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PREFACE: NON-CLASSICAL LOGICS. THEORY AND APPLICATIONS (PART II)

The articles in the present and forthcoming issues are revised and extended versions of papers presented at the conference *Non-Classical Logics. Theory and Applications*, held in Łódź on 4–8 September 2024.[1]

Non-Classical Logics. Theory and Applications (NCL) is an international conference series devoted to novel results and survey work in broadly understood non-classical logics and their applications. The first two editions took place in Łódź, Poland, in 2008 and 2009. Subsequently, the conference was held alternately in Toruń (2010, 2012, 2015, 2018) and Łódź (2011, 2013, 2016, 2022). The tenth edition, organised by the University of Lodz in 2022, was the first to publish its proceedings in *Electronic Proceedings in Theoretical Computer Science*. This practice was continued in the most recent, eleventh edition, with all accepted long papers again appearing in an EPTCS volume. The 2024 edition was supported by the European Research Council as part of the project *Coming to Terms: Proof Theory for Definite Descriptions and Other Terms* (ExtenDD), and featured four

[1]Due to the high number of accepted post-conference submissions, the editors decided to divide them into two sets that have been published in two separate issues.

invited talks, eighteen contributed talks, and eighteen short presentations accepted through a light reviewing process.

Given that non-classical logics form a broad and diverse area of research within logic, the contributions collected in this issue address a wide range of topics. They include, among others, a non-standard definition of identity, semi-substructural logic, Lambek calculus, or multi-dimensional modal logics.

The paper "Qualified Definiteness" by Bartosz Więckowski revisits Russell's analysis of definite descriptions and targets a well-known tension: natural language permits "loose" uses of "the F" even when strict uniqueness fails. Rather than abandoning Russell's framework or shifting to a purely model-theoretic treatment of incomplete descriptions, the author refines the uniqueness clause itself. The key move is to replace primitive identity in Russell's analysis with a defined notion of qualified identity—identity relative to a specified set of predicates Q. This yields a graded notion of definiteness: maximal (strict) definiteness when Q includes all predicates, and restricted (loose) definiteness when Q is a proper subset. The resulting system formally distinguishes between genuine uniqueness and indiscernibility relative to selected respects.

The main contribution is a proof-theoretic treatment of qualified definiteness within an intuitionistically acceptable framework. Building on a bipredicational language with a primitive notion of predication failure, the paper develops natural deduction systems, establishes normalisation and subformula (more generally, subexpression) properties, and provides a proof-theoretic semantics via canonical derivations. The technical novelty lies in internalising identity conditions through introduction/elimination rules that mirror the definitional structure of qualified identity, avoiding second-order quantification while preserving fine-grained control over discernibility. The framework is then applied systematically to complete, incomplete, generic, nested, predicative, and possessive definite descriptions.

Cheng-Syuan Wan's article "Semi-Substructural Logics à la Lambek with Symmetry" addresses a structural mismatch in the proof theory of skew monoidal closed categories. Earlier sequent calculi (with stoups, à la Girard) successfully captured left skew monoidal closed categories, but

did not extend smoothly to right skew or bi-closed variants, especially in the presence of symmetry. Wan identifies the source of the difficulty in the implicit left-associative structure of antecedents and proposes more flexible syntactic frameworks: a tree-based sequent calculus (in the style of non-associative Lambek calculus) and an equivalent axiomatic calculus with single-formula antecedents.

The principal contributions are twofold. First, the paper proves equivalence between the stoup-based and tree-based systems and establishes cut elimination. Second, it provides soundness and completeness results for the axiomatic calculi with respect to ternary relational semantics, and proves a correspondence theorem linking frame conditions to structural laws in the categorical setting. The technically interesting move is the shift from flat antecedents to tree-structured ones, which makes the skew (non-symmetric, non-associative) behaviour explicit and enables a uniform treatment of left, right, and bi-closed skew structures—including symmetric extensions. This yields an algebraically transparent account of how categorical coherence corresponds to relational constraints.

The article "Complexity of Nonassociative Lambek Calculus with Classical and Intuitionistic Logic" by Paweł Płaczek investigates the computational complexity of consequence relations for extensions of the non-associative Lambek calculus (NL). While pure NL has a polynomial-time decidable finitary consequence relation, adding unrestricted additive connectives (as in full NL) leads to undecidability. Interestingly, the distributive version restores decidability at exponential time. The author examines what happens when NL is combined not merely with distributive lattice structure, but with full Boolean (BFNL) or Heyting (HFNL) algebraic structure.

The main result is that both BFNL and HFNL retain decidability with an ExpTime upper bound (in the unital case), despite incorporating classical or intuitionistic logic into a non-associative, non-commutative setting. The proof strategy is algebraic: the author develops machinery for partial residuated Boolean and Heyting algebras, analyses embeddability conditions, and uses filter-extension techniques (adapted to partial structures) to control model construction. A key technical insight is that, in the Boolean

case, working with the full power set rather than families of upsets simplifies the treatment of negation while preserving the necessary structural properties. The complexity bounds are thus obtained by careful algebraic model analysis rather than purely syntactic proof-theoretic methods.

Finally, Yuki Nishimura's paper "Agent-Knowledge Logic for Alternative Epistemic Logic" introduces *agent-knowledge logic*, a two-dimensional modal-hybrid system designed as an alternative to standard epistemic logic. Inspired by Facebook logic and the Logic of Hide and Seek Game, the framework separates the dimensions of agents and epistemic alternatives. It incorporates two modal operators (over agents and knowledge), two kinds of propositional variables (agent-dependent and agent-independent), and two kinds of nominals, enabling explicit reference to particular agents and epistemic states.

The first main contribution is a formal embedding of standard epistemic logic into agent-knowledge logic, demonstrating that the new system properly generalizes the classical one. The second is the construction of a tableau calculus with a termination property, establishing decidability via finite proof search. Conceptually, the innovative aspect lies in decoupling agent-dependent and agent-independent propositions while preserving hybrid reference mechanisms. This yields expressive power beyond standard epistemic logic (e.g., formalising "one of an agent's friends knows p") while maintaining a well-behaved proof-theoretic framework.

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