A C T A U N I V E R S I T A T I S L O D Z I E N S I S
FOLIA OECONOMICA 286, 2013

*Dominik Krężołek**

NON-CLASSICAL RISK MEASURES ON THE WARSAW STOCK EXCHANGE – THE APPLICATION OF ALPHA-STABLE DISTRIBUTIONS

Abstract. The subject of this paper is the presentation of application of the alpha-stable distributions methodology in investment risk measurement. The non-classical risk measures based on Value-at-Risk are presented: Expected Shortfall and Median Shortfall, for given quantiles. The analysis is made using daily log-returns from the WSE. The results show better estimation of empirical risk values using probability distributions belonging to the family of stable ones.
Key words: alpha-stable distributions, Value-at-Risk, Expected Shortfall, Median Shortfall, heavy tails.

I. INTRODUCTION

All economic processes observed in the last few years are characterized by a high level of unpredictability. No matter what the area in which this phenomenon occurs, these processes are related to the increasing uncertainty and risk (in a broad sense).

There are many definitions of risk. One of them says that risk is a state, when the decision maker is able to define variants of the results of some decision and determine the probability of their occurrence. If uncertainty is taken into account, it is defined as the level of ignorance regarding the reality and what is the impact of thin ignorance on the potential actions of decision maker. Both risk and uncertainty show unambiguous difference – the risk can be measured. This feature determines the division of risk measures into classical (Markovitz (1952)) and non-classical risk measures (Artzner, *et al*. (1997), Rachev (2004)).

II. NON-CLASSICAL INVESTMENT RISK MEASURES

In this paper non-classical risk measures are considered in terms of coherent risk measures (based on quantiles of some distribution). Artzner *et al*. defined the properties of coherent risk measure: subadditivity, positive homogeneity,

* Ph.D., Department of Demography and Economic Statistics, University of Economics in Katowice.

monotonicity and translation invariance. All these axioms which define the properties of coherent risk measure possess explicit economic interpretation. Not derogating from the other properties, of particular importance in risk measurement is subadditivity which says that total risk of an investment is not higher than the sum of risks of its components. This axiom is especially important in terms of diversification of investment portfolio, as the total risk of portfolio is the sum of its components only if the risk factors are generated from common source. In other cases, the total risk of the portfolio is lower than the sum of the partial risks thanks to diversification.

As far as quantile measures are concerned, in this paper only some measures based on Basel methodology are presented:

– Value-at-Risk – which answer the question what is the maximum loss/profit which may be incurred (with certain probability) by the institution/investor, from an investment of a certain value within a specified period of time,

– Expected Shortfall – which answer the question what is the expected loss/profit from an investment beyond the VaR level (in terms of expected value):

$$ES^{(\alpha)}(X) = CVaR^{(\alpha)}(X) = E\left[X - VaR^{(\alpha)}(X) \middle| X > VaR^{(\alpha)}(X)\right] \quad (1)$$

– Median Shortfall – which answer the question what is the expected loss/profit from an investment beyond the VaR level (in terms of median):

$$MS^{(\alpha)}(X) = Median\left[X - VaR^{(\alpha)}(X) \middle| X > VaR^{(\alpha)}(X)\right] \quad (2)$$

Both Expected Shortfall and Median Shortfall are coherent risk measures.

Apart from measures mentioned above one of brand new quantile measures is presented – the Rachev Ratio:

$$R - ratio = \frac{E\left(|X| \mid X \geq -VaR_{\alpha}(-X)\right)}{E\left(|X| \mid X \leq VaR_{\beta}(X)\right)} \quad (3)$$

This measure can be defined as the expected tail return divided by expected tail loss. Rachev ratio is the ratio between expected profit above the level of VaR (at any given $\alpha -$quantile) to expected loss below the level of VaR (at any given $\beta -$quantile). In other words this measure can be defined as the probability area of profit divided by the probability area of loss. One of the most important features of this ratio is that, for being calculated, the values from both tails of the distribution are required.

III. THE METHODOLOGY OF ALPHA-STABLE DISTRIBUTIONS

The main theoretical works on this class of probability distributions are dated on the 60's when Mandelbrot (1963) and Fama (1965) discovered that the analyzed time series are characterized by a high, statistically significant level of kurtosis. This phenomenon resulted in rejection of hypothesis, that empirical distribution follows normal distribution, in favor of the alternative one.

The distributions which belong to the alpha-stable family are characterized by certain shape parameter, which enables for modeling of asymmetry and thickness of the tail of distribution. This allows to estimate the probability that a random variable takes values at the level significantly outlying from its expected value. Due to lack of explicit form of stable density function (except three special cases), to present the distribution of stable random variable the characteristic function approach is used:

$$\varphi_S(t) = E(\exp\{itX\}) = \\ = \begin{cases} \exp\left\{-\gamma^\alpha |t|^\alpha \left[1 + i\beta \tan\frac{\pi\alpha}{2}\operatorname{sgn}(t)\left(|\gamma t|^{1-\alpha} - 1\right)\right] + i\delta t\right\}, & \alpha \neq 1 \\ \exp\left\{-\gamma|t|\left[1 + i\beta\frac{2}{\pi}\operatorname{sgn}(t)\ln(\gamma|t|)\right] + i\delta t\right\}, & \alpha = 1 \end{cases} \tag{4}$$

where $0 < \alpha \leq 2$, $-1 \leq \beta \leq 1$, $\gamma > 0$, $\delta \in \Re$ and $\operatorname{sgn}(t) = \begin{cases} 1 \Leftrightarrow t > 0 \\ 0 \Leftrightarrow t = 0 \\ -1 \Leftrightarrow t < 0 \end{cases}$.

The most important parameter describing the alpha-stable distribution is the index of stability α which measures the thickness of the tail of distribution. The other parameters are skewness parameter β, scale parameter γ and location parameter δ. All these parameters are unknown and have to be estimated. Popular classical estimation methods, as MLE or Method of Moments, are supported by non-classical ones: Quantile Method or Tail Exponent Estimation.

IV. THE EMPIRICAL ANALYSIS

The application of alpha-stable distribution methodology together with non-classical risk measures is presented using daily log-returns from 36 assets listed in the WSE from July 2007 to June 2012. The theoretical distributions used in analysis are: normal distribution ($\alpha = 2$), alpha-stable distribution ($\alpha < 2$) and asymmetric Laplace distribution (belonging to the family of geo-stable distributions, described in details in Mittnik, Rachev (1993). To calculate non-classical risk measures given by formulas (1) – (3), the quantiles of order 0,01, 0,05, 0,95 and 0,99 are used. The average values of all estimated stable parameters for 36 assets in period 01/07/2007 – 31/12/2011 are presented in Table 1.

Table 1. Average values of estimated stable parameters

Period	$\bar{\alpha}^*$	$\bar{\beta}^*$	$\bar{\gamma}^*$	$\bar{\delta}^*$
01/07/2007 – 31/12/2011	1,6495	0,0639	0,0139	-0,0010

As presented in Table 1, the log-returns observed on Polish stock exchange are characterized (in average sense) by heavy-tailed distributions. The skewness parameter takes positive value which means that the empirical distributions are right-hand skewed. The location parameter takes negative value which means that within analyzed period average values of assets were negative. The fitted normal (dotted line), stable (thin solid line) and asymmetric Laplace (thick solid line) probability density functions (PDF) for MAGNA (the lowest value of index of stability) and KGHM (the highest values of index of stability) are presented in Figures 1 and 2.

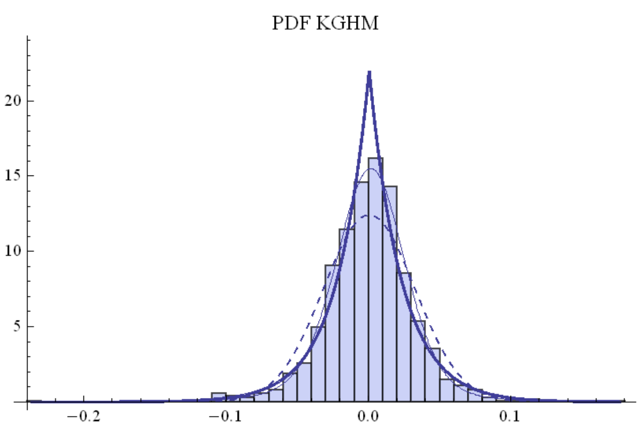

Fig. 1. PDF of KGHM

Source: own calculations.

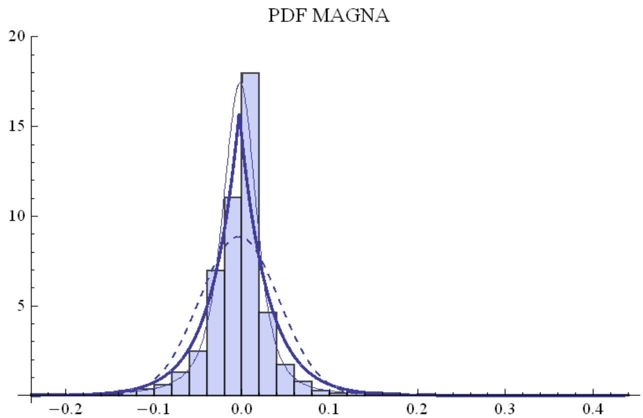

Fig. 2. PDF of MAGNA
Source: own calculations.

The estimated parameters of theoretical distributions fitted to the empirical data are presented in Tables 2-4.

Table 2. Estimated parameters of normal distribution

Asset	μ^*	σ^*
MAGNA	-0,0029	0,0450
KGHM	0,0006	0,0321

^ parameters statistically insignificant at level 0,01
Source: own calculations.

Table 3. Estimated parameters of alpha-stable distribution

Asset	α^*	β^*	γ^*	δ^*
MAGNA	1,3728	-0,1066	0,0167	-0,0051
KGHM	1,7176	-0,1188	0,0183	0,0005

Source: own calculations.

Table 4. Estimated parameters of asymmetric Laplace distribution

Asset	σ^*	γ^*	κ^*
MAGNA	-0,0029	0,0317	1,0475
KGHM	0,0006	0,0227	0,9868

Source: own calculations.

Estimated parameters of normal distribution are statistically insignificant while these, related to alpha-stable and asymmetric Laplace distribution are significant, at level 0,01. Using formulas 1-3 and estimated parameters of theoretical distributions presented in Tables 2-4, the non-classical risk measures, based on data from period 01/01/2012–30/06/2012 have been calculated. Theoretical quantiles have been calculated from simulated data derived from given theoretical distributions. Comparison of empirical and theoretical quantiles is presented in Table 5.

Table 5. Non-classical risk measures – comparison of empirical and theoretical quantiles

Measure	Quantile	Asset	Empirical distribution	Alpha-stable distribution	Normal distribution	Asymmetric Laplace distribution
VaR	0,01	MAGNA	–0.1342	–0.1883	–0.1075	**–0.1271**
		KGHM	–0.0935	**–0.0941**	–0.0741	–0.0882
	0,05	MAGNA	–0.0622	**–0.0649**	–0.0769	–0.076
		KGHM	–0.0483	**–0.048**	–0.0522	–0.0517
	0,95	MAGNA	0.0518	**0.0511**	0.071	0.0701
		KGHM	0.0488	**0.0476**	0.0534	0.0529
	0,99	MAGNA	0.1567	**0.1549**	0.1017	0.1212
		KGHM	0.0793	0.0871	**0.0753**	0.0894
ES	0,01	MAGNA	–0.1752	**–0.2023**	–0.1203	–0.1476
		KGHM	–0.1182	–0.1249	–0.0839	**–0.1226**
	0,05	MAGNA	–0.107	–0.1383	–0.0937	**–0.1085**
		KGHM	–0.081	–0.0909	–0.0775	**–0.0785**
	0,95	MAGNA	0.1127	**0.1114**	0.0947	0.1092
		KGHM	0.071	**0.0717**	0.0646	0.073
	0,99	MAGNA	0.2143	0.2803	0.1263	**0.1614**
		KGHM	0.1108	**0.1184**	0.0848	0.1002
MS	0,01	MAGNA	–0.1714	–0.2515	–0.1144	**–0.1522**
		KGHM	–0.1075	–0.1167	–0.0826	**–0.1122**
	0,05	MAGNA	–0.0947	**–0.0952**	–0.0893	–0.0995
		KGHM	–0.065	**–0.0676**	–0.0597	–0.0727
	0,95	MAGNA	0.0848	0.0745	**0.0889**	0.0958
		KGHM	0.063	0.0565	**0.0596**	0.0668
	0,99	MAGNA	0.1805	0.2598	0.1211	**0.1607**
		KGHM	0.1093	**0.0993**	0.0808	0.09
R-Ratio	0,01-0,99	MAGNA	1.2234	1.3859	1.0503	**1.0934**
		KGHM	0.9367	**0.9483**	1.0106	0.8172
	0,05-0,95	MAGNA	1.0535	0.8054	1.0111	**1.0165**
		KGHM	0.8772	0.7892	**0.833**	0.9301

Source: own calculations.

In Table 5, the smallest absolute differences between empirical and theoretical values of risk measures are marked in bold. As is shown, in most cases the risk measures obtained from alpha-stable or asymmetric Laplace distributions take values closest to the empirical ones, if compared to normal distribution. These results confirm that the application of probability distributions, which take into account some special features of empirical distributions of financial assets (like asymmetry, leptokurtosis or heavy tails), is more appropriate, especially if compared to normal distribution. Moreover, the use of normal distribution in risk assessment may lead to wrong investment decisions.

V. CONCLUSIONS

The purpose of this paper was to present some features and characteristics of alpha-stable distributions in investment risk measurement. Fitting theoretical distributions to empirical data it has been shown that the hypothesis of normality has to be rejected. Therefore, the risk measures based on alpha-stable distribution has been applied. The results shows that, for presented risk measures, the best approximation of Value-at-Risk obtained using alpha-stable approach (for quantile 0,05 and 0,95), while taking into account non-classical risk measures, the best approximation was obtained for Asymmetric Laplace distribution. Thus, basing on the mentioned results, the stable distributions approach allows to asses extreme investment risk better than if normal distribution is used.

REFERENCES

Artzner P., Delbaen F., Eber J-M., Heath D. (1999), Coherent Measures of Risk. *Mathematical Finance*, Vol. 9, No. 3, p. 203-228.

Biglova A., Ortobelli S., Rachev S.T., Stoyanov S. (2004), Different Approaches to Risk Estimation in Portfolio Theory, *Journal of Portfolio Management*, Vol. 31, No 1, p. 103-112.

Fama E.F. (1965), The behavior of stock market prices, *Journal of Business*, Vol. 38, No. 1, p. 34-105.

Mandelbrot B. (1963), The variation of certain speculative prices, *Journal of Business*, Vol. 36, No. 4, p. 394-419.

Markowitz H (1952), Portfolio selection, *The Journal of Finance*, Vol. 7, p. 77-91.

Mittnik S., Rachev S.T. (1993), Modeling Asset Returns with Alternative Stable Distributions, *Econometric Review*, Vol. 12, No 3, p. 261-330.

Dominik Krężołek

NIEKLASYCZNE MIARY RYZYKA NA PRZYKŁADZIE GIEŁDY PAPIERÓW WARTOŚCIOWYCH W WARSZAWIE – ZASTOSOWANIE ROZKŁADÓW ALFA-STABILNYCH

Przedmiotem artykułu jest prezentacja zastosowania metodologii rozkładów alfa-stabilnych w pomiarze ryzyka inwestycyjnego. Wykorzystano nieklasyczne mierniki ryzyka bazujące na metodologii Value-at-Risk: Expected Shortfall oraz Median Shortfall dla wybranych rzędów kwantyli. Badanie przeprowadzono dla dziennych logarytmicznych stóp zwrotu wybranych walorów Giełdy Papierów Wartościowych w Warszawie. Wyniki pokazują lepsze oszacowania rzeczywistych miar ryzyka przy wykorzystaniu rozkładów prawdopodobieństwa z rodziny stabilnych.