A C T A U N I V E R S I T A T I S L O D Z I E N S I S
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SPATIAL ASPECTS OF THE MULTILEVEL MODELS CONSTRUCTION[**]

1. INTRODUCTION

The multilevel models allow to analyse the data variation simultaneously at the different levels[1]. As the relationship between observations can differ by the groups, multilevel models are useful for exploring the clustered, nested or hierarchical data structure. Let consider the generalization of the 2-level linear multilevel model[2] (with random and fixed effects, without cross-level interactions),which can be written as (Goldstein 1999):

$$\begin{aligned} &Y_{ij} = \beta_{0,j} + \sum_{k=1}^{K} \beta_{k,j} X_{k,ij} + \sum_{l=1}^{L} \beta_l X_{l,ij} + \varepsilon_{ij}, \\ &\beta_{0,j} = \beta_0 + \mu_{0,j}, \qquad . \\ &\beta_{k,j} = \beta_k + \mu_{k,j}, \end{aligned} \tag{1}$$

where: $i = (1,..., N)$ refers to the level 1 and $j = (1,..., J)$ level 2 unit, Y_{ij} is the dependent variable, X_{ij} represents (K+L) different level 1 regressors. It is assumed that the residuals:

$$(\varepsilon_{ij}) : E(\varepsilon_{ij}) = 0, \mathrm{var}(\varepsilon_{ij}) = \sigma_\varepsilon^2 \quad \text{and} \quad \forall i \neq i^{'} : \mathrm{cov}(\varepsilon_{ij}, \varepsilon_{i^{'}j}) = 0 .$$

The fixed effects (across level 1) arerepresenting by the elements of the vector β_l. Also β_0 and the elements of the vector β_k are treated as the fixed coefficients in the estimation. The intercept $\beta_{0,j}$ and the coefficients $\beta_{k,j}$ are allowed

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1 In the literature several different names are using e.g.: *hierarchical models*, *random-effects regression models*, *mixed-effects models*. There is also multiple notations used for describing models (see: Ferron 1997) and no agreement if the existence of the interaction effect is the necessary condition constituting the multilevel model.

2 The general multilevel model contains different submodels (see: Steenbergen M., Jones B. 2002, p. 224).

to vary randomly across level 1 and are using as the dependent variable at the level 2. In the level 2 $\mu_{0,j}$ and $\mu_{k,j}$ are treated as random variables with:

$$E(\mu_{k,j}) = E(\mu_{0,j}) = 0 \quad \text{and} \quad \mathrm{var}(\mu_{0,j}) = \sigma^2_{\mu_0},$$

$$\mathrm{var}(\mu_{k,j}) = \sigma^2_{\mu_k}, \quad \mathrm{cov}(\mu_{0,j}, \mu_{k,j}) = \sigma_{\mu_{0k}},$$

$$\forall j \neq j' : \mathrm{cov}(\mu_{0,j'}, \mu_{0,j}) = \mathrm{cov}(\mu_{k,j'}, \mu_{k,j}) = 0,$$

$$\mathrm{cov}(\mu_{0,j'}, \varepsilon_{ij}) = \mathrm{cov}(\mu_{k,j'}, \varepsilon_{ij}) = 0.$$

In the contrast to the level 1, in the upper levels coefficients are not estimated explicitly[3]. The parameters estimated e.g. by restricted maximum likelihood (REML) are: the variance-covariance matrix of the random coefficients, the variance of the error terms and the fixed coefficients. This approach is the result of the assumption that the observations from each level are the random sample and comes from the larger population (Zeilstra 2008, p. 21).

The existence of the hierarchical or clustered data structure can be noted not only in the social or medical science, where multilevel modelling is the most popular, but also in the regional context[4]. In the most obvious case spatial units representing the lower level of data aggregation are grouped at the higher levels, e.g. in the three-level structure regions formed countries which are the part of the international organizations. As the consequence individual observations are affected by the group (membership, context) effect[5].

In the regional studies the membership effect might be the result of i.a. the legislation, social and institutional environment, culture, historical background, economic policies or nation's competitiveness which are common for the regions located in the same country but notice cross-countries differences. The explanation of it refers to the concept of the absolute location effect (Capello, Nijkamp 2009, p. 375).

According to Abreu et. al (2005) in the empirical research the non-spatial techniques to the absolute location effect has dominated. Within this literature to capture the effect of unit location the controlling variable (country-specific) are using (Armstrong 1995). Newly, the membership effect significance started be proved in the spatial econometrics modelling using regimes (Anselin 1988), spatial ANOVA (Griffith 1992) or by the way of the spatial weights matrix

[3] According to this, if K=1 total residual variance is decomposing into: $\mathrm{var}(\mu_{0,j} + \mu_{1,j} x_{ij} + \varepsilon_{ij})$ (Snijders, Bosker 2011, pp. 123–124).

[4] As noted by Corrado and Fingleton (2012, p. 226), existing the cross-sectional or spatial data and common locations (such natural in the economic geography) results the multilevel modelling became an obvious starting point.

[5] Due to this, the assumption of the data interdependence (necessary in OLS regression) has been broken.

formulation (Arbia et al. 2010). The common for all those method is capturing the spatial heterogeneity without its further explanation.

In the opposite to above multilevel models except controlling the group effect, are able to identify its causes. For the spatial analysis the main advantages of multilevel models are:

- simultaneously modelling the micro and macro levels,
- preventing decreasing degrees of freedom by resignation from the dummy variables which control the group effects,
- ability to decomposition the total random variation in the individual and group components;
- calculating dependencies between units using: the proportion of the variance explained at the upper levels (national or supranational effects) by the variance partitioning coefficients and degree of the correlations between observations in the same group by the intra-class correlation[6],
- allowing units to be cross-classified or multiple membership,
- understanding how the phenomena and processes are related or nested.

Despite above the multilevel modelling technique – such common for the social and medical science – seems to be marginalized in the regional studies which are concentrated on the spatial autocorrelation.

On the one hand, ignoring heterogeneity leads to mistakeas the spatial dependence might be the result of the unmodelled parameter instability. Additionally, hierarchical data structure might causes the spatial interaction differences in each level of data aggregation. On the other hand, in the presence of the spatial autocorrelation, modelling only spatial heterogeneity might cause the spatially correlated errors and upward bias of the estimated upper-level variance. As the traditional multilevel model are unable to jointly explaining both spatial effects: dependence and heterogeneity, it requires to be rebuild.

The aim of this paper is to present the evolution of the multilevel models construction towards the spatial effects identification. Section 2 concentrates on the models with cross-interdependencies and interaction-based. Section 3 contains the discussion about the spatial multilevel models while the conclusions are in the last one.

2. SPACE DIMENSION IN MULTILEVEL MODELLING

In the contrast to the method of the spatial heterogeneity capturing, different approaches to incorporating the spatial dependence into multilevel models exist in the literature. In the first group model with the interaction effects (Manski 1993) and cross-interdependencies (Cohen-Cole 2006) could be classified. In both cases the concept of the endogenous interaction effects is similar to the spatially lagged dependent variable. And although the modification toward the

[6] In the random intercept model both measures are equivalently. See: Gräb 2009, pp.45–47.

spatial autoregressive models (Corrado and Fingleton 2012, pp. 229–234) is possible in both, the traditional specification might be also useful to consider the spatial relationships.

2.1. INTERACTIONS-BASED MANSKI'S MODEL

Let start from the pioneering Manski's linear-in-expectations model, which can be expressed as following[7]:

$$
\begin{aligned}
Y_{ij} &= \beta_{0,j} + \sum_{l=1}^{L} \beta_l X_{l,ij} + \varepsilon_{ij}, \\
\beta_{0,j} &= \beta_0 + \rho \tilde{Y}_j + \sum_{l=1}^{L} \gamma_l \tilde{X}_{l,j} + \mu_{0,j},
\end{aligned}
\tag{2}
$$

where: $\tilde{Y}_j = \frac{1}{m_j} \sum_{i' \in j} E(Y_{i'} \mid v_i)$, $\tilde{X}_{l,j} = \frac{1}{m_j} \sum_{i' \in j} E(X_{l,i'} \mid v_i)$, m_j, is the j group size and v_i represents the set of i's information. In contrast to (1), in (2) individual i outcomes additionally depends on the group average expected outcomes[8]. Also it is assumed that unit i knows only the expected values of the cross-level variables. According to (2) there are three effects: endogenous (1 in Graph 1), correlated (2 in Graph 1) and exogenous (contextual). As the first one reflect to the situation when the observations are influenced by the others form the same group, e.g. herd behaviours (ρ), the second one corresponds to the unobserved ($\mu_{0,j}$) within-group similarities. The third one measures the observed characteristics of the individuals (β_l) and group levels (γ_l).

Such construction suffers both for the endogeneity (due to the self-selection or the common group effect) and simultaneity, what means that the correct distinguish between the endogenous and contextual effect from the correlated in (2) is complicated (Manski's "*reflection problem*")[9]. Several different approaches of solving those obstacles bring multilevel models closer to the spatial econometrics. The reason why the identification problem occurs lie on the model assumptions.

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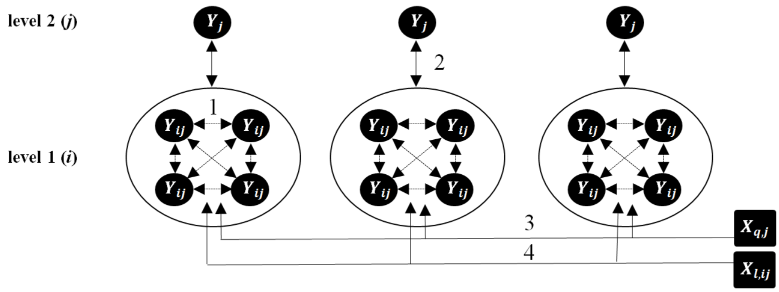
[7] In the literature there are several variations of the original Manski's formulation, e.g. Blume et al. (2011, p. 863) presented the model without the correlated effect. In contrast in Grahamand Hahn (2005) units have the common set of information, all variables are measure as the deviations from the mean ($\beta_0 = 0$) and the correlation effect exists.

[8] Under the additional assumptions (2) is equal to (1).

[9] The absence of the correlated effect in the model causes the collinearity between endogenous and contextual effects (Braumollé et al. 2009).



Graph 1. Relationships in the interactions-based multilevel model

Source: own studies.

Firstly, in the original Manski's model the contextual effects are equal to the average of the exogenous variables. That causes the parameters cannot be identified[10]. Iannides and Topa (2010) pointed that partially identification is possible in such situation, but breaking this reflection might be done even if at least one variable treated as exogenous (4 in Graph 1) is different from the contextual (the separate variables $X_{q,j}$ are noticed as 3 in Graph 1).

Secondly, in (2) the conditional expectations are linear. Brock and Durlauf (2001) proposed (under the assumption of no correlation effect) allowing nonlinearities in the model, what solved the reflection problems. Blumeet al. (2011, pp. 871–872) showed that the modification of the Manski's model toward the hierarchical can guarantee the full identification without any further assumption. Such model (with the correlated effects represented by the random terms) can be expressed as following:

$$\begin{aligned} Y_{ij} &= \beta_{0,j} + \sum_{k=1}^{K} \beta_{k,j} X_{k,ij} + \varepsilon_{ij}, \\ \beta_{0,j} &= \beta_0 + \rho \tilde{Y}_j + \sum_{k=1}^{K} \gamma_k \tilde{X}_{k,j} + \mu_{0,j}, \\ \beta_{k,j} &= \beta_k + \psi \tilde{Y}_j + \sum_{k=1}^{K} \varphi_k \tilde{X}_{k,j} + \mu_{k,j}, \end{aligned} \tag{3}$$

In contrast to (2) there are additional cross-products (in the third equation) which provide the nonlinearities in the relationship. As the result full identification of the parameters is possible, even under the assumption that the contextual variables are the average of the exogenous. Although such method resolving the reflection problem the practical interpretation of the ψ might be difficult.

10 In such case $\tilde{Y}_j$ is linear dependent on the other regressors.

In the opposite to the traditional spatial econometrics in (2) and(3) there are no cross-groups dependencies and the within-group interactions are the same for each pair of the units. The special form of the connections, where all units have the same reference group ($\tilde{Y}_j$),is the third source of the identification problem in (2). According to this the identification is possible due to the modification of the interaction structure. Technically it is achieved by incorporating the idea of the spatial weight matrix or quasi form of it. In opposite to (2) Moffitt (2001) proposed to exclude *i* from the reference group (self-selection). As the result $\tilde{Y}_j = \frac{1}{m_j - 1}\sum_{i \in j} E(Y_i \mid v_i)$ and $\tilde{X}_{l,j}$ are modified analogously. The identification in such model is unable as long as each group *j* has the same size[11]. Using the spatial weight matrix notation this relationship structure can be expressed as:

$$\mathbf{W_1} = \begin{bmatrix} \mathbf{M_1} & & 0 \\ & \ddots & \\ 0 & & \mathbf{M_J} \end{bmatrix}, \tag{4}$$

where: $\mathbf{W_1}$ is the block-diagonal $N \times N$ spatial weight matrix, each of the matrix $\mathbf{M_J}$ is symmetric with $m_j \times m_j$ elements equal to $\frac{1}{m_j - 1}$ and 0's on its main diagonal. More complicated relations, which guarantee the model identification, are also possible.

2.2. CROSS-INTERDEPENDENCIES COHEN-COLE'S MODEL

Let now expand the idea of the endogenous interaction toward the multiple reference groups. Cohen and Cole (2006) pointed that not only the neighbours from the same group might influence on the unit *i*, but also the other group members. Their model can be expressed as:

$$\begin{aligned} Y_{ij} &= \beta_{0,j} + \sum_{k=1}^{K} \beta_{k,j} X_{k,ij} + \varepsilon_{ij}, \\ \beta_{0,j} &= \beta_0 + \rho_1 \tilde{Y}_j + \rho_2 \tilde{Y}_I + \sum_{l=1}^{L} \gamma_l \tilde{X}_{l,I} + \mu_{0,j}, \end{aligned} \tag{5}$$

where:

[11] The influence of the group size variation was consider in Section 3.1.

$$\tilde{Y}_I = \tilde{Y}_j - \frac{1}{N - m_j} \sum\nolimits_{i' \notin j} E(Y_{i'} \mid v_i) \quad \text{and} \quad \tilde{X}_{l,I} ,$$

is calculated analogously. The endogenous effect is divided into: within-group (ρ_1) and between-group (ρ_2) to incorporating the additional across group expectations. Also in (5) appears the between-group contextual effects[12] (γ_l) instead of the within-group as in (2) and (3). In contrast to (3) Cohen and Cole's model has also more intuitive to interpret the contextual effects. The fully identification in (5) is achieving thanks to the additional information from the cross-group relations[13].

Such construction might be modified in the spatial econometrics manner. Firstly, as showed by Corrado and Fingleton (2012, pp.233–234) the spatial Durbin specification has the similar feature. The result of their rebuilding is as following:

$$\begin{aligned} Y_{ij} &= \beta_{0,j} + \sum_{k=1}^{K} \beta_{k,j} X_{k,ij} + \varepsilon_{ij}, \\ \beta_{0,j} &= \beta_0 + \rho_1 \mathbf{W_1} Y_{ij} + \rho_2 \mathbf{W}_2 Y_{ij} + \sum_{l=1}^{L} \gamma_l \mathbf{W}_3 X_{l.j} + \mu_{0,j}, \end{aligned} \tag{6}$$

where: $\mathbf{W_1}$ is specified as in (4), $\mathbf{W}_2$ is $N{\times}N$ block-diagonal spatial weight matrix with the between-group relations represented by $J{\times}J$–J off-diagonal blocks of $\mathbf{M} = \frac{1}{N - m_j}(\mathbf{l}_{m_j}\mathbf{l}'_{m_j} - \mathbf{I}_{m_j})$ and $\mathbf{l}_{m_j}$ is m_j-dimensional column vector of 1, $\mathbf{I}_{m_j}$ is m_j-dimensional identity matrix (see: Lee 2007).

$$\mathbf{W}_2 = \begin{bmatrix} 0 & \mathbf{M}_{21} & 0 \\ \mathbf{M}_{21} & \ddots & \mathbf{M}_{2J} \\ \mathbf{M}_{J1} & \mathbf{M}_{J2} & 0 \end{bmatrix}. \tag{7}$$

The third spatial weight matrix $\mathbf{W}_3 = \frac{1}{J-1}(\mathbf{l}_J\mathbf{l}'_J - \mathbf{I}_J)$ is $J{\times}J$ (according to the higher level predictors). As all of the non-diagonal elements have the same weight it is assumed (like in 5) that each member of the other groups influenced the unit i in the same way. In this case $\mathbf{W}_1$, $\mathbf{W}_2$, $\mathbf{W}_3$ are naturally row-standardized and $\rho_1, \rho_2, \gamma_l \in \langle -1,1 \rangle$. In the opposite to the traditional spatial Durbin

[12] Which are different from the average of the exogenous variables.

[13] The only assumptions are that the observations from each level represent the random sample from the larger population and the group sizes are different.

model, where only one type of the spatial weight matrix is taking into account, in (6) three different weighting schemes are incorporating. On the one hand this moves model toward multiparametric spatial models (Hays et al. 2009). On the other hand, existing of the random part brings multilevel context of the analysis. As long as the elements of the spatial weight matrix are the same, it is still unusual situation for the spatial econometrics perspective.

3. SPATIAL MULTILEVEL MODELS

In the spatial econometrics framework the spatial dependence, can be expressed by: spatially lagged dependent variable ($\mathbf{W}y$), spatially lagged explanatory variables ($\mathbf{W}x$) or spatially lagged error terms ($\mathbf{W}\mu$), where $\mathbf{W}$ is the spatial weights matrix. The spillover effect is then calculated using spatial multiplier (Anselin 2003) and can be written using "Leontief expansion":

$$(\mathbf{I}-\rho\mathbf{W})^{-1}=\mathbf{I}+\rho\mathbf{W}+\rho^{2}\mathbf{W}+\rho^{3}\mathbf{W}+\ldots, \tag{8}$$

where $\mathbf{I}$ is the identity matrix. According to (8) each observation is correlated with the others but, in opposite to the earlier presented, the influence decays over the space, referred to Tobler's First Law of Geography. Let now introduce the multilevel conception from the spatial econometrics perspective. As the starting point let consider the reflection problem again.

3.1. SPATIAL PANEL DATA MODELS APPROACH

The restricted version of the multilevel models[14]is the equivalent of the random effects model in which instead of the cross-sectional data the spatial hierarchy exists. According to this, Graham and Hahn (2005) noticed that separating the endogenous effect from the correlated(under the assumption of no contextual effects) in Manski's model is possible by adopting the quasi-panel data model. To handle with the identification problem they add an extra source of the information from the between-group variations and adopt the idea of the Hausman-Taylor estimator. Similar to this Corrado and Distante (2012) rebuild (2) toward the spatial dynamic panel data model, also achieving the identification.

The common for both are using the linear-in-means model. Let us concentrate on the idea of the rational expectations. In Manski's model it as assumed that due to the large number of units in groups information is incomplete and the expectations play role in the interactions (linear-in-expectations model). In opposite to this, in more common version of (2) it is assumed that the groups are small and units have complete information about other behaviours (linear-in-

[14] Without the higher level predictors as in (1).

means model). According to above Lee (2007) noticed that the identification is possible only if there is the variations in the group sizes and the groups are small[15]. At least three different group sizes was identified as obligatory to obtain the satisfactory variations and guarantee the identification in (9).

In contrast to the mentioned above, Lee (2007) modified the Manski's model toward the spatial autoregressive model with the contextual effects (in the form of the spatial dependences $\mathbf{W}_3 X_{l,j}$) and group interactions (expressed as fixed effects $\beta_{0,j}$)[16]. This model can be expressed as:

$$Y_{ij} = \beta_{0,j} + \rho \mathbf{W}_1 Y_{ij} + \sum_{l=1}^{L} \gamma_l \mathbf{W}_3 X_{l,j} + \sum_{k=1}^{K} \beta_{k,j} X_{k,ij} + \varepsilon_{ij}, \tag{9}$$

where $\mathbf{W_1}$ is as in (4) and $\mathbf{W}_3$ as in (6). By using the fixed effects to capture the heterogeneity, Lee (2007) handled with the collinearity between the correlated and exogenous effects. It is worth to mentioned that relaxing the assumptions of the relationship structure (from the restrictive network to more common in the spatial econometrics) might determines the identification.

As long as the restricted version of the multilevel model are using, analysis of the spatial dependencies might be done *via* spatial random (or fixed) effects panel data model. Unfortunately, in more advanced cases panel data framework seems to be insufficient. To deal with it let consider the further modification of (3), rather than the spatial panel data models.

3.2. SPATIAL MULTILEVEL MODELS

The wide range of the formulation based on (3) and incorporating spatial weight matrix are possibly. As the example consider Chasco and Le Gallo (2012) multilevel model with the spatial interactions designed for analysing Madrit's house market. Those specification can be treated as the variation of (6). The2-level generalization of the construction can be written as:

$$\begin{aligned} Y_{ij} &= \beta_{0,j} + \sum_{k=1}^{K} \beta_{k,j} X_{k,ij} + \sum_{k=1}^{K} \gamma_k \mathbf{W} X_{k,ij} + \sum_{l=1}^{L} \beta_l X_{l,ij} + \sum_{l=1}^{L} \gamma_l \mathbf{W} X_{l,ij} + \varepsilon_{ij}, \\ \beta_{0,j} &= \beta_0 + \sum_{q=1}^{Q} \beta_q X_{q,j} + \sum_{q=1}^{Q} \gamma_q \mathbf{W} X_{q,j} + \mu_{0,j}, \\ \beta_{k,j} &= \beta_k + \mu_{k,j}, \end{aligned} \tag{10}$$

15 In small groups the endogenous effect is stronger than in the large ones and achieving the converge, e.g. via conditional MLE and IV estimation, is easier (Lee 2007).

16 It refers to the spatial Durbin panel data model(with fixed effects).

where: $\mathbf{W} = [w_{ij}]$ and the elements of the spatial weight matrix w_{ij} might be based on the spatial contiguity or geographical/socio-economic distance[17]. Apart from the previous, (10) represents the advanced example of the spatial cross-regressive multilevel models (without the spatial endogenous effects). It means that the only source of the spatial interactions between units and groups is *via* the exogenous and contextual effects. Just like in the original Chasco and Le Gallo construction all explanatory variables at each level has been analysed as spatially lagged. The main advantage of (10) is the possibility of the model modification, e.g. by add or omit some variables. One of the most interesting variation of such model seems to treat one of the spatial lag parameters (γ_k, γ_l) as random.

In the opposite to (9) correlated unobservables are modelled here as the random effects. According to this, all multilevel modelling advantages are in force, what is useful especially when the observations come from the random sample survey[18]. Additionally, as the result of using the spatial weight matrix, with the structure respecting the Tobler's First Law of Geography, combining the within-group and between-group endogenous effects was possible. Such model can be estimated *via* REML or RIGLS, just like in (1). As the endogenous effects were eliminated in (10) the estimation problems not occurred.

4. CONCLUSIONS

In the spatial econometrics the correct distinguish between the two main spatial effects (heterogeneity and dependence) might help to answer the question about the sources of the spatial variability. An attractive solution of the simultaneously modelling both spatial effects is to combining the traditional spatial models with the multilevel modelling framework. In this paper a brief review of the spatial modification of the multilevel models was presented. The main concentration was put on the analyzing of the spatial effects in the interaction-based multilevel model and its variations. As the spatial multilevel models constructions were also mentioned, it is still need to exploring this field, e.g. spatial multilevel CAR models. Although incorporating spatial dependence into multilevel context makes deeper the spatial analysis, using such technique requires to solve the methodological problem, like the endogeneity and simultaneity.

[17] In Chasco and Le Gallo the inverse squared distance matrix was used.

[18] When all units from population are taking into account using the fixed effects rather than the random is recommend.

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SPATIAL ASPECTS IN THE MULTILEVEL MODELS CONSTRUCTION

Multilevel (hierarchical) models are used for analysing data for which getting a few levels of the aggregation is possible. In the simplest case it is possible to present the way of organizing the levels in the form of the hierarchical structure or applying the cross-classification of data. The multilevel model construction might be used in the spatial analyses. The purpose of this article is to present the possibility of spatial processes analyses using multilevel models. The implementation techniques of the already existing multilevel models to the spatial structure were discussed. Additionally, the possibility of the traditional multilevel models rebuilding, towards taking into account spatial interactions, was present.

ASPEKTY PRZESTRZENNE BUDOWY MODELI WIELOPOZIOMOWYCH

Modele wielopoziomowe (hierarchiczne) wykorzystywane są w celu analizy danych, dla których możliwe jest uzyskanie kilku poziomów agregacji. W najprostszych przypadkach sposób zorganizowania kolejnych poziomów można przedstawić w postaci struktury hierarchicznej lub stosując agregację poprzeczną danych. Sposób budowy modeli wielopoziomowych sprawia, że mogą one być również wykorzystywane na gruncie analiz przestrzennych. Celem artykułu jest zaprezentowanie możliwości zastosowania modeli wielopoziomowych w analizach procesów przestrzennych. W pracy omówiono dotychczasowe techniki implementacji modeli wielopoziomowych w analizach struktur przestrzennych. Dodatkowo, zaprezentowano możliwości rozszerzenia tradycyjnych modeli wielopoziomowych w kierunku uwzględnienia interakcji przestrzennych.