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Jerzy Pelczewski

ON AN OPTIMIZATION PROBLEM DESCRIBED BY SOME INTEGRAL EQUATIONS

The paper gives the necessary condition of optimality in the case where the optimization problem is described by some integral equations.

Introduction

There are many optimal control problems with a performance index being a functional given by a differential equation (called the state equation). The results obtained in [4] enable one to consider the case where the equality constraints are represented by some integral equation. If the functions appearing in the functional and in the above-mentioned integral equation are of class L_2, the necessary condition of optimality can be found.

Formulation of the problem

In the space $\mathcal{X} = L_2^n \times L_2^n$ let us consider the following optimization problem

$$(1) \qquad I(x, u) = \int_0^1 g(x(t), u(t), t)dt - \min$$

under the conditions

$$(2) \qquad x(t) = \int_0^1 f(x(\tau), u(t), t, \tau)d\tau$$

(3) $$u(t) \in V \quad \text{for} \quad t \in [0, 1] \text{ a.e.}$$

where g is the scalar function

$$g : R^{n+n+1} \to R$$

of the form

(4) $$g(x(t), u(t), t) = v^T(t)H_o(t)v(t) + p_o(t)v(t)$$

and f is the vector function $f : R^{n+n+1+1} \to R$ of the form

(5) $$f(x(\tau), u(t), t, \tau) =$$

$$= (f_1(x(\tau), u(t), t, \tau), \ldots, f_n(x(\tau), u(t), t, \tau))$$

$$f_i(x(\tau), u(t), t, \tau) = x^T(\tau)H_i(\tau)x(\tau) +$$

$$+ p_i(\tau)x(\tau) + u^T(t)G_i(t)u(t)$$

$$i = 1, 2, \ldots, n$$

where: $t \in [0, 1], \tau \in [0, 1]$, $v(t) = (x(t), u(t))^T$, $H_o = H_o(t)$ is an $(n + n) \times (n + n)$ matrix, $p_o = p_o(t)$ is a $1 \times (n + n)$ matrix, $H_i = H_i(\tau)$, $G_i = G_i(t)$, $i = 1, \ldots, n$, are $(n \times n)$ matrices, $p_i = p_i(\tau)$, $i = 1, \ldots, n$, are $(1 \times n)$ matrices. $V \subset R^n$ is a closed convex set in R^n, the vectors $x(\cdot), u(\cdot) \in L_2^n$. We assume that the elements of the matrices H_o, H_i, G_i are measurable and bounded, and that the coordinates of the vectors p_o, p_i belong to L_2^n. The functions f, g have continuous derivatives f_u, f_x, g_u, g_x with respect to u and x.

According to the results obtained in [4], we shall prove the following

Theorem. If $v_o(t) = (x_o(t), u_o(t))$ is the solution of the above-mentioned problem and the following additional assumptions hold:

a) the function f has the property

(6) $$\int_0^1 \int_0^1 |f_x(\mu(t,\tau), t, \tau)|^2 dt\, d\tau < 1$$

b) the matrix

(7) $$A(t) = \int_0^1 f_u(\mu(t,\tau),\ t,\tau)\, d\tau$$

is nonsingular, then there exist a real number $\lambda_0^* \geq 0$ and a vector function $\lambda_1(\cdot) \in L_2^n$, such that

(8) $$|\lambda_0| + \|\lambda_1\| > 0$$

and

(9) $$\lambda_0 g_x(v_0(t),\ t) - \int_0^1 (f_x^T(\mu(\tau,\ t)\ \tau t)\lambda_1(\tau))\, d\tau + \lambda_1(t) = 0$$

(10) $$(\lambda_0 g_u(v_0(t),\ t) - \lambda_1(t))^T u_0(t) = \min_{u \in V} (\lambda_0 g_u(v_0(t),\ t) - \lambda_1(t))^T u(t)$$

where

$$\mu(t,\tau) = (x_0(\tau),\ u_0(t))$$

$$\mu(\tau,\ t) = (x_0(t),\ u_0(\tau))$$

$$f(\mu(t,\tau),\ t,\tau) = f(x_0(\tau),\ u_0(t),\ t,\tau)$$

$$g(v_0(t),\ t) = g(x_0(t),\ u_0(t),\ t)$$

$$f(\mu(\tau,\ t),\ \tau t) = f(x_0(t),\ u_0(\tau),\ \tau,t)$$

P r o o f. First, we shall specify the characteristic cones [1] which enable us to obtain some Euler equation and to find the necessary condition of optimality. We consider the Cartesian product of L_2-spaces

$$X = L_2^n \times L_2^n$$

and denote by Z_1, Z_2 the following sets:

(12) $$Z_1 = \{(x,\ u) \in X;\ \ u(t) \in V\}$$

(13) $$Z_2 = \{(x,\ u) \in X : x(t) = \int_0^1 f(x(\tau),\ u(t),\ t,\tau)\, d\tau\}$$

we notice that our problem becomes

$$I(x, u) \to \min; \quad (x, u) \in Z_1 \cap Z_2 \tag{14}$$

We observe that Z_1 and Z_2 are sets with empty interiors.

The cone of directions of decrease of the functional $I(x,u)$ at the point $(x_0(\cdot), u_0(\cdot)) = v_0(\cdot)$ is (according to [1]) of the form

$$C_0 = \left\{(\bar{x}, \bar{u}) \in X : \int_0^1 (g_x^T(v_0(t), t)\bar{x}(t) + g_u^T(v_0(t), t)\bar{u}(t))dt < 0\right\} \tag{15}$$

and the conjugate cone C_0^* is

$$C_0^* = \left\{f_0 \in X^*: f_0(\bar{x}, \bar{u}) = -\lambda_0 \int_0^1 (g_x^T(v_0(t), t)\bar{x}(t) + g_u^T(v_0(t), t)\bar{u}(t))dt, \ \lambda_0 \geq 0\right\} \tag{16}$$

Next, we denote by C_1^* the set of functionals supporting Z_1 at the point $\mu(\cdot,\cdot)$. As we know (theorem 10.5 in [1]), C_1^* is the cone conjugate to the cone of tangent directions of the set Z_1 at the point $v_0(\cdot,\cdot)$. Hence

$$C_1^* = \left\{f_1 \in X': f_1(\bar{x}, \bar{u}) = f_1'(\bar{u})\right\} \tag{17}$$

where f_1' - functional supporting the set $U = \left\{u \in L_2^n : u(t) \in V\right\}$ at the point $u_0(\cdot)$.

Applying the Lusternik theorem [1], we shall find the cone of directions tangent to Z_2 at the point $\mu(\cdot,\cdot)$. Let us consider the operator

$$P : L_2^n \times L_2^n \to L_2^n$$

of the form

$$P(x, u) = x(t) - \int_0^1 f(x(\tau), u(t), t, \tau)\, d\tau \tag{18}$$

We have

$$P(x + \bar{x}, u + \bar{u}) - P(x, u) = \tag{19}$$

$$= \bar{x}(t) - \int_0^1 (f_x(x(\tau), u(t), t, \tau)\bar{x}(\tau) +$$

$$+ f_u(x(\tau), u(t), t,\tau)\bar{u}(t))d\tau +$$

$$+ 0\sqrt{(\|\bar{x}\|^2_{L_2} + \|\bar{u}\|^2_{L_2})}$$

Note that the operator $S : L_2^n \times L_2^n \to L_2^n$ of the form

$$S(\bar{x}, \bar{u}) = \bar{x}(t) - \int_0^1 (f_x(x(\tau), u(t), t,\tau)\bar{x}(\tau) + f_u(x(\tau), u(t), t,\tau)\bar{u}(t))d\tau \tag{20}$$

is a linear operator with respect to $(\bar{x}, \bar{u})$. Hence we find that $P(\bar{x}, \bar{u})$ is differentiable and

$$P'(x, u)(\bar{x}, \bar{u}) = \bar{x}(t) - \int_0^1 (f_x(x(\tau), u(t), t,\tau)\bar{x}(\tau) + f_u(x(\tau), u(t), t,\tau)\bar{u}(t))d\tau \tag{21}$$

We shall show that $P'(x, u)$ maps $L_2^n \times L_2^n$ onto the whole space L_2^n. This means that the equation

$$\bar{x}(t) - \int_0^1 (f_x(x(\tau), u(t), t,\tau)\bar{x}(\tau) + f_u(x(\tau), u(t), t, \tau)\bar{u}(t))d\tau = a(t) \tag{22}$$

has the solution $(\bar{x}, \bar{u})$ for any function $a(t) \in L_2^n$. If we put $\bar{u}(t) \equiv 0$, formula (22) takes the form

$$\bar{x}(t) - \int_0^1 f_x(x(\tau), u(t), t,\tau)\bar{x}(\tau)d\tau = a(t) \tag{23}$$

It is known [3] that Fredholm's linear integral equation (23) has a unique solution $\bar{x}(t)$ for any function $a(t) \in L_2^n$ in the case where assumption (6) is satisfied.

According to [1], the cone of directions tangent to Z_2 at the point $\mu(\cdot, \cdot)$ is of the form

(24)
$$C_2 = \Big\{(\bar{x}, \bar{u}) \in X : \bar{x}(t) = \int_0^1 (f_x(\mu(t,\tau), t,\tau)\bar{x}(\tau) + f_u(\mu(t,\tau), t,\tau)\bar{u}(t))\, d\tau\Big\} =$$
$$= \Big\{(\bar{x}, \bar{u}) \in X : \bar{x}(t) = \int_0^1 f_x(\mu(t,\tau), t,\tau)\bar{x}(\tau)d\tau + \Big(\int_0^1 f_u(\mu(t,\tau), t,\tau)d\tau\Big)\bar{u}(t)\Big\}.$$

and, with A(t) from (7), we obtain

(25)
$$C_2 = \Big\{(\bar{x}, \bar{u}) \in X : \bar{x}(t) = \int_0^1 f_x(\mu(t,\tau), t,\tau)\bar{x}(\tau)d\tau + A(t)\bar{u}(t)\Big\}$$

The conjugate cone becomes

(26)
$$C_2^* = \Big\{(f_2^x, f_2^u) \in X : f_2^x(\bar{x}) + f_2^u(\bar{u}) = 0, \quad \forall (\bar{x}, \bar{u}) \in C_2\Big\}$$

where the functionals f_2^x and f_2^u belong to L_2^n.

We denote the values of those functionals on the elements $\bar{x}$ and $\bar{u}$, respectively, by

(27)
$$f_2^x(\bar{x}) = \int_0^1 (\psi_2^x(t))^T \bar{x}(t)dt$$

(28)
$$f_2^u(\bar{u}) = \int_0^1 (\tilde{\psi}_2^u(t))^T \bar{u}(t)dt$$

where $\psi_2^x \in L_2^n$, $\tilde{\psi}_2^u \in L_2^n$.

Putting $f_2^x(\bar{x})$ from (27) and $f_2^u(\bar{u})$ from (28) into (26) we obtain

(29) $$C_2^* = \Big\{f_2 \in X^*: \ f_2(\bar{x}, \bar{u}) = \int_0^1 (\Psi_2^x(t))^T \bar{x}(t)dt + \\ + \int_0^1 (\tilde{\Psi}_2^u(t))^T \bar{u}(t)dt = 0, \quad \forall(\bar{x}, \bar{u}) \in C_2\Big\}$$

We rewrite it in the form

(30) $$C_2^* = \Big\{f_2 \in X^*: \ f_2(\bar{x}, \bar{u}) = \\ = \int_0^1 \Psi_2^x(t)^T \bar{x}(t)dt + \\ + \int_0^1 (\tilde{\Psi}_2^x(t))^T A^{-1}(t)A(t)\bar{u}(t)dt = 0, \quad \forall(\bar{x}, \bar{u}) \in C_2\Big\}$$

with $A(t)$ from (7).

By $\Psi_2^u \in L_2^n$ we denote the vector satisfying the formula

(31) $$(\Psi_2^u(t))^T = (\tilde{\Psi}_2^u(t))^T A^{-1}(t)$$

Then we find from (30)

(32) $$C_2^* = \Big\{f_2 \in X^*: \ f_2(\bar{x}, \bar{u}) = \int_0^1 ((\Psi_2^x(t))^T \bar{x}(t) + \\ + (\Psi_2^u(t))^T A(t)\bar{u}(t))dt, \ \forall(\bar{x}, \bar{u}) \in C_2\Big\}$$

According to (25), we have

$$A(t)\bar{u}(t) = \bar{x}(t) - \int_0^1 f_x(\mu(t,\tau),t,\tau)\bar{x}(\tau)d\tau$$

Hence

(33) $$f_2(\bar{x}, \bar{u}) = \int_0^1 ((\Psi_2^x(t))^T \bar{x}(t) + (\Psi_2^u(t))^T (\bar{x}(t) - \\ - \int_0^1 f_x(\mu(t,\tau),t,\tau)\bar{x}(\tau)d\tau)dt = 0$$

We rewrite the right side of (33) as

$$\int_0^1 ((\psi_2^x(t))^T \bar{x}(t) + (\psi_2^u(t))^T \bar{x}(t))dt -$$

$$- \int_0^1 (\int_0^1 < \psi_2^u(t),\ f_x(\mu(t,\ \tau), t, \tau)\bar{x}(\tau) > d\tau)dt =$$

$$= \int_0^1 ((\psi_2^x(t))^T \bar{x}(t) + (\psi_2^u(t))^T\ \bar{x}(t))dt -$$

$$- \int_0^1 (\int_0^1 < f_x^T(\mu(t,\tau),\ t, \tau)\psi_2^u(t),\ \bar{x}(\tau) > d\tau)dt =$$

$$= \int_0^1 ((\psi_2^x(t))^T \bar{x}(t) + (\psi_2^u(t))^T \bar{x}(t))dt -$$

$$- \int_0^1 (\int_0^1 < f_x^T(\mu(\tau,\ t), \tau,\ t)\psi_2^u(\tau),\ \bar{x}(t) > dt)d\tau =$$

$$= \int_0^1 ((\psi_2^x(t))^T \bar{x}(t) + (\psi_2^u(t)^T\ \bar{x}(t))dt -$$

$$- \int_0^1 (\int_0^1 (f_x^T(\mu(\tau,\ t), \tau,\ t)\psi_2^u(\tau))^T\ \bar{x}(t)dt)d\tau$$

Denoting by $B(t)$ the integral

$$\int_0^1 (f_x^T\ (\mu(\tau,\ t), \tau,\ t)\ \psi_2^u(\tau))^T d\tau = B^T(t) \tag{34}$$

we obtain

$$\int_0^1 ((\psi_2^x(t))^T + (\psi_2^u(t))^T - B^T(t)\bar{x}(t))dt = 0, \tag{35}$$

$$\forall \bar{x} \in L_2^n$$

Hence

$$(\psi_2^x(t))^T + (\psi_2^u(t))^T - B^T(t) = 0 \quad \text{for } t \in [0,\ 1] \text{ a.e.} \tag{36}$$

or, with $B^T(t)$ from (34),

(37) $$(\psi_2^x(t))^T + (\psi_2^u(t))^T - \int_0^1 (f_x^T(\mu(\tau, t), \tau, t)\psi_2^u(\tau))^T d\tau = 0$$

Next, we observe that:

the cone C_0 is open and convex,

the cones C_1 and C_2 are convex,

the cones C_1^* and C_2^* are of the same sense (according to theorem 3.4 in [4]).

We shall prove (in the lemma presented afterwards) that the intersection of the cones C_1 and C_2 is a subset of a cone tangent to $Z_1 \cap Z_2$. Then we conclude that the assumptions of theorem 4.1 in [4] are satisfied.

This enables us to apply the Euler equation of the form

(38) $$-\lambda_0 \int_0^1 (g_x^T(v_0(t), t)\bar{x}(t) + g_u^T(v_0(t), t)\bar{u}(t))dt + f_1'(\bar{u}) + \int_0^1 (\psi_2^x(t))^T \bar{x}(t)dt + \int_0^1 (\psi_2^u(t))\bar{u}(t)dt = 0$$

where f_1' - as in formula (17).

Equation (38) is satisfied for an $(\bar{x}, \bar{u}) \in X$; hence

(39) $$-\lambda_0 g_x^T(v_0(t), t) + (\psi_2^x(t))^T = 0$$

(40) $$f_1'(\bar{u}) = \int_0^1 (\lambda_0 g_u(v_0(t), t) - \psi_2^u(t))^T \bar{u}(t)dt$$

Since f_1 is a functional supporting the set $U = \{u \in L_2^n; u(t) \in V\}$ at the point $u_0(\cdot)$, we conclude from (40) that

(41) $$(\lambda_0 g_u(v_0(t), t) - \psi_2^u(t))^T u_0(t) = \min_{u \in V} (\lambda_0 g_u(v_0(t), t) - \psi_2^u(t))^T u(t)$$

With $(\psi_2^x(t))^T$ from (37), equation (39) becomes

(42) $$\lambda_0 g_x^T(v_0(t), t) + (\psi_2^u(t))^T - \int_0^1 (f_x^T(\mu(\tau, t), \tau, t)\psi_2^u(\tau))^T d\tau = 0$$

With $\Psi_2{}^u(\cdot) = \lambda_1(\cdot)$, we obtain from (41) and (42)

$$(43)\qquad (\lambda_0 g_u(\nu_0(t),\ t) - \lambda_1(t))^T u_0(t) =$$

$$= \min_{u\in V} (\lambda_0 g_u(\nu_0(t),\ t) - \lambda_1(t))^T u(t)$$

$$(44)\qquad \lambda_0 g_x(\nu_0(t),\ t) + \lambda_1(t) -$$

$$- \int_0^1 f_x{}^T(\mu(\tau,\ t),\tau,\ t)\lambda_1(\tau)d\tau = 0$$

In the case where $\lambda_0 = 0$ and $\lambda_1(\cdot) \equiv 0$, we conclude that $f_0(\bar{x},\ \bar{u}) = f_1(\bar{u}) = f_2(\bar{x},\ \bar{u}) = 0$. In that case, the condition of the existence of functionals f_0, f_1, f_2 not all equal to zero is not satisfied. Hence

$$|\lambda_0| + \|\lambda_1\| > 0$$

That completes the proof.

<u>Lemma</u>. The intersection of the cones C_1 and C_2 is a subset of a cone tangent to $Z_1 \cap Z_2$.

P r o o f. It has been shown that the operator P from formula (18) is differentiable and the operator $P'(x,\ u)$ from (21) maps $L_2{}^n \times L_2{}^n$ onto the whole space $L_2{}^n$. Hence we conclude that P satisfies the assumptions of the implicit function theorem in some neighbourhood V_0 of $(x_0,\ u_0)$, which implies that the set Z_2 can be represented in this neighbourhood in the form

$$(45)\qquad Z_2 = \left\{(x,\ u) \in X :\ x = \varphi(u)\right\}$$

where $\varphi : L_2{}^n \to L_2{}^n$ is the C^1-class operator satisfying the condition

$$P(\varphi(u),\ u) = 0$$

for all u such that $(\varphi(u),\ u) \in V_0$. We deduce that the cone C_2 can be represented in the form

$$(46)\qquad C_2 = \left\{(\bar{x},\ \bar{u}) \in X :\ \bar{x} = \varphi_u(u_0)\bar{u}\right\}$$

Let $(\bar{x}, \bar{u})$ be an arbitrary element of the set $C_1 \cap C_2$. Then there exists an operator $\nu_u{}^2 : R \to V$ such that

$$\frac{\nu_u{}^2(\varepsilon)}{\varepsilon} \xrightarrow[\varepsilon \to 0^+]{} 0$$

and the formula

(47) $$(x_0, u_0) + \varepsilon(\bar{x}, \bar{u}) + (\nu_x^2(\varepsilon), \nu_u{}^2(\varepsilon)) \in Z_1$$

holds for sufficiently small ε and any $\nu_x{}^2(\varepsilon)$, such that

$$\frac{\nu_x{}^2(\varepsilon)}{\varepsilon} \xrightarrow[\varepsilon \to 0^+]{} 0$$

Hence, according to (45), we observe that, for sufficiently small ε, the following formula holds:

(48) $$(\varphi(u_0 + \varepsilon\bar{u} + \nu_u{}^2(\varepsilon)), u_0 + \varepsilon\bar{u} + \nu_u{}^2(\varepsilon)) \in Z_2$$

$\varphi(u)$ is a differentiable operator, hence

(49) $$\varphi(u_0 + \varepsilon\bar{u} + \nu_u{}^2(\varepsilon)) = \varphi(u_0) + \varepsilon\varphi_u(u_0)\bar{u} + \nu_x{}^1(\varepsilon)$$

for some $\nu_x{}^1$ such that $\lim\limits_{\varepsilon \to 0^+} \dfrac{\nu_x{}^1(\varepsilon)}{\varepsilon} = 0$.

From (48) and (49) we obtain

(50) $$(\varphi(u_0) + \varepsilon\varphi_u(u_0)\bar{u} + \nu_x{}^1(\varepsilon), u_0 + \\ + \varepsilon\bar{u} + \nu_u{}^2(\varepsilon)) \in Z_2$$

and, since

(51) $$(\varphi(u_0) + \varepsilon\varphi_u(u_0)\bar{u} + \nu_x{}^1(\varepsilon), u_0 + \\ + \varepsilon\bar{u} + \nu_u{}^2(\varepsilon)) = (x_0, u_0) + \varepsilon(\bar{x}, \bar{u}) + \\ + (\nu_x{}^1(\varepsilon), \nu_u{}^2(\varepsilon))$$

we find

(52) $$(x_0, u_0) + \varepsilon(\bar{x}, \bar{u}) + (\nu_x{}^1(\varepsilon), \nu_u{}^2(\varepsilon)) \in Z_2$$

If we take $\nu_x^{2}(\varepsilon) = \nu_x^{1}(\varepsilon)$, we conclude from (47) and (52) that the vector $(\bar{x}, \bar{u}) \in C_1, \cap C_2$ is tangent to the set $Z_1 \cap Z_2$. The arbitrariness of $(\bar{x}, \bar{u})$ completes the proof.

Example

Consider the minimization of the functional

$$I(x, u) = \int_0^1 \left([3,\ 2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + [1,\ 2] \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \right) dt$$

with the equality constraint

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \int_0^1 \left(\begin{bmatrix} 1/2 & 0 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} x_1(\tau) \\ x_2(\tau) \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2\ (t) \end{bmatrix} \right) d\tau$$

and the inequality constraints imposed on u(t):

$$|u_k(t)| \le \alpha_k, \quad k = 1,\ 2$$

From (9) we obtain

$$\begin{bmatrix} \lambda_{1,1}(t) \\ \lambda_{1,2}(t) \end{bmatrix} \begin{bmatrix} -3\lambda_0 + & 1/2\lambda_{1,1}(\tau)d\tau \\ -2\lambda_0 + & 1/3\lambda_{1,2}(\tau)d\tau \end{bmatrix}$$

the solution of the above integral equation is

$$\begin{bmatrix} \lambda_{1,1}(t) \\ \lambda_{1,2}(t) \end{bmatrix} = \begin{bmatrix} -6\lambda_0 \\ -3\lambda_0 \end{bmatrix}$$

According to (10), we find

$$\min_{u \in V} \left[\left(\begin{bmatrix} \lambda_0 \\ 2\lambda_0 \end{bmatrix} - \begin{bmatrix} -6\lambda_0 \\ -3\lambda_0 \end{bmatrix} \right)^T \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \right] =$$

$$= \min_{u \in V} \lambda_0 (7u_1(t) + 5u_2(t))$$

Hence

$$u_{0,1} = -\alpha_1, \quad u_{0,2} = -\alpha_2$$

and, from

$$x_{0,1}(t) = -2\alpha_1 + \int_0^1 1/2x_{0,1}(\tau)d\tau$$

$$x_{0,2}(t) = -\alpha_2 + \int_0^1 1/3x_{0,2}(\tau)d\tau$$

we find

$$x_{0,1} = -4\alpha_1, \quad x_{0,2} = -3/2\alpha_2$$

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Institute of Mathematics
The University of Łódź

Jerzy Pelczewski

O ZAGADNIENIU OPTYMALIZACYJNYM
OPISANYM PRZEZ PEWNE RÓWNANIA CAŁKOWE

W pracy uzyskano warunek konieczny optymalności dla układu opisanego za pomocą pewnych równań całkowych.