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METHODS OF TWO DIMENSIONAL IMAGES RESTORATION

ABSTRACT. In the paper the problems of segmentation and restoration of two dimensional images on the basis of possessed distorted version of images are considered. Bayesian methods of image analysis, ICM Besag algorithm, mathematical morphology methods and Bayesian morphology methods are discussed. All methods are assessed from the point of view of three criteria: quality of the image restored, the speed of algorithms used and the quality of mathematical and statistical foundations. A new algorithm is also proposed and the results of applying all the methods discussed to some images are presented. The algorithm may be assessed as competitive especially as the speed and the quality of the image restored is concerned.

Key words: image analysis, Bayesian morphology, computer algorithms.

I. BAYESIAN MORPHOLOGY

Let S be the set of all pixels which constitute the true but unknown image and the very image itself will be written as $x=\{x_i, i \in S\}$ and it will be treated as a realization of a random vector X. The observed image y is interpreted as a realization of a random vector Y which is a degraded or contaminated version of X. The vector Y depends on X through a known conditional probability density function $L(y/x)$ which incorporates both the image model and the noise model. We are looking for an estimator $\hat{X} = \hat{X}(Y)$ of X that will allow us to restore true image X. Let us assume that $P(x)$ is the distribution of X. Then the restored image $\hat{x}$ is based on the posterior density of x, i.e. $P(x/y)$ which is proportional to $L(y/x)\ P(x)$. If we maximize this density we will arrive at the maximum a posteriori (MAP) estimate of x. In order to simplify maximization we have to make a few assumptions. Firstly, we assume that x is the realization

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of a Markov random field. From this it follows that for all pixels i $P(x_i|x_{S\setminus\{i\}}) = P(x_i|x_{N(i)})$ i.e. the conditional distribution depends only on the pixels in a subset $N(i)$ called the neighbourhood of pixel i. Secondly, we assume that Y_i are conditionally independent and have the same conditional density function $f(y_i/x_i)$ that depends only on x_i. Then we can write $L(y/x) = \prod_{i \in S} f(y_i|x_i)$.

To simplify heavy computations needed to find MAP we can apply the Iterated Conditional Modes algorithm of B e s a g (1986). It is an iterative algorithm. Given a current estimate $\hat{x}$ of the image we compute a new one. We visit each pixel i and the current value of that pixel is replaced by the value that maximizes the conditional density $P(x_i|\hat{x}_{S\setminus\{i\}}, y)$ This choice is motivated by the following equality

$$P(x|y) = P(x_i|x_{S\setminus\{i\}}, y)P(x_{S\setminus\{i\}}|y) \tag{1}$$

Under the assumptions that were made, maximizing the conditional density is equivalent to maximizing $f(y_i|x_i)P(x_i|\hat{x}_{N(i)})$.

If we consider binary (two coloured) images then for all pixels i, $x_i \in \{0, 1\}$ and $y_i \in \{0,1\}$. The prior distribution for the true image X is the usually used Ising model

$$P(x) = Z(\beta)^{-1} \exp(\beta\, v(x)), \tag{2}$$

where $v(x) = \sum_{i \approx j} \delta(x_i, x_j)$ is the number of pairs of neighbouring pixels having the same colour. Notation $i \approx j$ means that the pixels i and j are neighbours and δ is the Kronecker delta function. The quantity $Z(\beta)$ is the normalizing constant $Z(\beta) = \sum_x \exp(\beta\, v(x))$ and β is a parameter that will be estimated later. The conditional distributions of $P(x)$ i.e. $P(x_i|x_{S\setminus\{i\}})$ have the form which is proportional to $\exp(\beta\ u_i(x_i))$, where $u_i(x_i) = \sum_{j \in N(i)} \delta(x_i, x_j)$ is the number of neighbours of pixel i having colour x_i. The true images are assumed to be degraded by the so called channel noise characterized by the two parameters p_{01} and p_{10} understood in the following way

$$p_{01} = P(y_i = 1/x_i = 0) \qquad p_{10} = P(y_i = 0/x_i = 1).$$

Thus, the expression $f(y_i / x_i)$ can be written as

$$f(y_i|1) = (1 - p_{10})^{y_i}(p_{10})^{1-y_i} \qquad f(y_i|0) = (1 - p_{01})^{1-y_i}(p_{01})^{y_i}$$

For an Ising model we can change the colour of each pixel i according to the formulae given by Forbes and Raftery (1997). The theorem is the following .

For an Ising model with the channel noise given by parameters p_{10} and p_{01}, the current ICM estimate of the true image at pixel i is updated by changing $\bar{x}_i$ to x_i^* according to the rule

$$x_i^* = \begin{cases} 1 & \text{if} \quad u_i(1) - u_i(0) \geq 2w_i \\ 0 & \text{if} \quad u_i(1) - u_i(0) \geq -2w_0 \\ y_i & \text{if} \quad u_i(1) - u_i(0) \geq 2w_i \end{cases}$$

where w_0 and w_1 are positive integers that depend on the noise and model parameters p_{10} , p_{01} and β through

$$w_0 = \left[\frac{1}{2\beta} \log\left(\frac{1 - p_{10}}{p_{01}} \right) \right]$$

$$w_1 = \left[\frac{1}{2\beta} \log\left(\frac{1 - p_{01}}{p_{10}} \right) \right]$$

where parameter β has to be estimated.

II. PARAMETER ESTIMATION

To estimate parameter β for the model (2) we may try different approaches. The maximum likelihood estimator can be found by maximizing the following log-likelihood

$$\log(P(x)) = \beta\, v(x) - \log(Z(\beta))$$

The derivative of this function is given by the formula

$$\frac{d\log(P(x))}{d\beta} = \nu(x) - E_\beta(\nu(X))$$

where $E_\beta(\nu(X)) = \sum_x \nu(x)P(x)$. From that it follows that $E_\beta(\nu(X)) = \nu(x)$.

These basic properties allow us to investigate the behaviour of the expectation considered more closely and to establish an algorithm making use of the formulae given by Forbes and Raftery (1997) to estimate β. However, the exact computation of the appearing expectations is impossible because of the amount of computations needed and one would have to use some other algorithms which accelerate this process e.g. the Swendsen-Wang algorithm (1987).

Another approach is the pseudo-likelihood estimation. In this method we maximize the expression

$$\prod_{i\in S} P(x_i | x_{S\setminus\{i\}}) \tag{3}$$

For the model (2) we have $p_i(x_i) = Z_i(\beta)^{-1}\exp(\beta\, u_i(x_i))$ where $p_i(x_i) = P(x_i | x_{S\setminus\{i\}})$ and $Z_i(\beta) = \sum_{c\in\{1,\dots,C\}} \exp(\beta\, u_i(c))$. Maximizing expression (3) is equivalent to maximizing the log-pseudo-likelihood $F(\beta) = \sum_{i\in S}(\beta\, u_i(x_i) - \log(Z_i(\beta))$.

We can calculate derivatives and present them in a form similar to that of the maximum likelihood case

$$\frac{dF(\beta)}{d\beta} = \sum_{i\in S}\left(u_i(x_i) - E_{p_i}(u_i(X_i))\right)$$

where

$$E_{p_i}(u_i(X_i)) = \sum_{c\in\{1,\dots,C\}} u_i(c)\,p_i(c).$$

Similarly as in the case of the maximum likelihood estimation we find the estimate of β, the difference and advantage of this method being that all the expectations appearing can be computed exactly because they are restricted to some pixel's neighbourhood.

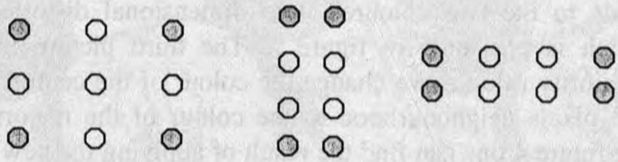

Fig. 1. Three kinds of pixel neighbourhoods considered in the new algorithm

III. NEW ALGORITHM PROPOSAL

Quite different method of restoring distorted two dimensional images is looking for algorithms which have more in common with the methods termed mathematical image morphology. The idea of such algorithms is to consider small neighbourhoods of every pixel and to look for patterns of colours distribution in the neighbourhoods that would justify changing some pixels colours. For example we can change the colour of the central pixel if all other pixels have the other of the two colours. We were trying many combinations of neighbourhoods shapes and pixels-to-be-changed. Interesting results can be achieved if we try three kinds of neighbourhoods depicted in figure 1. The rule to change colours is the following : we change only one pixel which is marked as white (does not lie in the corner) if all other pixels have the other of the two colours. We ran each of these neighbourhoods three times to the Mickey mouse picture.

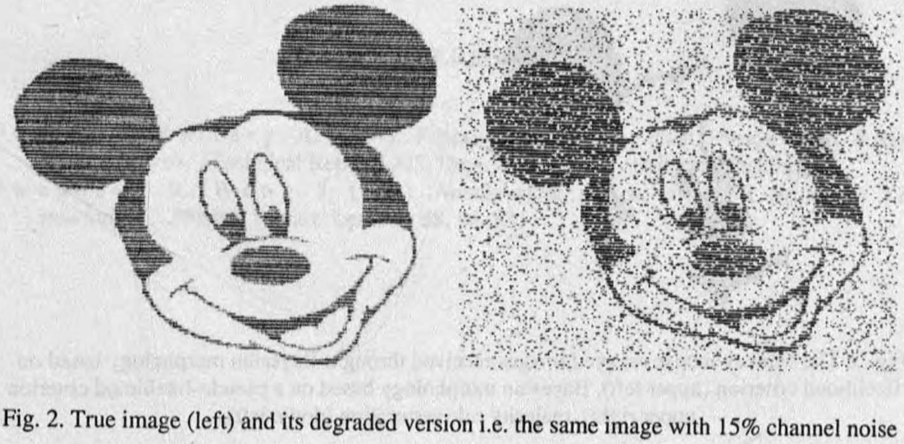

Fig. 2. True image (left) and its degraded version i.e. the same image with 15% channel noise

IV. COMPARISON OF PERFORMANCE

In figure 3 one can see the results of applying bayesian morphology restoration methods to the two coloured, two dimensional distorted Mickey mouse image which is presented in figure 2. The third picture in figure 3 presents simple majority rule i.e. we change the colour of the central pixel of a symmetric 3 by 3 pixels neighbourhood to the colour of the majority of this neighbourhood. In figure 4 one can find the result of applying the new algorithm to the similar image. Intuitive visual examination, as well as comparison through the criterion of the percentage of the number of pixels incorrectly restored, allows to state that the new algorithm is competitive. One has to remember however that the speed of work of the new algorithm is probably much faster (more precise comparison would require the same conditions for all methods).

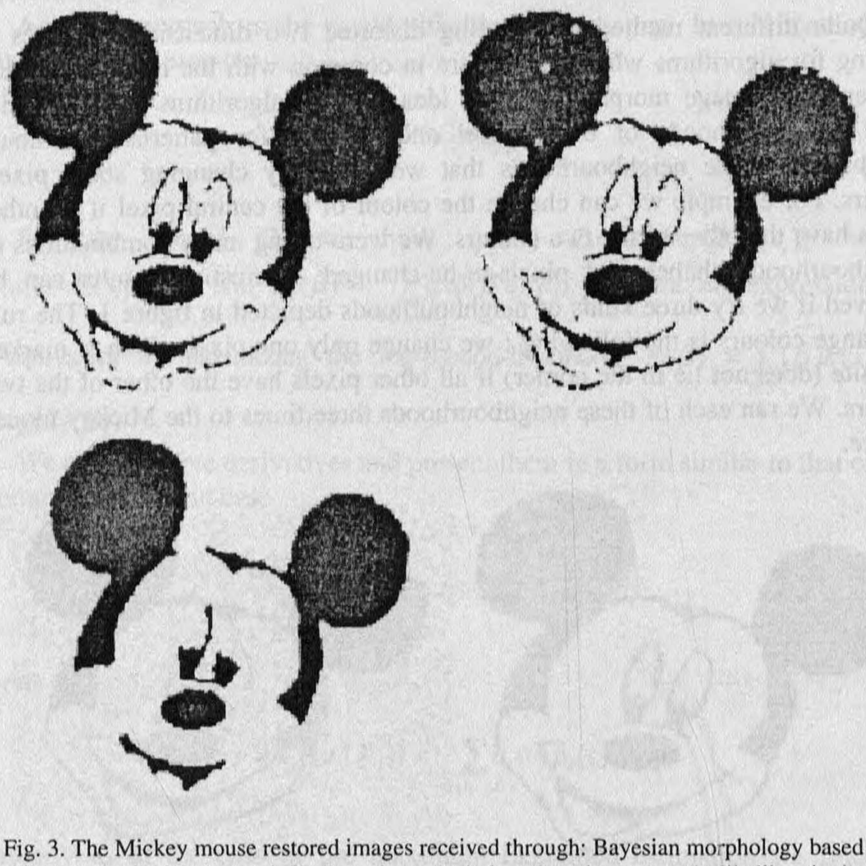

Fig. 3. The Mickey mouse restored images received through: Bayesian morphology based on a likelihood criterion (upper left), Bayesian morphology based on a pseudo-likelihood criterion (upper right), majority rule restoration (down left).

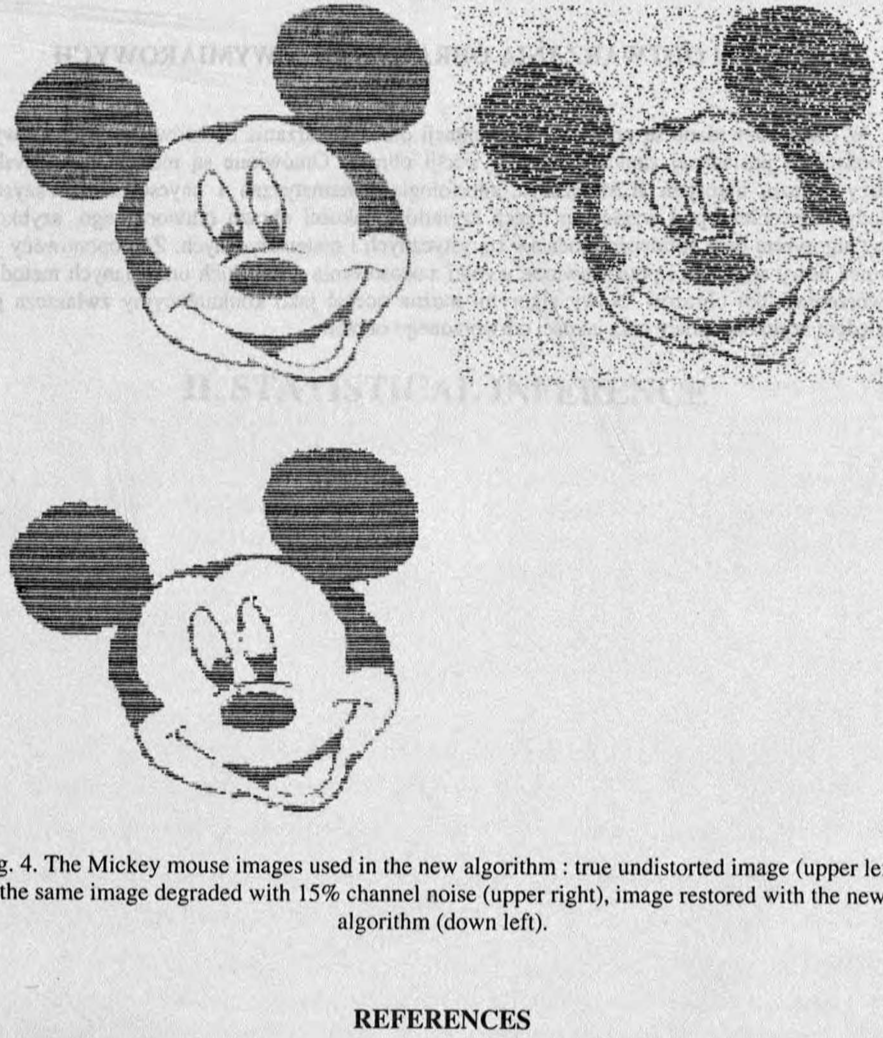

Fig. 4. The Mickey mouse images used in the new algorithm : true undistorted image (upper left), the same image degraded with 15% channel noise (upper right), image restored with the new algorithm (down left).

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METODY ODTWARZANIA OBRAZÓW DWUWYMIAROWYCH

W pracy rozważane są problemy segmentacji oraz odtwarzania obrazów dwuwymiarowych na podstawie posiadanej zanieczyszczonej wersji obrazu. Omówione są metody bayesowskiej analizy obrazu, algorytm ICM Besaga, morfologia matematyczna i bayesowska. Wszystkie metody są oceniane pod względem trzech kryteriów: jakości obrazu odtworzonego, szybkości pracy algorytmu oraz solidności podstaw statystycznych i matematycznych. Zaproponowany jest również nowy algorytm i przedstawione wyniki zastosowania wszystkich omawianych metod do odtworzenia kilku obrazów. Nowy algorytm można ocenić jako konkurencyjny zwłaszcza pod względem szybkości pracy oraz jakości odtworzonego obrazu.