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OPTIMAL STOCK PORTFOLIO – APPLICATION OF MULTIVARIATE STATISTICAL ANALYSIS

Abstract. Investigating relationship between risk of the Markowitz optimal portfolio and the strength of interdependence for the set of rates of return for portfolio components we state (Konarzewska, 2008, 2012) that the risk measured as variance/standard deviation is slightly sensitive on small disturbance in data set when the series of data are strongly interrelated. What more, portfolio risk rises as the strength of interdependence declines. We have found that if strong linear relationship is present among series, it is important to control the direction between the portfolio weights vector and the eigenvector corresponding to maximal eigenvalue of the correlation/covariance matrix – the ideal situation being orthogonality of the two vectors. These results can be utilized in:

- the algorithm for pre-selection of investment portfolio components,
- construction of the optimal investment portfolio models.

Both propositions utilize eigenvalue decomposition of the rates of return correlation or covariance matrix.

Theoretical results are illustrated by empirical examples for medium-sized firms being components of mWIG40 index on Stock Exchange in Warsaw. We compare optimal portfolios obtained for Markowitz and PCA – aided models.

Key words: optimal portfolio models, principal component analysis of rates of return.

I. INTRODUCTION

Analyzing rates of return on stocks on Stock Exchange in Warsaw since 16 April 1991, the day of the first session, we observe dynamic changes of number of firms, transactions, volume of sales, price trends etc. Investors try to allocate their funds optimally, which usually means that they look for such investment portfolios which give the maximal return with limited risk or to with minimal risk guarantee realization of satisfactory rate of return. Empirical research conducted since 1991 revealed existence of strong linear relationships among rates of return on stocks. We are interested in identification of such relationships, measurement and analysis of their consequences on the portfolio

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investment results. Some insight into the problem of measuring the strength of relationships among series of rates of return on stocks and analysis of their influence on sensitivity of the optimal portfolios properties was done in Konarzewska (2008, 2012). In this paper we would like to present, besides some theoretical considerations, results of the empirical research for portfolios prepared with application of eigenvalue decomposition of correlation and covariance matrices for series of rates of return on stocks for medium-sized segment of firms on Stock Exchange in Warsaw observed in the period 2009-2011.

Let us introduce some formal notations:

$\mathbf{R} = [R_{tj}]$ j=1,…,N, t= 1,…,T, T≥N – the matrix of N time series of the rates of return on stocks,

$\mathbf{R_s} = [R^{\mathrm{s}}_{tj}]$ – the standardized matrix of the rates of return, where

$$R^{\mathrm{s}}_{tj} = \frac{R_{tj} - \overline{R}_j}{\hat{\sigma}_j} \tag{1}$$

$\overline{R}_j$ – sample mean for j-th series, j = 1,…,N,

$\hat{\sigma}_j$ – sample standard deviation for j-th series, j = 1,…,N,

$\hat{\mathbf{\Sigma}} = [\hat{\sigma}_{ij}]$, $\hat{\sigma}_{ij} = \frac{1}{\mathrm{T}}\sum_{t=1}^{\mathrm{T}}(R_{ti} - \overline{R}_i)(R_{tj} - \overline{R}_j)$, i, j =1,...,N – sample covariance matrix estimated basing on N series of T observations,

$\mathbf{P}$ – the sample linear correlation matrix of the rates of return,

$$\mathbf{P} = \frac{1}{\mathrm{T}}\mathbf{R}_\mathbf{s}^{\mathrm{T}}\mathbf{R}_\mathbf{s} \tag{2}$$

We assume that $r(\hat{\mathbf{\Sigma}}) = \mathrm{N}$ and eigenvalues of $\hat{\mathbf{\Sigma}}$ being $\lambda_1^* > \lambda_2^* > ... > \lambda_\mathrm{N}^* > 0$. Using eigenvalue decomposition of $\hat{\mathbf{\Sigma}}$ we can present the sample covariance matrix as the product:

$$\hat{\mathbf{\Sigma}} = \mathbf{V}^*\mathbf{\Lambda}^*\mathbf{V}^{*\mathrm{T}}, \tag{3}$$

$\mathbf{\Lambda}^* = \mathrm{diag}(\lambda_1^*, \lambda_2^*, ..., \lambda_\mathrm{N}^*)$ – the diagonal matrix, with eigenvalues of $\hat{\mathbf{\Sigma}}$ on the main diagonal,

$\mathbf{V}^* = \left[\mathbf{v}_1^*,..., \mathbf{v}_N^*\right]$ – the matrix of normalized eigenvectors related to $\lambda_1^*, \lambda_2^*,..., \lambda_N^*$; $\mathbf{V}^{*T}\mathbf{V}^* = \mathbf{V}^*\mathbf{V}^{*T} = \mathbf{I}_N$.

Using the result that

$$\mathbf{\Lambda}^* = \mathbf{V}^{*\mathbf{T}}\hat{\mathbf{\Sigma}}\mathbf{V}^* \tag{3a}$$

we can present eigenvalues of the covariance matrix in the following way:

$$\lambda_l^* = \mathbf{v}_l^{*T}\hat{\mathbf{\Sigma}}\mathbf{v}_l^* = \sum_{i=1}^{N}\sum_{j=1}^{N} v_{il}^* v_{jl}^* \hat{\sigma}_{ij}\ ,\ l=1,2,..,N, \tag{4}$$

where $\mathbf{v}_l^* = [v_{il}^*]$, i=1,…,N – eigenvector of $\hat{\mathbf{\Sigma}}$ corresponding to λ_l^*.

Eigenvalue decomposition of the rates of return sample correlation matrix results in the formulas

$$\mathbf{\Lambda} = \mathbf{V}^T\mathbf{P}\mathbf{V} = \frac{1}{T}\mathbf{V}^T\mathbf{R}_s^T\mathbf{R}_s\mathbf{V} \tag{5}$$

$$\mathbf{P} = \mathbf{V}\mathbf{\Lambda}\mathbf{V}^T, \tag{5a}$$

where $\mathbf{V} = \left[\mathbf{v}_1,...,\mathbf{v}_N\right]$ – the matrix of normalized eigenvectors related to eigenvalues of $\mathbf{P}$ ordered in descending series $\lambda_1 > \lambda_2 > ... > \lambda_N > 0$; $\mathbf{V}^T\mathbf{V} = \mathbf{V}\mathbf{V}^T = \mathbf{I}_N$.

Eigenvalues of the correlation matrix can be presented as functions of standardized data series and eigenvectors in the following way:

$$\begin{aligned}\lambda_l &= \frac{1}{T}(\mathbf{R_s v}_l)^T(\mathbf{R_s v}_l) = \\ &= \frac{1}{T}\sum_{t=1}^{T}(\sum_{j=1}^{N} R_{tj}^s v_{jl})^2, l = 1,..., N\end{aligned} \tag{6}$$

Exact linear relationships among rates of return resulting in zero-valued eigenvalues of $\hat{\mathbf{\Sigma}}$ or $\mathbf{P}$ are unlikely among data series. Very strong linear relationships can be identified using the measure known as matrix condition number – in case of quadratic symmetric matrices as covariance or correlation matrix it is defined as a ratio of the matrix maximal eigenvalue to the minimal one:

$$\kappa(\hat{\mathbf{\Sigma}}) = \frac{\lambda_1^*}{\lambda_N^*} \qquad \kappa(\mathbf{P}) = \frac{\lambda_1}{\lambda_N}. \tag{7}$$

Taking into account that $\hat{\mathbf{\Sigma}} = \mathbf{SPS}$, where $\mathbf{S} = \mathrm{diag}(\hat{\sigma}_i)$, i=1,…,N is a diagonal matrix with sample standard deviations of the rates of return on the main diagonal, the relationship between eigenvalues of the sample covariance matrix and sample correlation matrix is the following[1]:

$$\mathbf{\Lambda} = \mathbf{V}^{\mathrm{T}}\mathbf{S}^{-1}\mathbf{V}^{*}\mathbf{\Lambda}^{*}\mathbf{V}^{*\mathrm{T}}\mathbf{SV} = \mathbf{D}^{-1}\mathbf{\Lambda}^{*}(\mathbf{D}^{\mathrm{T}})^{-1} \tag{8}$$

where $\mathbf{D} = \mathbf{V}^{*\mathrm{T}}\mathbf{SV}$.

II. PORTFOLIO OPTIMIZATION – SOME THEORETICAL RESULTS

We consider the solution of the following optimization model:

$$\begin{aligned} &\min \hat{\sigma}_{\mathrm{P}}^2 \\ &\sum_{i=1}^{\mathrm{N}} x_i \overline{R}_i \ge r \\ &\sum_{i=1}^{\mathrm{N}} x_i = 1 \\ &x_i \ge 0, \quad i = 1,\ldots,\mathrm{N} \end{aligned} \quad , \tag{9}$$

where

$\mathbf{x} = [x_i]$, i=1,...,N; x_i – weight (fraction of wealth invested) of i-th security in the portfolio,

N – number of securities in the portfolio,

$\hat{\sigma}_{\mathrm{P}}^2$ – portfolio sample variance,

r – minimal rate of return satisfactory for the investor.

Portfolio sample variance can be formulated applying eigenvalue decomposition of sample covariance matrix as

[1] See Konarzewska (2012).

$$\begin{aligned}\hat{\sigma}_{\mathrm{P}}^2 &= \mathbf{x}^{\mathrm{T}}\hat{\mathbf{\Sigma}}\mathbf{x} = \\ &= \mathbf{x}^{\mathrm{T}}\mathbf{V}^*\mathbf{\Lambda}^*\mathbf{V}^{*\mathrm{T}}\mathbf{x} = \\ &= \boldsymbol{\omega}^{*\mathrm{T}}\mathbf{\Lambda}^*\boldsymbol{\omega}^* = \quad , \\ &= \sum_{j=1}^{\mathrm{N}} \lambda_j^* \omega_j^{*2},\end{aligned} \tag{10}$$

where $\boldsymbol{\omega}^* = \mathbf{V}^{*\mathrm{T}}\mathbf{x}$

$$\omega_j^* = \sum_{l=1}^{\mathrm{N}} X_l v_{lj}^* \text{ ,} j\text{=1,…,N.}$$

Similar to (10) representation can be obtained using eigenvalue decomposition of sample correlation matrix[2]:

$$\begin{aligned}\hat{\sigma}_{\mathrm{P}}^2 &= \mathbf{x}^{\mathrm{T}}\hat{\mathbf{\Sigma}}\mathbf{x} = \\ &= \boldsymbol{\omega}^{\mathrm{T}}\mathbf{\Lambda}\boldsymbol{\omega} = \sum_{j=1}^{\mathrm{N}} \lambda_j \omega_j^2,\end{aligned} \tag{11}$$

where $\boldsymbol{\omega} = \mathbf{V}^{\mathrm{T}}\mathbf{S}\mathbf{x}$,

$$\omega_j = \sum_{l=1}^{\mathrm{N}} X_l \hat{\sigma}_l v_{lj} \text{ ,} j\text{=1,…,N.}$$

Multivariate statistical analysis methods, like principal component analysis (PCA) are intensively applied nowadays in finance. Alexander (2008) presents such application to develop a statistical factor model for stock returns. We present here another application. Analyzing empirical data when strong linear relationships among rates of return on securities are present[3] using (PCA) leads to the result that most of variability measured by portfolio variance can be expressed by a small number of components.

The complete TxN matrix of principal components, denoted by **Y** is defined in the following way[4]:

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[2] It is worth mentioning here that conclusions drawn for PCA of covariance and correlation matrices are different. Krzyśko (2009) presents the illustrative example that only analysis of covariance matrix is correct because results for correlation matrices are not unique. Nevertheless, many statistical computer packages (Statistica, Statgraphics) conduct PCA for correlation matrices – standardized data.

[3] Relationship is usually considered as strong if the condition number for correlation matrix exceeds 30.

[4] Computer packages, f.e. Statistica, Statgraphics calculate principal components as columns of $\mathbf{Y} = \mathbf{R}_s\mathbf{V}$.

$$\begin{aligned} &\mathbf{\Lambda} = \frac{1}{\mathrm{T}}\mathbf{V}^{\mathrm{T}}(\mathbf{R}_s)^{\mathrm{T}}\mathbf{R}_s\mathbf{V} = \mathbf{Y}^{\mathrm{T}}\mathbf{Y} \Rightarrow \\ &\mathbf{Y} = \frac{1}{\sqrt{\mathrm{T}}}\mathbf{R}_s\mathbf{V} \end{aligned} \tag{12}$$

In our empirical study we modify the model (9) taking as an objective of minimization not the complete value of portfolio sample variance but only elements in the sum in (10) or (11) corresponding to limited number of principal components. We have found conducting former research on similar topics that when multicollinearity is present among series of data only the element corresponding to the first principal component was important to describe most portfolio variability – the structures of the optimal portfolios obtained for the model (9) and for the following model with the objective of minimizing the impact of the first principal component on portfolio variance were almost the same.

$$\begin{aligned} &\min \omega_1^2 \\ &\sum_{i=1}^{\mathrm{N}} x_i \overline{R}_i \geq r \\ &\sum_{i=1}^{\mathrm{N}} x_i = 1 \\ &x_i \geq 0, \quad i = 1,\dots,\mathrm{N} \end{aligned} \tag{13}$$

Our research is to check the properties of the optimal structures of portfolio weights for a set of q models defined in the following way:

$$\begin{aligned} &\min \sum_{j=1}^{q} \lambda_j \omega_j^2 \\ &\sum_{i=1}^{\mathrm{N}} x_i \overline{R}_i \geq r \\ &\sum_{i=1}^{\mathrm{N}} x_i = 1 \\ &x_i \geq 0, \quad i = 1,\dots,\mathrm{N} \end{aligned} \tag{14}$$

where $q \in [1, \mathrm{N}]$ is an integer, which value will be suggested by PCA – number of principal components significant to explain total variance of the set of rates of return.

III. MULTIVARIATE STATISTICAL ANALYSIS OF RATES OF RETURN – EMPIRICAL STUDY FOR THE STOCK MARKET OF MEDIUM-SIZE FIRMS

We analyzed weekly rates of return on stock in the period 2009-2011 years for 37 firms classified on Stock Exchange in Warsaw as medium-sized firms. The behavior of the firms stock prices is synthetically represented by the quotations of stock index mWIG40. Figure 1 illustrates the data series for mWIG40 index during the sample period – general rising tendency up to the half of the year 2011 when a big fall in prices was observed, then we observe stabilization of stock prices in this market segment the mean level lower than observed in the second half of the year 2009.

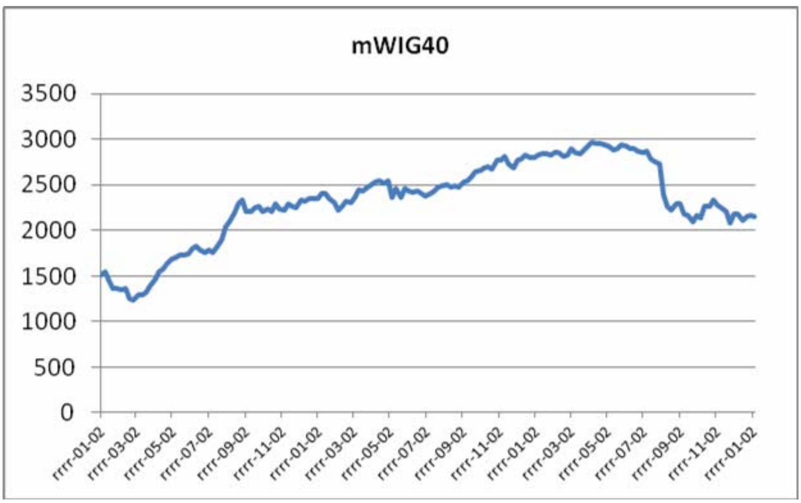

Fig. 1. Index MWIG40 – weekly data for the period 2009-2011

Figure 2 presents the relative profitability as a ratio of mean rate of return to risk, measured by standard deviation of rates of return, observed in sample period for individual securities as well as for the index mWIG40.

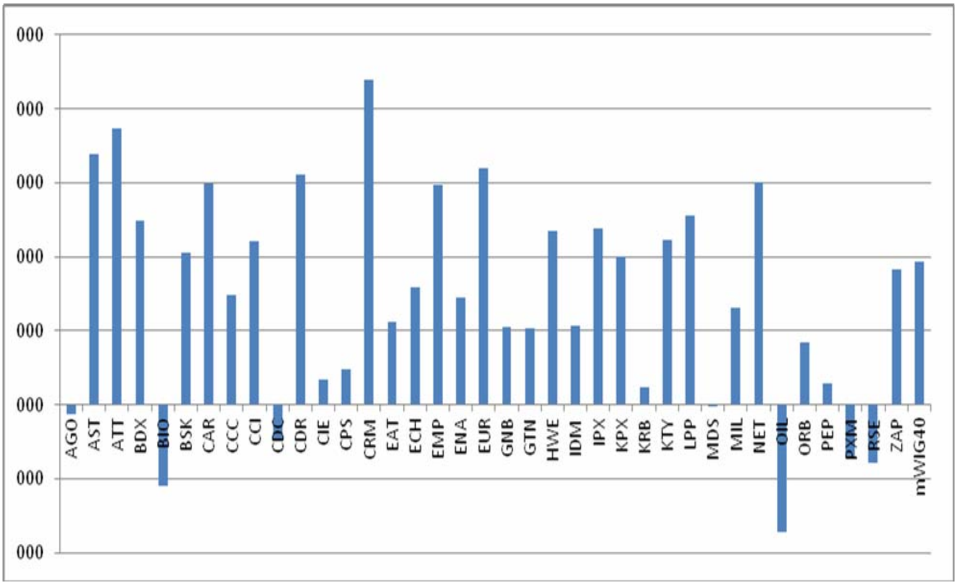

Fig. 2. Mean profit on medium-sized stock in the units of risk – weekly data 2009-2011

We analyzed the properties of sample covariance and correlation matrices for the rates of return in three chosen periods: whole sample 2009-2011, 2-year period 2010-2011 and for the year 2011. Some results are included in tables Table 1 to 3 and on graphs Figure 3 to Figure 9. Condition numbers in Table 1 were calculated using Excel[5]. It can be observed that condition number depends on the length of the sample period – less data available results in worse conditioning of the matrices. Condition numbers for the matrices of sample covariance were greater than for corresponding correlation matrices.

Table 1. Conditioning of the empirical data in sample periods

Condition number for the correlation matrix	Sample period	Condition number for the covariance matrix
59,91	2009-2011 *157 observations*	90,72
92,54	2010-2011 *106 observations*	183,79
1415,40	2011 *53 observations*	1980,00

[5] For eigenvalues and eigenvectors calculation we used Excel Add-In: Matrix and Linear Algebra for Excel v.2.3.2 – SVD (Singular Value Decomposition) procedure, http://digilander.libero.it/foxes, author: Leonardo Volpi.

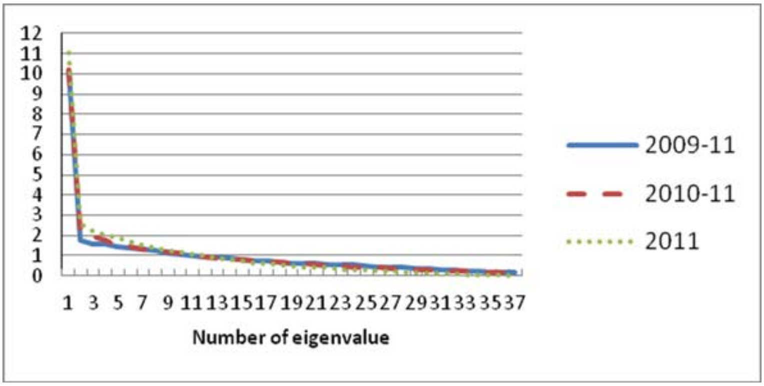

Fig. 3. Factorial scree for correlation matrices for the three sample periods

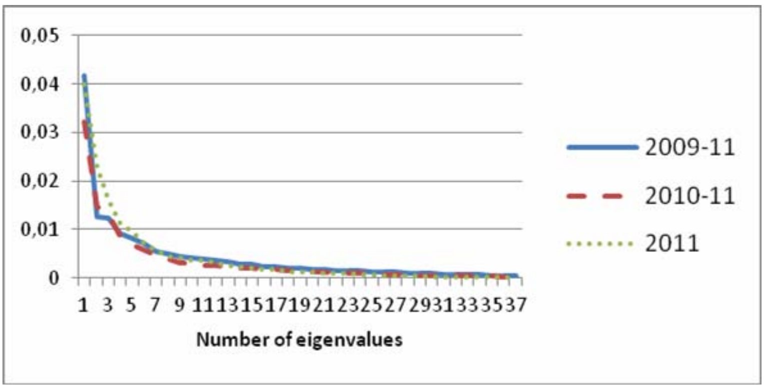

Fig. 4. Factorial scree for covariance matrices for the three sample periods

Figures 3 and 4 help us to make decision on how many principal components are important to describe the total data variability. The common criterion is to neglect eigenvalues lower than one – in case of correlation matrix analysis (called Kaiser criterion). In our analysis the number of eigenvalues greater than one for correlation matrices is 11 in all sample cases. Looking at Figure 4 prepared for covariance matrices, we also notice that 11 can also be the rational cut-off number. Another criterion to be used is to explain at least 75% of total variability of the set of rates of return. Graphs in Figures 5 and 6 present cumulative percentage of total variance explained for each case considered.

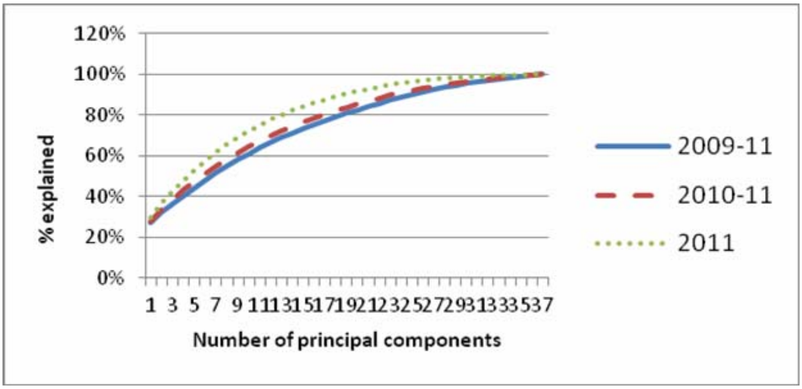

Fig. 5. Cumulative percentage of total variance explained – for correlation matrices

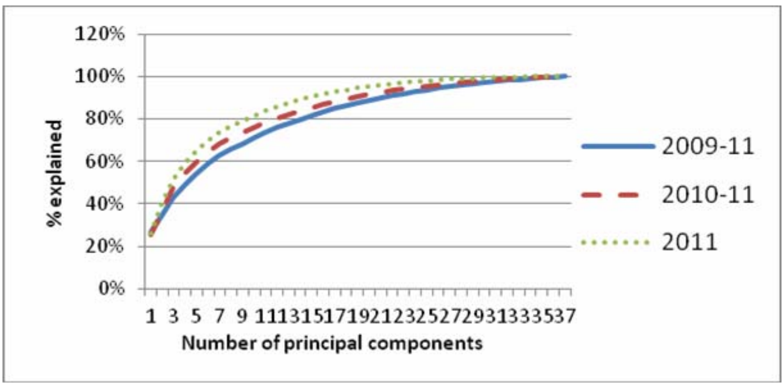

Fig. 6. Cumulative percentage of total variance explained – for covariance matrices

Comparing the concavity of lines in Figures 5 and 6 we conclude that:

– with greater condition number we observe stronger concavity – it means that we need less principal components to explain the same amount of total variability of the rates of return;

– lines in Figure 5 showing the relationship between number of principal components included and percentage of total variance explained based on covariance matrices are more concave than the corresponding lines in Figure 6 prepared using principal component analysis for correlation matrices – it means that applying the criterion to explain at least 75% of total variance for covariance matrix results in a lower number of principal components needed than in the case of correlation matrix.

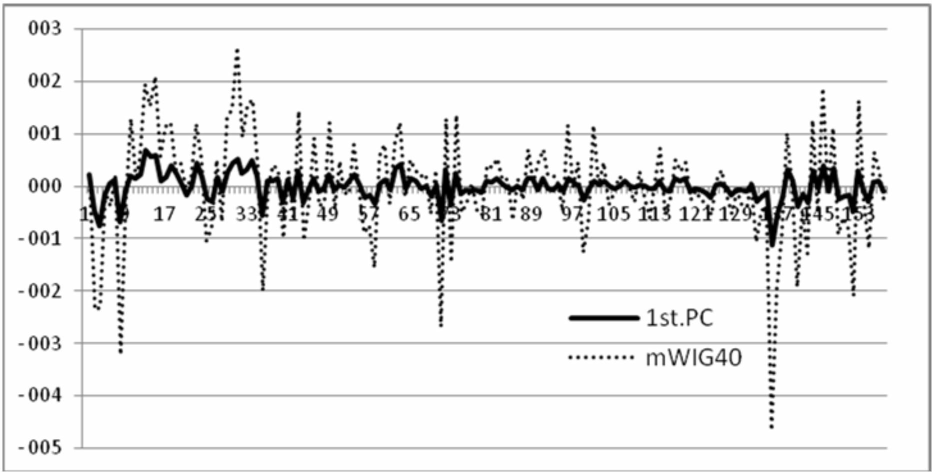

Fig. 7. First principal component and standardized rates of return on mWIG40

Graph in Figure 7 shows first principal component calculated according to formula (12) and rates of return on mWIG40 in the analyzed sample period 2009-2011. We can observe almost perfect consistency of peaks and dales of the two lines.

The scatter plot in Figure 8 presents the securities on the map constructed by 2 eigenvectors corresponding to the two greatest eigenvalues of the correlation matrix.

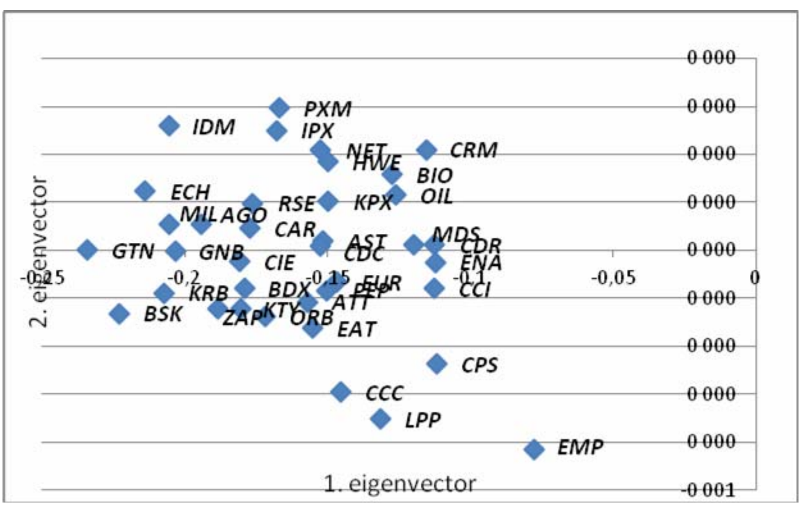

Fig. 8. Eigenvectors corresponding to 2 greatest eigenvalues of correlation matrix for the sample period 2009-2011

Such maps can be used in classification of securities, taking into account their impact on total variability on the market. Securities with the smallest impact, taking into account 1 eigenvector, are as follows: EMP, CPS, ENA, CDR, CCI[6]. We expect them to be present as components of optimal portfolios.

IV. OPTIMAL PORTFOLIOS BASED ON PCA – EMPIRICAL RESULTS

The aim of our research was to check how optimal portfolio structures react to reduced dimension of data set, while strong relationships among rates of return are observed. The dimension suggested was 11 principal components out of 37 series of rates of return. We calculated 12 portfolios solving the problems defined in (9) and (14) after PCA based on correlation matrix (PCA-corr) and 12 corresponding portfolios after PCA based on covariance matrix (PCA-cov).

Table 2. Optimal portfolios structures – based on PCA for correlation matrix

	Model											
Firm	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC37
BDX	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**5%**
CCC	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**8%**
CCI	0%	0%	**29%**	**26%**	0%	0%	0%	0%	0%	0%	0%	**9%**
CDR	0%	0%	0%	0%	**12%**	0%	0%	0%	0%	0%	0%	**1%**
CPS	0%	0%	0%	**27%**	**36%**	0%	0%	0%	0%	0%	0%	**11%**
EAT	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	**2%**
EMP	**100%**	**7%**	**3%**	0%	0%	0%	0%	0%	**4%**	**5%**	**10%**	**10%**
ENA	0%	**93%**	**68%**	**34%**	**52%**	**20%**	**24%**	**23%**	**19%**	**15%**	**11%**	**12%**
EUR	0%	0%	0%	0%	0%	**7%**	**3%**	**2%**	**3%**	**11%**	**11%**	**4%**
IPX	0%	0%	0%	0%	0%	0%	0%	**2%**	**2%**	**1%**	**1%**	**1%**
KPX	0%	0%	0%	0%	0%	0%	**4%**	**1%**	0%	0%	0%	**2%**
LPP	0%	0%	0%	0%	0%	**45%**	**48%**	**51%**	**46%**	**39%**	**36%**	**14%**
NET	0%	0%	0%	**13%**	0%	**21%**	**18%**	**19%**	**21%**	**23%**	**26%**	**11%**
PEP	0%	0%	0%	0%	0%	**7%**	**3%**	**2%**	**5%**	**7%**	**5%**	**10%**

Source: own calculations.

[6] All elements of the first eigenvector are negative. For the indicated securities we have noticed the smallest absolute value of the corresponding coordinates.

Let us summarize the results on components of the optimal portfolios. Optimal portfolio structure minimizing portfolio variance took into account shares of 15 firms. Optimal weights for shares if we consider only first principal component, resulted in non-diversified portfolio – only EMP shares. Portfolios obtained with the help of PCA based on covariance matrix, presented in Table 3, chose less types of shares than those based on correlation matrix: only EMP, CPS, CRM and CCI. Portfolios in Table 2 based on correlation matrix are very different although base on the same sample data.

In the applied models we omitted constraint about required minimal rate of return. We can compare portfolios looking at estimated risk – Figure 9. The conclusion is that models based on PCA of correlation matrix produced, in general, lower risk.

Table 3. Optimal portfolios structures – based on PCA for covariance matrix

	Model										
Firm	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11
CCI	0%	0%	0%	0%	0%	0%	**22%**	**32%**	**50%**	**49%**	**52%**
CPS	0%	0%	0%	0%	0%	0%	0%	0%	0%	**10%**	**5%**
CRM	0%	0%	0%	0%	**5%**	**7%**	**5%**	**5%**	**3%**	**2%**	**2%**
EMP	**100%**	**100%**	**100%**	**100%**	**95%**	**93%**	**73%**	**63%**	**47%**	**39%**	**41%**

Source: own calculations.

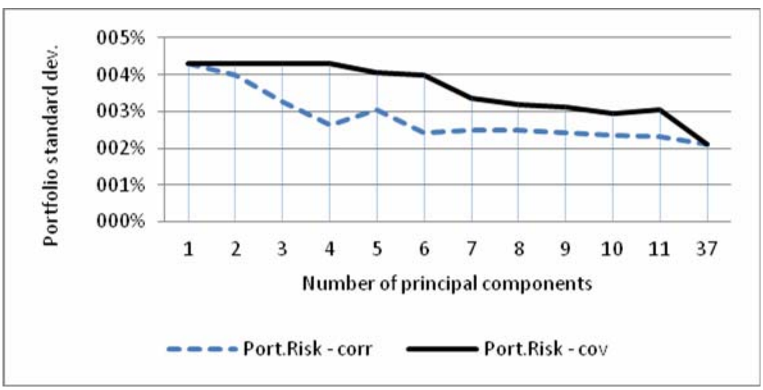

Fig. 9. Risk of the optimal portfolios measured by standard deviation

All vectors of optimal weights for portfolio models were almost orthogonal to eigenvectors corresponding to the first eigenvalue of correlation/covariance matrix – in case of models based on correlation matrix the angular distance was in the range [0,9082 - 0,9970], in case of covariance matrix – in the range [0,9698 - 0,9993].

V. FINAL CONCLUSIONS

The series of rates of return on stock are found to be highly multicollinear. We tried to investigate in what way and how strongly this fact influences the optimal investment portfolio structures. Principal component analysis was helpful to verify our hypothesis that in case of strong linear interdependence among rates of return we can reduce the dimension of data set without a big loss in the sense of risk extension. Our research was conducted on the base of correlation and covariance matrices. The results showed that sample correlation matrix compared with sample covariance matrix, both estimated on the same sample data, lose some amount of interdependence measured by condition number. It is mainly the consequence of data standardization. We have found that in the case of covariance matrix PCA suggests less sufficient number of principal components needed to explain total variance. Solving a set of optimization models, each of them including more principal components, we observed that more principal components forced, in general, more diversified solutions. This effect was much weaker for portfolios based on covariance matrix than for the ones obtained basing on correlation matrix. Portfolio risk measured by standard deviation was lower in cases utilizing PCA of correlation matrix than covariance for the same number of principal components.

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**OPTYMALNY PORTFEL AKCJI –
ZASTOSOWANIE WIELOWYMIAROWEJ ANALIZY STATYSTYCZNEJ**

W artykule przedstawiamy wybrane wyniki teoretyczne na temat konstrukcji optymalnego portfela akcji z wykorzystaniem informacji dostepnej w wyniku przeprowadzenia analizy głównych składowych macierzy kowariancji czy też macierzy korelacji stóp zwrotu z akcji. Wyniki teoretyczne prowadzą do konstrukcji modeli optymalizacyjnych uwzględniajacych redukcję przestrzeni danych do określonej liczby głównych składowych, co udaje się skutecznie przeprowadzić w warunkach silnych związków o charakterze liniowym między szeregami stóp zwrotu z akcji. W pracy prezentujemy wyniki analiz dla modeli Markowitza oraz modeli opartych o analizę głównych składowych na przykładzie sektora średnich spółek na GPW w Warszawie w latach 2009-2011. Badanie empiryczne pokazuje różnice wyników optymalizacji oraz ryzyka portfeli w przypadkach, kiedy korzystamy z macierzy kowariancji albo korelacji.