RYZYKA NA RDN A MODLE ZMIENNOŚCI WARIANCJI

W pracy przeprowadzono analizę ryzyka na Rynku Dnia Następnego (RDN) Towarowej Giełdy Energii. Do pomiaru ryzyka zmiany ceny na RDN wykorzystano wartości zagrożone: *Value-at-Risk (VaR)* oraz *Conditional Value–at–Risk (CVaR)*, oszacowane na podstawie modeli z warunkową wariancją: *Generalized Autoregressive Conditional Heteroscedasticity (GARCH)*. Do oceny efektywności oszacowanych wartości *VaR* oraz *CVaR* wykorzystano test przekroczeń Kupca. Analizę ryzyka przeprowadzono na szeregach czasowych dziennych logarytmicznych stóp zwrotu cen energii elektrycznej notowanej na RDN w okresie od marca do października 2003.