ACTA UNIVERSITATIS LODZIENSIS
FOLIA MATHEMATICA 2, 1987

Urszula Łędzewicz-Kowalewska

THE EXTREMUM PRINCIPLE FOR PROBLEMS OF OPTIMAL CONTROL WITH MIXED CONSTRAINTS

In the paper the extremum principle for problems of optimal control with equality constraints on the phase coordinates and the control is proved by using the generalization of the Dubovitskii-Milyutin method from [14].

Introduction

The problem of optimal control with equality and inequality constraints on the phase coordinates and the control was investigated in [3], [7], [1], [4], [5], [12]. In these papers the extremum principle for this problem was obtained by making use of the variational method under the assumption of "regular controllability" but what is important is that there was no nonoperator constraint in the form $u(\cdot) \in U$ where U - some set. The impossibility of addition of these constraints follows from the properties of the variational method applied in the above papers.

Another method, which is applied to obtain the necessary conditions in optimal control is the Dubovitskii-Milyutin method presented in [9]. But using this method, we can obtain the necessary conditions for the optimal control problems with only one equality constraint. In [14] there was obtained some generalization of the Dubovitskii-Milyutin method for the case of n equality constraints. The result was obtained under the assumption of the same sense of cones which implies the weak* closure of the algebraic sum.

By using the results from [14], in [15] Walczak obtained the extremum principle for problems of optimal control with equality constraints on the phase coordinates and in [16] for problems of optimal control with the nonoperator equality constraint.

In the present paper the extremum principle for problems of optimal control with equality constraints on the phase coordinates and the control is proved by using the generalization of the Dubovitskii-Milyutin method from [14]. The problem considered here differs from that of [15] in that the equality constraint depends on the control (not only on the phase coordinates) and in the method of calculations of cones. The extremum principle for this problem is proved under the assumption of "strong regular controllability" which (in a weaker form) was presented in [1], [3] and [7], but the problems from these papers do not contain the nonoperator constraint $u(\cdot) \in U$ and the method of proof of the extremum principle is different.

1. Basic definitions

D e f i n i t i o n 1. Let $L_1^p(0, 1)$ be the space of functions Lebesgue - integrable on the interval $[0, 1]$ with values from R^p, with the norm

$$\|x\| = \int_0^1 |x(t)|\,dt$$

D e f i n i t i o n 2. Denote by $L_\infty^p(0, 1)$ the space of functions measurable and essentially bounded on the interval $[0,1]$ with values from R^p, with the norm

$$\|x\| = \sup_{0 \le t \le 1} \operatorname{vrai} |x(t)|$$

D e f i n i t i o n 3. Denote by $W_{11}^p(0, 1)$ the space of absolutely continuous functions whose derivatives $\dot{x} \in L_1^p(0, 1)$. The norm in $W_{11}^p(0, 1)$ is defined by the formula

$$\|x\| = |x(0)| + \int_0^1 |\dot{x}(t)|\,dt$$

- Definition 4. Let $\overline{W}^P_{11}(0, 1)$ be a subspace of the space $W^P_{11}(0, 1)$ which contains functions satisfying the condition $x(0) = 0$. The norm in $\overline{W}^P_{11}(0, 1)$ is of the form

$$\|x\| = \int_0^1 |\dot{x}(t)|\, dt$$

Definition 5. Let (Σ, μ) stand for a space with the Lebesgue measure defined on the Σ-field of subsets of $[0, 1]$. Denote by $b.a\ (0, 1)$ the family of additive functions $\varphi : \Sigma \longrightarrow R$ satisfying the conditions:

a) if $A \in \Sigma$ and $\mu(A) = 0$, then $\varphi(A) = 0$,

b) the variation of the function φ is bounded, i.e. $|\varphi|_{(0,1)} < \infty$.

The space $b.a\ (0, 1)$ is a Banach space with the norm $\|\varphi\| = |\varphi|_{(0, 1)}$ (cf. [6], Part III, § 7).

Remark 1. Let $b^P.a\ (0, 1)$ be the space of functions $\varphi : \Sigma \longrightarrow R^P$ in the form

$$\varphi(A) = (\varphi_1(A) \ldots \varphi_p(A))$$

where $A \in \Sigma$, $\varphi_i \in b.a(0, 1)$ for $i = 1, 2, \ldots, P$.

It is easy to show that the space $b^P.a\ (0, 1)$ is a Banach space with the norm

$$\|\varphi\| = \sum_{i=1}^{P} |\varphi_i|_{(0, 1)}$$

Remark 2. It can be proved that the space dual to $L^P_\infty(0, 1)$ is the space $b^P.a\ (0, 1)$ (cf. [6], Part III, § 7 and [17], pp. 41, 42).

Making use of the formula for a linear and continuous functional on $L^1_\infty(0, 1)$, we obtain that any linear and continuous functional on $L^P_\infty(0, 1)$ has the form

$$\int_0^1 f\, d\omega = \sum_{i=1}^{P} \int_0^1 f_i\, d\omega_i$$

where $f = (f_1, f_2, \ldots f_p) \in L^P_\infty(0, 1)$, $\omega \in b^P.a(0, 1)$, $\omega_i \in b.a(0,1)$ for $i = 1, 2, \ldots P$.

2. Some properties of cones - auxiliary lemmas

We prove some properties of cones of the same sense and the dual cones.

Lemma 1. Let X be a linear normed space, $\{C_i\}_{i=1}^n$ a system of cones in X. If $A: X \xrightarrow{onto} X$ is a linear homeomorphism, then the system of cones $\{C_i\}_{i=1}^n$ is of the same sense if and only if the system of cones $\{AC_i\}_{i=1}^n$ is of the same sense.

P r o o f. Let us assume that the system $\{C_i\}_{i=1}^n$ is of the same sense. From the definition of cones of the same sense (cf.[14], def. 1) we have

$$\forall M>0 \quad \exists M_1>0, \dots M_n>0 \quad \forall x=\sum_{i=1}^n x_i \tag{1}$$

where $x_i \in C_i$ from the fact that $\|x\| \le M$ it follows that $\|x_i\| \le M_i$ for $i = 1, 2, \dots, n$.

We want to prove that

$$\forall M'>0 \quad \exists M_1'>0, \dots, M_n'>0 \tag{2}$$

$$\forall y = \sum_{i=1}^n y_i, \; y_i \in AC_i$$

from the fact that $\|y\| \le M'$ it follows that $\|y_i\| \le M_i'$ for $i = 1, 2, \dots, n$.

Consider an arbitrary $M'>0$ and put $M = \frac{M'}{\|A\|}$ (from the assumption on the operator A it follows that $\|A\| \neq 0$). Then condition (1) has the form $\exists \tilde{M}_i > 0, \dots, \tilde{M}_n > 0 \quad \forall x = \sum_{i=1}^n x_i, \; x_i \in C_i$ from the fact that $\|\sum_{i=1}^n x_i\| \le \frac{M'}{\|A\|}$ it follows that $\|x_i\| \le \tilde{M}_i$ for $i = 1, 2, \dots, n$.

Let us multiply the first inequality of condition

(3) by $\|A\|$. Then

$$\|A\| \cdot \|\sum_{i=1}^n x_i\| \le M' \tag{4}$$

From the properties of linear operators we have

$$\|\sum_{i=1}^n A x_i\| = \|A \sum_{i=1}^n x_i\| \le \|A\| \cdot \|\sum_{i=1}^n x_i\| \tag{5}$$

Using (4) and (5), we obtain

(6) $$\left\| \sum_{i=1}^{n} A x_i \right\| \leqslant M' \text{ hence } \left\| \sum_{i=1}^{n} y_i \right\| \leqslant M'$$

where $y_i \in AC_i$ for $i = 1, 2, \dots n$.

Consider the second inequality of condition (3). Multiplying it by $\|A\|$ and using the properties of the operator A, we obtain the conditions

(7) $$\|A\| \cdot \|x_i\| \leqslant \|A\| \cdot \tilde{M}_i$$

(8) $$\|A x_i\| \leqslant \|A\| \cdot \|x_i\| \leqslant \|A\| \cdot \tilde{M}_i$$

Using (7) and (8), we have

(9) $$\|y_i\| = \|Ax_i\| \leqslant \|A\| \cdot \|x_i\| \leqslant \|A\| \cdot \tilde{M}_i$$

where $y_i \in AC_i$ for $i = 1, 2, \dots, n$, hence $M_i' = \|A\|\ \tilde{M}_i$, then condition (2) is satisfied. The system $\{AC_i\}_{i=1}$ is of the same sense.

Now, let us assume that the system $\{AC_i\}_{i=1}^{n}$ is of the same sense. Then from the assumptions on the operator A^{-1} it follows that the system $\{C_i\}_{i=1}^{n}$ is of the same sense, too.

Let us now consider the properties of the dual cones. We prove some analogue of the Minkowski-Farkasz theorem in the case when the operator A is a linear homeomorphism.

Lemma 2. Let C be an arbitrary cone of the Banach space X. If $A : X \xrightarrow{\text{onto}} X$ is a linear homeomorphism, then

(10) $$(AC)^* = (A^*)^{-1} C^*$$

where A^* is the operator dual to A.

P r o o f. First, let us notice that if A is a linear homeomorphism, then A^* and $(A^{-1})^*$ are linear and continuous operators (cf. [16], § 27) and the following equality holds:

(11) $$(A^*)^{-1} = (A^{-1})^*$$

(cf. [6], Part VI, § 2), thus the operator $(A^*)^{-1}$ exists, is linear and continuous.

Consider an arbitrary element $y^* \in (AC)^*$. From the definition of the dual cone (cf. [9], § 5) we have that

$$(y^*, A\,x) \geq 0 \tag{12}$$

for any $x \in C$.

Using the definition of the dual operator (cf. [6], § 0.1 from (12) we obtain that

$$(A^*y^*, x) \geq 0 \tag{13}$$

for any $x \in C$, where A^* is the operator dual do A.

From inequality (13) it follows that the element $A^*y^* \in C^*$ thus $y^* \in (A^*)^{-1} C^*$. Hence the following inclusion holds:

$$(AC)^* \subset (A^*)^{-1} C^* \tag{14}$$

To obtain the proposition, we must show the opposite inclusion. Let us consider an arbitrary element $y^* \in (A^*)^{-1} C^*$. Then there exists $x^* \in C^*$ such that

$$y^* = (A^*)^{-1}x^* . \tag{15}$$

By the definition of the dual cones, $(x^*, x) \geq 0$ for any $x \in C$. From (15) we have that $x^* = A^*y^*$, thus

$$(A^*y^*, x) \geq 0 \tag{16}$$

for any $x \in C$.

By the definition of the dual operator, $(y^*, Ax) \geq 0$ for any $x \in C$, hence $y^* \in (AC)^*$. The opposite inclusion has thus been proved.

3. Formulation of the problem. The local extremum principle

Let us consider the following optimal control problem minimize the functional

$$I(x, u) = \int_0^1 f^0(x, u, t)\,dt \tag{17}$$

under the constraints

(18) $$\dot{x} = f(x, u, t)$$

(19) $$g(x, u, t) = 0$$

(20) $$u(\cdot) \in U$$

where $x(\cdot) \in \overline{W}^n_{11}(0, 1)$, $u(\cdot) \in L^r_\infty(0, 1)$, $u(t) \in M \subset R^r$ for $t \in [0,1]$; the set $U = \{u(\cdot) \in L^r_\infty(0, 1) : u(t) \in M\}$, the functions $f^0 : R^n \times R^r \times R \longrightarrow R$, $f : R^n \times R^r \times R \quad R^n$, $g : R^n \times R^r \times R \longrightarrow R^k$, $k \leq r$.

We assume that:

(21) there exist derivatives f^0_x, f^0_u, f_x, f_u, g_x, g_u which are bounded for any (x, u);

(22) the functions f^0, f, g, f^0_x, f_x, f_u are continuous with respect to (x, u) for any $t \in [0, 1]$ and measurable with respect to t and the Fréchet derivative $(g_x(x, u, t), g_u(x, u, t))$ is continuous with respect to (x, u) in the topology of the space $L(\overline{W}^n_{11} \times L^r_\infty, L^k_\infty)$ (cf. [10], § 0.1);

(23) the set M is closed, convex and possesses a nonempty interior.

Remark 3. Problem (17)-(20) under assumptions (21)-(23) will be called problem I.

Let us put $X = \overline{W}^n_{11} \times L^r_\infty$.

Denote by $F_2 : X \longrightarrow \overline{W}^n_{11}$ the operator defined by the formula

(24) $$F_2(x, u)(t) = x(t) - \int_0^t f(x(t), u(t), t)dt$$

By $F_3 : X \longrightarrow L^k_\infty$ let us denote the operator of the form

(25) $$F_3(x, u)(t) = g(x(t), u(t), t).$$

The function given by the formula

(26) $$L(x, u, \lambda_0, y^*_2, y^*_3) =$$
$$= \lambda_0 \, I(x, u) + (y^*_2, F_2(x, u)) + (y^*_3, F_3(x, u))$$

where $y^*_2 \in (\overline{W}^n_{11})^*$, $y^*_3 \in (L^k_\infty)^*$, will be called then Lagrange function for problem I.

Using the generalization of the Dubovitskii-Milyutin method from [14] and the properties of cones given before, we shall prove the local extremum principle for problem I.

Theorem 1. If

1^o (x^o, u^o) is an optimal process for problem I;

2^o there exists a minor of rank k of the matrix $g_u(x^o(t)\ u^o(t)\ t)$ and a constant $\alpha > 0$ such that $|m\ g_u(x^o(t), u^o(t), t| > \alpha$ for $t \in [0, 1]$ a.e., then there exist $\lambda_0 \geq 0$ and functions $\Psi \in L^n_\infty(0, 1)$, $\omega \in b.^k a\ (0, 1)$ such that

1^o

$$\|\omega\| + |\lambda_0| + \|\Psi\| > 0,$$

2^o

$$L_x\,\bar{x} = \lambda_0 \int_0^1 f_x(x^o(t), u^o(t), t)\,\bar{x}\,dt + \int_0^1 (\dot{\bar{x}} - f_x(x^o(t), u^o(t), t)\,\bar{x})\,\Psi(t)dt + \int_0^1 g_x(x^o(t), u^o(t), t)\bar{x}\,d\omega = 0$$

for any $\bar{x} \in \bar{W}^n_{11}$,

3^o

$$\lambda_0 \int_0^1 f^o_u(x^o(t), u^o(t), t)\,u^o(t)\,dt + \int_0^1 f_u(x^o(t), u^o(t), t)\,\Psi(t)\,dt + \int_0^1 g_u(x^o(t), u^o(t), t)u^o(t)d\omega =$$

$$= \min_{u \in U} \Big(\lambda_0 \int_0^1 f_u(x^o(t), u^o(t), t)u(t)dt + \int_0^1 f_u(x^o(t), u^o(t), t)u(t)\Psi(t)dt - \int_0^1 g_u(x^o(t), u^o(t), t)u(t)d\omega\Big)$$

Remark 4. Assumption 2^o of this theorem is similar to the condition of "regular controllability" from [3], [4] and [7]. We shall call it a condition of "strong regular controllability".

P r o o f. Let us define the following sets:

(27) $$Z_1 = \{(x, u) \in X : u \in U\}$$

(28) $$Z_i = \{(x, u) \in X : F_i(x, u) = 0\}$$

for $i = 2, 3$, where the operators F_2, F_3 are given by formulae (24), (25), respectively.

Hence problem I formulated above may be represented in the form: determine the minimal value of the functional $I(x, u)$ under the condition $(x, u) \in \bigcap_{i=1}^{3} Z_i$. Problem I contains two equality constraints: the set Z_2 and Z_3 given by formula (28). In the proof we shall apply theorem 6 from [4] generalizing the Dubovitskii-Milyutin theorem in the case of n equality constraints.

We shall find the following cones:

$C_0 = DC\,(I(x^0, u^0))$ - the cone of directions of decrease of the functional I at the point (x^0, u^0),

$C_1 = FC\,(Z_1, (x^0, u^0))$ - the cone of feasible directions for the set Z_1 at the point (x^0, u^0),

$C_i = TC\,(Z_i, (x^0, u^0))$ - the cone of tangent directions to the set Z_i at the point (x^0, u^0) for $i = 2, 3$,

and the cones dual to them C_0^*, C_1^*, C_2^*, C_3^* ([9], § 5-9).

Proceeding identically as in ([9] § 7, 8), we derive formulae for the cones C_{o1} and C_1. We have

(29) $$C_0 = \{(\bar{x}, \bar{u}) \in X : \int_0^1 (f_x^0(x^0, u^0, t)\bar{x} + f_u^0(x^0, u^0, t)\bar{u})dt < 0\}$$

(30) $$C_1 = \{(\bar{x}, \bar{u}) \in X ; \bar{u} = \lambda(u - u^0)$$

where $\lambda \geq 0$, $u \in \operatorname{int} U\}$, and we assume temporarily that $C_0 \neq \emptyset$.

The cones C_0^* and C^*_1 are given in the form (cf. [9], § 10):

(31) $$C_0^* = \{f_0 \in X^* : f_0(x, u) = -\lambda_0 \int_0^1 (f_x^0(x^0, u^0, t)\bar{x} + f_u(x^0, u^0, t)\bar{u})dt,\ \lambda_0 \geq 0\}$$

(32) $$C_1^* = \{f_1 \in X^* : f_1(\bar{x}, \bar{u}) = f_1'(\bar{u})$$

where f_1' is a functional supporting the set $U = \{u \in L_\infty^r : u(t) \in M$ at the point $(x^0, u^0)\}.\}$.

The set Z_2 is an equality constraint. The operator F_2 given in form (24) is known to be strongly continuously differentiable at the point (x^0, u^0) and its differential is defined by the formula (cf. [9], § 9):

$$F_2'(x^0, u^0)(\bar{x}, \bar{u}) = \bar{x}(t) - \tag{33}$$

$$- \int_0^t (f_x(x^0, u^0, t)\bar{x}(t) + f_u(x^0, u^0, t)\bar{u}(t))dt$$

It is easy to show that F_2 is regular at the point (x^0, u^0), i.e. the equation

$$\bar{x} - \int_0^t (f_x \bar{x} + f_u \bar{u})dt = a(t)$$

has a solution for any $a(t) \in \bar{W}_{11}^n$.

Really, it is enough to differentiate the last equation and, afterwards, to put $\bar{u}(t) \equiv 0$. We obtain the equation in the form

$$\dot{\bar{x}} - f_x x - \dot{a} = 0$$

where $\dot{a} \in L_1^n$, thus it is a nonhomogeneous linear differential equation with respect to x which with the integrable functions f and $\dot{a}$ has a solution (cf. [17], Part II, § 4).

Thus the operator F_2 satisfies the assumptions of the Lusternik theorem and the cone C_2, is of the form

$$C_2 = \{(x, u) \in X : F_2'(x^0, u^0)(\bar{x}, \bar{u}) = 0\} = \tag{34}$$

$$= \{(\bar{x}, \bar{u}) \in X : \bar{x} - \int_0^t (f_x \bar{x} + f_u \bar{u})dt = 0\}$$

After differentiation of the equation from formula (34) we obtain that

$$C_2 = \{(\bar{x}, \bar{u}) \in X : \dot{\bar{x}} - f_x \bar{x} - f_u \bar{u} = 0\} \tag{35}$$

Now, we shall find the cone dual to C_2. The cone C_2 is a subspace, hence the cone C_2^* is of the form (cf. [9], § 10)

$$C_2^* = \{f_2 \in X^* : f_2(\bar{x}, \bar{u}) = 0$$

$$\text{for any } (\bar{x}, \bar{u}) \in C_2\}.$$

Denote by Λ the operator of the form

(36) $$\Lambda(\bar{x}, \bar{u}) = \dot{\bar{x}} - f_x \bar{x} - f_u \bar{u}$$

$\Lambda : \bar{W}^n_{11} \times L^r_\infty \to L^n_1$ is a linear and continuous operator.

The cone C_2 can be written in the form

(37) $$C_2 = \{(\bar{x}, \bar{u}) \in X : \Lambda(\bar{x}, \bar{u}) = 0\}$$

From (36), (37) and from the definition of the annulator of a subspace (cf. [10], § 0.1) we have that

(38) $$C_2^* = (\ker\Lambda)^\perp$$

Making use of the annulator lemma (cf. [10], § 0.1), we obtain the equality

(39) $$C^*_2 = \mathrm{Im}\Lambda^*$$

where Λ^* is the operator dual to Λ (cf. [10], § 0.1).

Let us consider an arbitrary $f_2 \in C_2^*$. From (39) it follows that $f_2 \in \mathrm{Im}\,\Lambda^*$. The space dual to $L^n_1(0, 1)$ is $L^n_\infty(0, 1)$, hence $\mathrm{Im}\,\Lambda^* = \{y^* \in (\bar{W}^n_{11} \times L^r_\infty)^*$: there exists an element $x^* \in L^n_\infty(0,1)$ such that $\Lambda^* x^* = y^*\}$.

Thus, from the fact that $f_2 \in \mathrm{Im}\Lambda^*$ it follows that there exists an element $\Psi \in L^n_\infty(0, 1)$ such that

(40) $$f_2 = \Lambda^* \Psi$$

Using the definition of the dual operator and from (36) and (40) we obtain that

(41) $$f_2(\bar{x}, \bar{u}) = (\Lambda^* \Psi, (\bar{x}, \bar{u})) = (\Psi, \Lambda(\bar{x}, \bar{u})) = $$
$$= (\Psi, \dot{\bar{x}} - f_x\bar{x} - f_u\bar{u})$$

From the formula of a linear and continuous functional on L^n_1 (cf. [6], Part IV, § 5) and from (41) we have that

(42) $$f_2(\bar{x}, \bar{u}) = \int_0^1 (\dot{\bar{x}} - f_x\bar{x} - f_u \bar{u})\, \Psi(t)dt,$$

thus

$$C_2^* \subset \{f_2 \in X^* : f_2(\bar{x}, \bar{u}) = \int_0^1 (\dot{\bar{x}} - f_x\bar{x} - f_u\bar{u})\, \Psi(t)dt, \Psi \in L^n_\infty(0, 1)\}$$

The opposite inclusion follows from the definition of the cone C_2, hence

(43) $$C_2^* = \left\{f_2 \in X^* : f_2(\bar{x}, \bar{u}) = \int_0^1 (\dot{\bar{x}} - f_x\bar{x} - f_u\bar{u})\Psi(t)dt\right.$$

where $\left.\Psi \in L_\infty^n(0, 1)\right\}$.

The set Z_3 is an equality constraint, too. The operator F_3 given in form (25) is strongly continuosly differentiable (cf. assumption (22)) and its differential is given by the formula

(44) $$F_3'(x^0, u^0)(\bar{x}, \bar{u})(t) = g_x(x^0(t), u^0(t), t)\bar{x}(t) + g_u(x^0(t), u^0(t), t)\bar{u}(t)$$

It is easy to notice that assumption 2^0 of the theorem, i.e the condition of "strong regular controllability" (cf. Remark 4) is the sufficient condition for regularity of the operator F_3.

Really, if this condition holds, then, putting $x(t) \equiv 0$ and using the Cronecker-Cappelli lemma and the Cramer theorem (cf. [2], Part IV), we obtain that the equation

$$g_x\,\bar{x}(t) + g_u\,\bar{u}(t) = a(t)$$

has a solution $(\bar{x}(t), \bar{u}(t)) \in X$ for any $a(t) \in L_\infty^n$.

Thus the operator F_3 satisfies the assumptions of the Lusternik theorem (cf. [10], § 0.2) and the cone tangent to set Z_3 at the point (x^0, u^0) is of the form

(45) $$C_3 = \{(\bar{x}, \bar{u}) \in X : F_3'(x^0, u^0)(\bar{x}, \bar{u}) = 0\} = \{(\bar{x}, \bar{u}) \in X : g_x\bar{x} + g_u\bar{u} = 0\}$$

Proceeding analogously as in the case of the cone C_2, we shall calculate the cone dual to C_3.

The cone C_3 is a subspace, thus, as before, the cone C_3^* is of the form

(46) $$C_3^* = \{f_3 \in X^* : f_3(\bar{x}, \bar{u}) = 0 \text{ for any } (\bar{x}, \bar{u}) \in C_3\}$$

Let us denote by Γ the operator given by the formula

(47) $$\Gamma(\bar{x}, \bar{u}) = g_x \bar{x} + g_u \bar{u}$$

It is obvious that $\Gamma : \bar{W}^n_{11} \times L^r_\infty \to L^k_\infty$ is a linear and continuous operator. The cone C_3 can be written in the form

(48) $$C_3 = \{(\bar{x}, \bar{u}) \in X : \Gamma(\bar{x}, \bar{u}) = 0\}$$

From (46), (48) and the definition of the annulator of a subspace we have that

(49) $$C_3^* = (\ker \Gamma)^\perp$$

Analogously as before, making use of the annulator lemma, we obtain that

(50) $$C_3^* = \operatorname{Im}\Gamma^*$$

where Γ^* is the operator dual to Γ, i.e. $\Gamma^* : (L^k_\infty)^* \to (\bar{W}^n_{11} \times L^r_\infty)^*$.

Let f_3 be an arbitrary element of the cone C_3. Hence from (50) it follows that $f_3 \in \operatorname{Im}\Gamma^*$. The space dual to the space $L^k_\infty(0, 1)$ is $b^k.a\ (0, 1)$ (cf. Remark 2), thus, from the fact that $f_3 \in \operatorname{Im}\Gamma^*$ it follows that there exists an element $\omega \in b^k.a\ (0, 1)$ such that

(51) $$f_3 = \Gamma^* \omega$$

Using the definition of the dual operator and from (47) and (51) we obtain that

(52) $$f_3(\bar{x}, \bar{u}) = (\Gamma^*\omega, (\bar{x}, \bar{u})) = (\omega, \Gamma(\bar{x}, \bar{u})) = $$
$$= (\omega, g_x \bar{x} + g_u \bar{u})$$

Making use of the formula for a linear and continuous functional on L^1_∞ and of Remark 2, we have

(53) $$f_3(\bar{x}, \bar{u}) = \int_0^1 (g_x \bar{x} + g_u \bar{u})\, d\omega$$

From the last equality it follows that if $(\bar{x}, \bar{u}) \in C_3$, then $f_3(\bar{x}, \bar{u}) = 0$, thus, analogously as in equality (43), the cone C_3 is of the form

(54) $$C_3^* = \{f_3 \in X^*: f_3(\bar{x}, \bar{u}) = \int_0^1 (g_x \bar{x} + g_u \bar{u})\, d\omega$$

where $\omega \in b^k_{\cdot}a(0, 1)\}$.

Now, we shall show that the cones calculated above satisfy the assumptions of theorem 6 from [14].

Thus we must prove that

a) the cones C_2^* and C_3^* are of the same sense,

b) the following inclusion holds:

$$C_2 \cap C_3 \subset TC(Z_2 \cap Z_3)$$

First, we shall check condition a). We can show that the cones C_2^* and C_3^* are of the same sense by using theorem 3 from [14]. For this purpose, we must reduce the cones C_2 and C_3 to the special form.

Let us consider the case when $k = r$. In order to reduce the cones C_2 and C_3 given by formulae (35) and (45), respectively, to the form required in theorem 3 from [14], we shall apply the linear and continuous operator $\tilde{A} : X \xrightarrow{onto} X$ given by the formula

$$\tilde{A} = \begin{bmatrix} I & 0 \\ g_x & g_u \end{bmatrix} \tag{55}$$

where I denotes the unit matrix of rank n.

For an arbitrary fixed $t \in [0, 1]$, the operator $\tilde{A}$ is a matrix of rank $(n + r)$ and, according to assumption 2^o of the theorem the condition of "strong regular controllability" for $k = r$, the following condition holds:

$$\det \tilde{A} = \det g_u > \alpha \tag{56}$$

for $t \in [0, 1]$ a.e.

Thus, the operator $\tilde{A}^{-1}$ exists (cf. [2], Part IV, the Cramer theorem), is linear and continuous (cf. [13], § 5, the theorem on an inverse operator) and its of the form

$$\tilde{A}^{-1} = \begin{bmatrix} I & 0 \\ -g_u^{-1} g_x & g_u^{-1} \end{bmatrix} \tag{57}$$

Then, using formulae (35) and (45), it is easy to calculate that the cones $\tilde{A}C_2$ and $\tilde{A}C_3$ are of the form

(58) $$\tilde{A}C_2 = \left\{(\bar{x}, \bar{u}) \in X : \dot{\bar{x}} = (f_x - f_u g_u^{-1} g_x)\bar{x} + f_u g_u^{-1}\bar{u}\right\}$$

(59) $$\tilde{A}C_3 = \left\{(\bar{x}, \bar{u}) \in X : g_u g_u^{-1}\bar{u} + (g_x - g_u^{-1} g_x)\bar{x} = 0\right\} =$$

$$= \left\{(\bar{x}, \bar{u}) \in X : \bar{u} = 0\right\} = \bar{W}_{11}^n \times \{0\}$$

Now, let us consider the cone $\tilde{A}C_2$. The differential equation from condition (58) is, for a fixed control $\bar{u}(t)$, a nonhomogeneous linear equation with respect to $\bar{x}(t)$ thus, denoting

(60) $$E(t) = f_x - f_u g_u^{-1} g_x$$

(61) $$F(t) = f_u g_u^{-1}$$

we obtain that the solution of this equation is of the form (cf. [17], Part II, § 4):

(62) $$\bar{x}(t) = Y(t) \int_0^1 Y(t)^{-1} F(t)\bar{u}(t)dt$$

where $Y : [0, 1] \to B(R^n, R^n)$ is an absolutely continuous function satisfying the equation

(63) $$\dot{Y}(t) = E(t)Y(t), \ Y(0) = I$$

Hence, making use of (62), the cone $\tilde{A}C_2$ can be written in the form

(64) $$\tilde{A}C_2 = \{(\bar{x}, \bar{u}) \in X : \bar{x} = N\bar{u} \text{ where}$$
$$N\bar{u} = Y(t) \int_0^1 Y(t)^{-1} F(t)\bar{u}(t)dt,$$

$Y(t)$ satisfies equation (63), the operators $E(t)$ and $F(t)$ are given by formulae (60) and (61), respectively$\}$

Obviously, N is a linear and continuous operator mapping L_∞^r into $\bar{W}_{11}^n$.

The cones $\tilde{A}C_2$ and $\tilde{A}C_3$ of forms (64) and (59), respectively, satisfy the assumptions of theorem 3 from [14]. Using this theorem, we obtain that the cones $(\tilde{A}C_2)^*$ and $(\tilde{A}C_3)^*$ are of the same sense.

Now, we shall apply lemma 2 to the cones C_2, C_3 and the operator $\widetilde{A}$. We obtain the equalities

$$(\widetilde{A}C_2)^* = (\widetilde{A}^*)^{-1}C_2^*$$

$$(\widetilde{A}C_3)^* = (\widetilde{A}^*)^{-1}C_3^*$$

In this way we have obtained the condition that the system of cones $(\widetilde{A}^*)^{-1}C_2^*$, $(\widetilde{A}^*)^{-1}C_3^*$ is of the same sense. Applying lemma 1 to this system, we obtain that the system of cones C_2^*, C_3^* is of the same sense, too.

We shall also consider the case when $k < r$. In this case, to reduce the cones C_2 and C_3 given by formulae (35) and (45), respectively, to the form required in theorem 3 from [14], we shall apply the linear and continuous operator $\widehat{A} : X \xrightarrow{\text{on to}} X$ of the form

(65) $$A = \begin{bmatrix} I_1 & 0 \\ \widehat{A_{21}} & I_2 \end{bmatrix}$$

where $\widehat{A_{21}}$ is some linear and continuous operator on the space $\overline{W}^n_{11}$ with values from L^r_∞, I_1 is the unit matrix of rank n, I_2 is the unit matrix of rank r.

For any fixed $t \in [0, 1]$, $\widehat{A}$ is a matrix of rank (n + r) and

$$\det \widehat{A} = 1$$

thus the operator $\widehat{A}^{-1}$ exists, is linear and continuous (cf. [16] § 15, the theorem on an inverse operator).

Let us calculate, as before, the images of C_2 and C_3 by using the operator $\widehat{A}$. Making use of formulae (35), (45) and (65), we can easily calculate that

(66) $$\widehat{A}C_3 = \left\{(\overline{x}, \overline{u}) \in X : g_u\overline{u} + (g_x - g_u\widehat{A_{21}})\,\overline{x} = 0\right\}$$

(67) $$\widehat{A}C_2 = \left\{(\overline{x}, \overline{u}) \in X : \dot{\overline{x}} = (f_x - f_u\widehat{A_{21}})\,\overline{x} + f_u\overline{u}\right\}$$

Let us consider the equation

(68) $$g_x - g_u\,\widehat{A_{21}} = 0$$

with an unknown operator $\widehat{A_{21}}$ defined on $\overline{W}^n_{11}$.

Let us analyse assumption 2^0 of this theorem. We denote by $\widehat{g_u}$ the matrix made from the matrix g_u by omitting $(r-k)$ columns and such that the determinant of $\widehat{g_u}$ satisfies assumption 2^0 of the theorem. (We can assume that we omit the last $(r-k)$ columns and the problem will be of the same generality). Then assumption 2^0 means that there exists a constant $\alpha > 0$ such that

$$(69) \qquad |\det \widehat{g_u}(x^0(t), u^0(t), t)| > \alpha \text{ for } t \in [0, 1] \text{ a.e.}$$

hence the operator $\widehat{g_u}^{-1}$ exists (cf. [2], Part IV the Cramer theorem), is linear and continuous (cf. [13], § 15, the theorem on an inverse operator).

Hence, as can easily be een, it is enough to put

$$(70) \qquad \widehat{A_{21}} = \begin{bmatrix} \widehat{g_u}^{-1} \cdot g_x \\ 0 \end{bmatrix}$$

For any fixed $t \in [0, 1]$, $\widehat{A_{21}}$ is a matrix of rank $r \cdot n$, 0 is a zero matrix of rank $(r-k)\,n$. From the previous considerations it follows that $\widehat{g_u}^{-1} g_x$ is a linear and continuous operator from the space $\overline{W}^n_{11}$ into L^k_∞, thus, it is easy to see that $\widehat{A_{21}}$ of form (70) is a linear and continuous operator mapping the space $\overline{W}^n_{11}$ into L^r_∞.

Using the Cramer theorem (cf. [2], Part IV), after simple calculations we obtain that the operator $\widehat{A_{21}}$, of form (70) satisfies equation (68), thus the cone $\widehat{AC}_3$ can be written in the form

$$(71) \qquad \widehat{AC}_3 = \overline{W}^n_{11} \times \left\{ \overline{u} \in L^r_\infty : g_u \overline{u} = 0 \right\}$$

Proceeding analogously as in the case $k=r$, we can reduce the cone $\widehat{AC}_2$ to the form

$$(72) \qquad \widehat{AC}_2 = \left\{ (\overline{x}, \overline{u}) \in X : \overline{x} = S\overline{u} \right.$$

where $S : L^k_\infty \longrightarrow \overline{W}^n_{11}$ is some linear and continuous operator$\}$.

Analogously as before, we apply theorem 3 from [14] and, next, lemmas 2 and 1 to the cones $\widehat{AC}_2$ and $\widehat{AC}_3$ given by formulae (72) and (71), respectively. We obtain that in the case $k < r$ the cones C_2^* and C_3^* are of the same sense, too.

In this way we have checked completely that condition a) is satisfied.

Now, we must verify condition b), i.e. the inclusion

$$C_2 \cap C_3 \subset TC(Z_2 \cap Z_3)$$

where C_2 and C_3 are given by formulae (34), (45), respectively, Z_2 and Z_3 by (28).

For this purpose, it is enough to show that the operator $F : X \longrightarrow \overline{W}^n_{11} \times L^k_\infty$ of the form

(73) $$F(x, u) = (F_2(x, u), F_3(x, u))$$

where F_2 and F_3 are of forms (24) and (25), respectively, is regular at the point $(x^0, u^0) \in X$.

Obviously, the operator F given in form (73) is Fréchet differentiable at the point (x^0, u^0) and its differential is of the form

(74) $$F'(x^0, u^0)(\overline{x}, \overline{u}) = (F_2'(x^0, u^0)(\overline{x}, \overline{u}), F_3'(x^0, u^0)(\overline{x}, \overline{u}))$$

From the definition of a regular operator (cf. [10], § 0.2) and from the formulae for differentials (33), (44) and (74) it follows that, to prove the regularity of the operator F, it is enough to show that the system of equations

(75) $$\overline{x}(t) - \int_0^t (f_x \overline{x}(t) + f_u \overline{u}(t))dt = y_1(t)$$

(76) $$g_x \overline{x} + g_u \overline{u} = y_2(t)$$

has a solution for any $y_1 \in \overline{W}^n_{11}$, $y_2 \in L^k_\infty$.

Let us consider, as before, two cases.

If k=r, then the condition of "strong regular controllability" implies, as we have shown before, the existence of the inverse operator g_u^{-1} and equation (76) can be written in the form

$$\overline{u} = -g_u^{-1} g_x \overline{x} + g_u^{-1} y_2$$

for any $y_2 \in L^k_\infty$.

We can put the last equation in (76) to obtain

(77) $$\overline{x}(t) - \int_0^t ((f_x - f_u g_u^{-1} g_x) \overline{x} + f_\mu g_u^{-1} y_2)dt = y_1$$

where $y_1 \in \overline{W}^n_{11}$, $y_2 \in L^k_\infty$.

After differentiation of (77) we get a linear differential equation with respect to x in the form

(78) $$\dot{\overline{x}} = A(t)\overline{x} + B(t)$$

where $A(t) = f_x - f_u g_u^{-1}$ and $B(t) = f_u g_u^{-1} y_2 + \dot{y}_1$ are integrable functions, thus equation (77) has a solution $\overline{x} \in \overline{W}_{11}^n$ for any $(y_1, y_2) \in \overline{W}_{11}^n \times L_\infty^k$ (cf. [17], Part II, § 4).

Let us consider the second case, i.e. assume that $k < r$.

Let $\widehat{g_u}$ be a square matrix of rank k which we considered in condition a), i.e. such that its determinant satisfies condition (69).

We denote

(79) $$\tilde{u}(t) = (\hat{u}(t), 0 \dots 0) \in L_\infty^r$$

where $\hat{u}(t) = (u_1(t) \dots u_k(t)) \in L_\infty^k$.

Equations (75) and (76) are satisfied for any $\overline{u} \in L_\infty^r$, thus, in the particular case, for $\tilde{u}(t)$ given by formula (79).

The operator $\widehat{g_u}$ is invertiable (cf. the proof of condition a)), thus, using (79), we can rewrite equation (76) in the form

$$\hat{u}(t) = -\widehat{g_u}^{-1} g_x\overline{x} + \widehat{g_u}^{-1} y_2$$

for any $y_2 \in L_\infty^k$.

After this, we can put a control of the form

$$\tilde{u} = (-\widehat{g_u}^{-1} g_x\overline{x} + g_u y_2, 0 \dots 0)$$

where $y_2 \in L_\infty^k$, in (75) to obtain, as in the previous case, the nonhomogeneous differential equation which has a solution $\overline{x} \in \overline{W}_{11}^n$ for any $(y_1, y_2) \in \overline{W}_{11}^n \times L_\infty^k$.

We have thus proved in both cases that the operator F of form (73) is regular.

Let us notice that

(80) $$Z_2 \cap Z_3 = \{(x, u) \in X : F(x, u) = 0\}$$

The operator F satisfies the assumptions of the Lusternik theorem (cf. [10], § 0.2), thus, making use of this theorem, we have that

(81) $$TC\ (Z_2 \cap Z_3) = \{(\overline{x}, \overline{u}) \in X : F'(x^0, u^0)(\overline{x}, \overline{u}) = 0\}$$

From the last condition, (74) and the formulae for the cones C_2 and C_3 we obtain

$$TC\ (Z_2 \cap Z_3) = \{(\bar{x}, \bar{u}) \in X : F_2'(x^0, u^0)(\bar{x}, \bar{u}) = 0\} \cap$$
$$\{(\bar{x}, \bar{u}) \in X : F_3'(x^0, u^0)(\bar{x}, \bar{u}) = 0\} = C_2 \cap C_3$$

whence condition b) holds.

We have thus checked all the assumptions we obtain that there exist functionals $f_i \in C_i^*$, $i = 0, 1, 2, 3$, not vanishing simultaneously and such that

(82) $$f_0 + f_1 + f_2 + f_3 = 0$$

After putting the formulae for the functionals $f_i \in C_i^*$, $i = 0, 1, 2, 3$ (31), (32), (43), (54), respectively in (82) we have the equation

(83) $$-\lambda_0 \int_0^1 (f_x^0(x^0(t), u^0(t), t)\bar{x}(t) +$$
$$+ f_u^0(x^0(t), u^0(t), t)\bar{u}(t))dt + f_1'(\bar{u}) +$$
$$+ \int_0^1 (\dot{\bar{x}} - f_x(x^0(t), u^0(t), t)\bar{x}(t) +$$
$$+ f_u(x^0(t), u^0(t), t)\bar{u}(t))\Psi(t)dt +$$
$$+ \int_0^1 (g_x(x^0(t), u^0(t), t)\bar{x}(t) +$$
$$+ g_u(x^0(t), u^0(t), t)\bar{u}(t))d\omega = 0$$

for any $(\bar{x}, \bar{u}) \in X$, where $\Psi \in L_\infty^n(0, 1)$, $\omega \in b^k a(0, 1)$.

Let us first put $(\bar{x}, \bar{u}) = (\bar{x}, 0) \in X$ in (83) and, next $(\bar{x}, \bar{u}) = (0, \bar{u}) \in X$. We obtain the following equations

(84) $$-\lambda_0 \int_0^1 f_x^0(x^0(t), u^0(t), t)\bar{x}(t)dt +$$
$$+ \int_0^1 (\dot{\bar{x}} - f_x(x^0(t), u^0(t), t)\bar{x}(t))\Psi(t)dt +$$
$$+ \int_0^1 g_x(x^0(t), u^0(t), t)\bar{x}(t)d\omega = 0$$

(85) $$\lambda_0 \int_0^1 f_u^0(x^0(t), u^0(t), t)\bar{u}(t)dt +$$

$$+\int_0^1 f_u(x^0(t),\ u^0(t),\ t)\bar{u}(t)\Psi(t)dt +$$

$$-\int_0^1 g_u(x^0(t),\ u^0(t),\ t)\bar{u}(t)d\omega = f'_1(\bar{u})$$

for any $(\bar{x},\ \bar{u}) \in X$, where f'_1 is a functional supporting the set Z_1 at the point u^0.

Equation (84) is condition 2^0 of the proposition. From equation (85) and the definition of a supporting functional (cf. [9], § 4) we obtain the extremum condition

(86)
$$\lambda_0 \int_0^1 f_u^0(x^0(t),\ u^0(t),\ t)\ u^0(t)dt +$$
$$+\int_0^1 f_u(x^0(t),\ u^0(t),\ t)u^0(t)\Psi(t)dt +$$
$$-\int_0^1 g_u(x^0(t),\ u^0(t),\ t)u^0(t)d\omega =$$
$$= \min_{u \in U} (\lambda_0 \int_0^1 f_u{}^0(x^0(t),\ u^0(t),\ t)\bar{u}(t)dt +$$
$$+\int_0^1 f_u(x^0(t),\ u^0(t),\ t)\bar{u}(t)\Psi(t)dt +$$
$$+\int_0^1 g_u(x^0(t),\ u^0(t),\ t)\bar{u}(t)d\omega)$$

Finally, we must show that $|\lambda_0| + \|\Psi\| + \|\omega\| > 0$. This condition follows from equality (85). Really, if $\lambda_0 = 0$, $\Psi = 0$, $\omega = 0$, then $f_0 = f_2 = f_3 = 0$, and, by equality (85), $f_1 = 0$, but this contradicts the proposition of theorem 6 from [14].

Thus, this theorem is proved under the assumption that the cone of directions of decrease of the functional

$$C_0 = \left\{(\bar{x},\ \bar{u}) \in X : \int_0^1 (f_x^0\ \bar{x} + f_u{}^0\bar{u})dt < 0\right\}$$

is nonempty.

Let us assume that $C_0 = \emptyset$.

Then, for any $(\bar{x},\ \bar{u}) \in X$.

(87)
$$\int_0^1 (f_x^0\ \bar{x} + f_u{}^0\bar{u})dt = 0$$

In this case, to prove the theorem, it is enough to put $\lambda_0 =$

$= 1$, $\Psi = 0 \in L^n_\infty$, $\omega = 0 \in b^k a\ (0, 1,\ f'_1 = 0$; then, from (87) we get the equation

$$\lambda_o \int_0^1 (f_x{}^o\bar{x} + f_u{}^o\bar{u})dt + f'_1(\bar{u}) +$$

$$+ \int_0^1 (\dot{\bar{x}} - f_x\bar{x} - f_u\bar{u})\Psi(t)dt + \int_0^1 (g_x\bar{x} + g_u\bar{u})d\omega = 0$$

for any $(\bar{x}, \bar{u}) \in X$.

Proceeding analogously as in the case $C_o \neq \emptyset$, we obtain the proposition.

Remark 5. Let us consider the situation in which the extremum principle can be written in a simpler form, i.e. the situation in which $\omega = (\omega_1, \omega_2, \ldots, \omega_k) \in b^k a\ (0, 1)$ is such that the functions ω_i, $i = 1, 2, \ldots k$ are measures. In this case we apply the Radon-Nikodym theorem (cf. [6]) and Remark 2 of this work. We obtain that there exists a function $v(\cdot) \in L^k_1$ such that

(88) $$\int_0^1 f(t)d\omega = \int_0^1 f(t)\ V(t)dt$$

for any $f \in L^k_1$.

Applying (88) to conditions 2^o and 3^o of the theorem and using the Dubois-Raymond lemma (cf. [8], Part I, § 3) and the properties of absolutely continuous functions (cf. [11], Part VII, § 4), after simple calculations we obtain that the proposition of Theorem 1 can be reduced to the form: there exist $\lambda_o \geq 0$, $v(\cdot) \in L^k_1$ and an absolutely continuous function $\varphi : [0, 1] \to R^n$, not vanishing simultaneously satisfying the equation

$$\dot{\varphi} = \lambda_o f_x - f^*_x\varphi - g^*_x v, \quad \varphi(1) = 0$$

and such that

$$(\lambda_o f_u{}^o - f^*_u\varphi - g^*{}_u V,\ u - u^o(t)) \geq 0$$

for any $u \in U$ and $t \in [0, 1]$ a.e.

References

[1] A n o r o v B. P., Princip maksimuma dla processov z ogranicheniami obshchevo vida, "Avtomatika i Telemechanika" 1967, no 3, pp. 5-15, 1967, no 4, pp. 5-15.

[2] B i a ł y n i c k i-B i r u l a A. A., Algebra liniowa z geometrią, Warszawa 1976.

[3] B o ł t i a ń s k i W., G a m k r e l i d z e S., Matematyczna teoria procesów optymalnych, Warszawa 1976.

[4] D u b o v i t s k i j A. Ya., M i l y u t i n A.A., Zadacha na ekstremum pri nalichii ogranichenji, "Zhurnal Vychislitelnoj Matematiki i Matematicheskoj Fiziki" 1965, T. 5, no 3.

[5] D u b o v i t s k i j A. Ya., M i l y u t i n A. A., Nieobkhodimyje uslovia slabovo ekstremuma dla zadach so smieshanymi ogranicheniami tipa nieravienstva, "Zhurnal Vychislitelnoj Matematiki i Matematicheskoj Fiziki" 1968, T. 8, no 4.

[6] D u n f o r d N., S c h w a r z J. T., Liniejnyje operatory. Obshschaja teorya. Moskwa 1974.

[7] D z h g a r k o v a D. T., O nieobkhodimykh uslovyakh optimalnosti w zadachakh so smieshanymi ogranicheniami, Biul. AN Gruzinskoj SSR 1980, 97, no 3.

[8] G e l f a n d J. M., F o m i n S. W., Rachunek wariacyjny, Warszawa 1975.

[9] G i r s a n o v J. V., Lekcji po matematicheskoj teorji ekstremalnykh zadach, Moskwa 1974.

[10] J o f f e A. D., T i k h o m i r o v V. M., Teorya ekstremalnych zadach, Moskwa 1974.

[11] Ł o j a s i e w i c z S., Wstęp do teorii funkcji rzeczywistych, Warszawa 1976.

[12] M a k o w s k i K. and Neustadt L. W., Optimal control problems with mixed control phase variable equality and inequality constraints, SJAM J. Control, May 1974, vol 12, no 2.

[13] M u s i e l a k J., Wstęp do analizy funkcjonalnej, Warszawa 1976.

[14] W a l c z a k S., On some properties of cones in normed spaces and their application to investigating extremal problems, "Journal of Optimization Theory and Application", April 1984.

[15] W a l c z a k S., The local extremum principle for pro-

blems with constraints upon phase coordinates Bull. Soc. Sci. Łódź 1984, vol. XXXII, 3.

[16] W a l c z a k S., On some control problem, Acta Univ. Lodz. 1984, Folia math., z. 1.

[17] W a r g a Dz., Optimalnoe uprawlenie differencyalnymi i funkcyonalnymi uprawleniami, Moskwa 1977.

Institute of Mathematics
The University of Łódź

Urszula Łędzewicz-Kowalewska

ZASADA EKSTREMUM DLA ZADAŃ STEROWANIA OPTYMALNEGO Z MIESZANYMI OGRANICZENIAMI

W niniejszej pracy wykazana jest zasada ekstremum dla zadań sterowania optymalnego z ograniczeniami typu równości na współrzędne fazowe i sterowanie w oparciu o uogólnienie metody Dubowickiego-Milutina zawarte w [14]. Zadanie badane w niniejszej pracy różni się od zadania rozważanego w [15] wystąpieniem sterowania w ograniczeniu typu równości oraz nieco inną metodą obliczania stożków. Zasada ekstremum przedstawiona tutaj zawiera założenie tzw. wzmocnionej regularnej sterowalności, które w nieco słabszej postaci występuje również w [1], [3], [7], jednak rozważane tam zadania pozbawione są ograniczenia $u(\cdot) \in U$ oraz, jak już nadmieniłam, zastosowana jest tam inna metoda dowodu.