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The Concept of Structural Equation Modelling for Measuring the Shadow Economy – International and Polish Perspectives

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Abstract

The goal of the article is to explore the potential of explicatory Structural Equation Modelling (SEM) and its specifications for measuring the shadow economy (SE). This is done from the perspective of various approaches in selected countries.
The article is a review and conceptual paper. The study is divided into three stages: a comprehensive description of the nature of the SE and the difficulties associated with measuring

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it; a comparative analysis of the approaches applied in selected EU countries (with particular emphasis on Italy and Poland), and finally, the concept of SE estimation based on SEM model is proposed.
One of the most important limitations regarding the SE is that it is not possible to measure the extent of this phenomenon directly. This leads to the use of non-standard estimation techniques based on latent variable models. The innovation of this approach is that it considers three factors that are not directly observable, i.e., tax morality, concealing salaries, and regulation of the economy.
The proposed model allows us to capture and explain empirical SE phenomena more precisely and effectively than with classical statistical and econometric methods. However, we are aware that it is highly probable that many SEMs will need to be tested and modified to achieve the final result.

Keywords: shadow economy, national statistical offices, structural equation models, unobserved variables, international economy

JEL: A10, B41, C01, C18, C51

Introduction. The shadow economy as a research challenge

The shadow economy (SE) is an important and contemporary research problem, both for individual experts involved in the analysis, assessment, and measurement of the economy, as well as for state authorities whose task is to counteract this phenomenon. Although the SE has always existed in all economies, regardless of their level of development or geographical context, it is still an area where methodological disagreement persists in terms of measurement. The basis for this dispute can be seen from the very beginning of the research process, i.e., at the stage of agreeing on a definition.

In the literature, the SE is also referred to as grey, informal, hidden, secondary, mysterious, concealed, parallel, black, dual, or under-the-table (cf. Orsi, Davide, and Turino 2014; Buehn and Schneider 2016; Medina and Schneider 2018; Piecuch and Szczygieł 2018). Attempts to define this phenomenon are made not only by economists, lawyers, and sociologists but also by psychologists, philosophers, those who work in management sciences, and ethics specialists.

The SE is an ambiguous and widespread phenomenon, and it is a major challenge for researchers around the world, among whom there is a consensus on the need to measure it. When analysing the methods of estimating the size of the SE, a literature review indicates a multitude of methods (e.g. Buehn and Schneider 2016; Dymarski 2016). In principle, two groups can be distinguished: direct and indirect. In the first group, Mróz (2002) lists household surveys, tax surveys, labour market surveys, and direct surveys of partial markets. In contrast, intermediate methods, which use the traces that the SE leaves in different sizes and economic indicators, include the analysis of discrepancies between expenditure and income statistics, microeconomic analyses of the labour market, monetary methods, econometric methods, methods based on the consumption balances of certain materials and raw materials, multivariate methods, indirect analytical partial methods, and Delphic methods (Mróz 2002). Each of them has its advantages and disadvantages.

An unquestionable advantage of direct methods is that information can be obtained directly from business owners using, for example, direct interviews, which ensures the desired level of information detail. However, this type of research raises the question of data quality. This is because there is a subjective nature to the information provided by those involved in the SE. In addition, direct methods, however detailed, are most often fragmented, and thus relate to selected areas of the economy, without meeting the condition of representativeness for the whole economy. Mróz (2002) points this out by indicating the problematic and dangerous moment of transition from estimating specific spheres of the SE to assessing its global size.

On the other hand, indirect methods, especially econometric methods, enable the construction of multivariate models that can describe the entire economy. They are easier to use because there is no need to perform primary research. They are also more transparent and make it easier to follow the dynamics of the phenomenon over longer time horizons. Mróz (2002) lists formal elegance among the advantages of econometric methods. At the same time, he draws attention to the risks arising from subjectivity and even arbitrariness in the selection of indicators and assumptions. In the discussion on the advantages and disadvantages of different methods of measuring SE, researchers agree in principle that when studying a phenomenon that is not subject to traditional statistical observation, several complementary research methods should be used simultaneously (Szewczyk-Jarocka 2011).

The article presents the idea of a methodology developed in the area of econometric methods based on the technique of structural equation models with latent variables. The aim is to present the unused potential of an explicative *Structural Equation Modelling* (SEM) and its specifications in the context of measuring the SE from international and Polish perspectives. This article serves as a conceptual review and contribution to the international discussion on improving SE estimations.

The shadow economy as a research problem for national statistical offices – an international overview

The size of the SE in public European statistics is estimated in the system of national accounts, which, within the framework of the European System of National Accounts[1] in force in EU countries, should aim at the completeness of GDP and GNI estimates. This means that, in addition to directly observed production (through statistical surveys or administrative data), they should include the SE as one of the elements of the non-observed economy (NOE). To unify the practices of individual countries, which would improve the completeness of national accounts, Eurostat has defined a universal seven-part scheme categorising data incompleteness (Table 1). The scheme is based on the division of producers (data providers) into four types, depending on the accuracy deficiencies that individual groups of producers may generate. Each type of producer is assigned different types of incompleteness (N1–N7).

Table 1. Types of incomplete data in the European System of National Accounts

Type of producers	Symbol	Name	Characteristic description
Unregistered	N1	Producers deliberately not registering – underground	Entrepreneurs want to avoid taxes and social security contributions. Type N1 does not cover all hidden activities, some of which are type N6.
	N2	Producers deliberately not registering – illegal	Entrepreneurs involved in illegal production. Type N2 does not include illegal activities conducted by legal entities or business owners who are engaged entirely or partially in illegal activities within a legally registered company.
	N3	Producers are not required to register	Business owners who do not have to register because they do not produce for the market. Usually, they are households producing for their private use, which undertake the production of goods for their consumption or their own fixed assets, construction, and renovations of dwellings. This group also includes business owners with a market production so small that there is no obligation to register it.

1 The European System of National and Regional Accounts in the European Union, implemented in the Polish system of official statistics in accordance with Regulation (EU) No. 549/2013 of the European Parliament and of the Council of May 21, 2013.

Type of producers	Symbol	Name	Characteristic description
Not surveyed	N4	Legal persons not surveyed	Legal persons not surveyed for various reasons, e.g. outdated registers, incorrect data classifying the type of activity, size of the company, etc. This leads to a systematic error in the exclusion of legal persons from the research in which they should be involved.
	N5	Registered business owners not surveyed	Registered enterprises not surveyed for various reasons, e.g. the statistics do not survey such enterprises, the incompleteness of reports for research, or incorrect classification data (type of activity, size, or territorial classification).
Subtracting Data	N6	Producers deliberately misreporting	Business owners who want to avoid taxes (income, VAT, other taxes) or social security contributions that understate production and/or overstate costs. Data distortion is often accompanied by double-entry accounting, 'envelope' salary payments, cash payments without a bill, and tax evasion (VAT fraud).
Others	N7	Other statistical deficiencies	Incomplete, not collected, or not directly collectable data; data entered, processed, or aggregated incorrectly by statisticians. Among others, the following areas should be included in this type: how to deal with no responses, production of market-producing entities for own use, tips, and salaries in kind.

Source: own compilation based on OECD (2002).

NOE size estimates, in particular of the SE, continue to be a challenge for statistical services in many countries. Despite the practices recommended by Eurostat, such as the use of the incompleteness scheme (N1–N7), periodic international revisions of national accounts are also conducted, including reviews of the completeness of GDP calculations. This process aims to improve the methodologies used by individual countries and ensure the correctness of determining GDP. For example, the revision conducted in 2005/2006 by the United Nations Economic Commission for Europe covered 43 countries: 18 member states of the then EU, five OECD member states outside the then EU, three candidate countries of the then EU, 12 countries of the then Commonwealth of Independent States, and five others. The general conclusions of this review regarding NOE were as follows:

- 34 countries revised their GDP estimates for underreporting (N6);
- 34 countries included the impact of deliberately unregistered units in their GDP estimates (N1);
- 32 countries revised their GDP estimates for units not requiring registration (N3);
- 7 countries included illegal activities in their GDP estimates (N2).

This review focused on the analysis of the following aspects of NOE estimates in the countries studied: data sources, estimation techniques, and presentation. In terms of data sources, almost all reviewed countries used data from general censuses (population, housing, and agriculture), enterprise research, household surveys, labour market research, tax and treasury databases, police services databases, social services databases, and foreign trade statistics. An overview of the data sources used in selected countries to estimate SE is presented in Table 2.

Table 2. Data sources used in selected countries to estimate SE

Country	Data sources
Belgium	Administrative records (company financial statements, VAT refund data, social security data)
Finland	Data from corporate tax records and tax audits, labour market data (Labour Force Survey)
Germany	Census data, household consumption data, agricultural survey data
The Netherlands	Labour Force Survey, Data from Business Registers, Data from the Social Security Register, Employment Data from Structured Surveys

Source: own compilation based on Urząd Statystyczny w Kielcach (2011).

The review of national practices in selected EU countries showed different approaches to estimating NOE and its components, and how it is published. Some countries published aggregate estimates for NOE (Austria and Finland), others published estimates only for the SE (Belgium, the Netherlands, and the Czech Republic), and still others published estimates for both NOE and the SE (Hungary, Latvia, Poland). Some countries, in turn, showed estimates of selected elements, e.g. extra estimates for avoiding VAT.

The diversity of approaches to estimating the SE means that one should be careful when comparing results between individual countries or even data from one country but at different times or provided by different researchers. Therefore, a decrease or increase in the share of NOE in GDP can mean changes in the scope of economic activities, although it may also be due to improved methods, the use of new statistical data sources, or both.

Estimation of the shadow economy – example of Italy[2] and Poland

As indicated earlier, EU member states use different methodologies to estimate the SE. This is not only due to differences in socioeconomic development but also different structures of national statistics systems (data sources). However, equally important are

2 The example was described on the basis of Urząd Statystyczny w Kielcach (2011, pp. 60–69).

the legal conditions (for example, prostitution, which is included in SE estimates, as it is legal in some countries but not in others), and even cultural differences (for example, the issue of tips for doctors – in Hungarian public statistics, a model was developed to estimate donations in the health service due to the universality of this phenomenon).

In this part of the article, we present the basic principles of the workload method developed by the Italian Statistical Office (ISTAT) and the Statistics Poland approach in this field. The ISTAT method was developed in the 1980s. However, since then, it has been revised several times and, under the recommendations of the European Commission, successively expanded to include further NOE elements. Its principles became the basis for methods developed in other EU countries, including Poland.

In Italy, it is assumed that part of the SE results from economic reasons, i.e. undeclared work, understating legal production, and overstating indirect costs by enterprises. The second reason for the SE is statistics, which is mainly related to the insufficient updating of registers of a large number of small enterprises and the growing group of part-time employees and other producers who are difficult to reach in surveys. To estimate the size of the grey economy, the following are used in Italy:

1) research on unregistered work,
2) adjustments to underestimated income (mainly small enterprises) resulting from adjustments to production per capita and value added,
3) consistency check of aggregated data through supply and use tables at the industry level.

The first two methods are identified with the labour input method mentioned in the introduction. The most important elements include the following:

1. Estimates of labour supply for selected industries and enterprise size classes are obtained based on the Labour Force Survey and other demographic studies.
2. Estimates of output per unit of labour input or value added per unit of labour input for similar industries and enterprise size classes are obtained from regular surveys or special surveys.
3. Estimates of production and value added by industry and enterprise size are obtained using appropriate coefficients related to unit estimates.

The adopted method minimises problems related to identifying enterprises that conduct economic activity and structural changes, which are the main cause of the statistical SE (thus reassigning this area to the observed economy).

As mentioned, an important element of the Italian approach to estimating the size of the SE, and more broadly, the unobserved economy, is estimating the total labour input. It involves calculating the equivalent of legal work (defined as registered in the relevant offices) in terms of full-time, illegal (unregistered) work of residents and the registered and unregistered work of non-residents. Estimates are made while comparing, harmonising, and integrating data from the above-mentioned sources. At the same time, the results on the demand and supply sides are compared.

In Poland, to ensure the quality of national statistics resulting from the European Statistics Code of Practice[3], research conducted in the area of national accounts ensures coherence of the statistical information system from the point of view of definitions of concepts, classifications, and estimation methods. It thus provides a basis for conducting reliable socio-economic statistical analyses in the country and the European Union. According to Eurostat's guidelines, surveys of Polish public statistics assume that the SE includes production activities in the economic sense that are completely legal (in terms of meeting standards and egulations) but which are hidden from public authorities to avoid paying income tax, value-added tax, and other taxes and social security contributions; to avoid the application of legal requirements such as minimum wages, maximum working hours, and work safety conditions; to avoid administrative procedures such as completing statistical questionnaires and other forms. It corresponds to type N6 in Eurostat's notation of incomplete data types (Table 1).

In estimating the size of the SE, Statistics Poland uses a direct method, i.e., an annual representative survey addressed to small entities, and indirect methods. The indirect methods include surveying undeclared work through a survey of the population's economic activity and a modular survey of undeclared work, a consumer questionnaire survey, estimates of activities related to providing sexual services, and the smuggling of cigarettes legally produced in Poland. Official estimates of the SE are made for registered small private-sector businesses in the scope of understated production and revenue in the information provided for statistics and VAT fraud and natural persons for performing undeclared work, mainly in the services sector. The direct method is used to estimate hidden production in registered business entities. It comprises estimating norms of average labour productivity and average remuneration per employee. These norms are then used to estimate average income per employee, which in turn is the basis for estimating global production, intermediate consumption, and gross value added (GUS 2022, p. 110).

As mentioned above, SE size estimates in Poland, prepared by public statistics services, are based on a direct survey of small enterprises and indirect methods of surveying

3 The European Statistics Code of Practice adopted by the European Statistical System Committee on 28.09.2011. The principles of the Statistics Code of Practice together with the general principles of quality management constitute a common quality framework for the European Statistical System.

undeclared work, as well as consumer surveys. In search of possibilities of using alternative measurement methods, especially in the field of econometric methods, the Statistical Office in Kielce[4] has tried to specify estimation techniques based on the idea of SEM for hidden variables.

To the best of our knowledge, only a special case of general SEM has been used in research to estimate the size of the SE, namely the MIMIC (Multiple Indicators Multiple Causes) rule, allowing the relatively simple identification of a class of models. The MIMIC model was first used to estimate the SE by Frey and Weck-Hanneman (1984) in 17 OECD countries. In 1988, Aigner, Schneider, and Ghosh proposed an extended form of the MIMIC model by adding a time factor, the dynamic MIMIC model, the DYMIMIC, which they used to model the SE for the USA. In subsequent years, Giles (1995; 1999) and Giles and Tedds (2002) estimated the size of the SE for New Zealand, Canada, Australia, and other countries of the Pacific. In 2003, Dell'Anno and Schneider (2005) estimated the SE for Italy compared to other OECD countries. In 2007, Dell'Anno, Gomez-Antonio, and Pardo estimated the size of the SE in Mediterranean countries, i.e., France, Spain, and Greece. Buehn and Schneider (2008) used the MIMIC model to estimate the SE in France, while Zagoršek, Jaklič, and Hribernik (2009) presented an analysis of informal activities in Slovenia. They pointed out institution that, in their opinion, were closely related to the development and functioning of the SE. The most recent research in this field was by Efendic, Pasovic, and Efendic (2018), who presented the results of a study on the size dynamics of the SE in Bosnia and Herzegovina from 1998 to 2016. The MIMIC model and its modification were also presented by Dybka et al. (2017), who estimated the SE for 43 countries.

Models of structural equations

In the literature on the subject, models of structural equations are often included among models described as influential statistical revolutions in social sciences (e.g. Cliff 1983; Staniec 2018). SEM is a multivariate technique, or rather a set of statistical procedures and tools that make it possible to describe direct and indirect connections between observable variables (explicit, measurable) and variables that are not directly observable (hypothetical constructs, latent/hidden variables).

In general, SEM can be understood as the combination of factoranalysis and multiple regression analysis, where latent variables or factors are an underlying cause of multiple observed (measured) variables. Consequently, in the simplest terms, SEM can be understood as a path analysis model in which latent variables are allowed (Kline 2011).

4 In Statistics Poland, the Statistical Office in Kielce is responsible for estimating the SE.

The SEM approach is a highly flexible and comprehensive methodology that differs from traditional approaches in several areas, as described below.

While traditional methods only account for measured variables, SEM incorporates observed and unobserved variables (latent constructs). The use of unobserved variables allows us to specify measurement error (to recognise the imperfect nature of the observations), while traditional techniques assume that measurement is free of error. The analytical part of the model allows for measurement errors, while the structural part determines the prediction error of the model. The idea of the approach is to recreate the structure of the substantive process of the variables studied using a simplified theoretical structure of covariance (or correlation) of observable variables. Although the original idea of SEM was based on linear statistical models, modifications can be introduced that allow the inclusion of nonlinear causal relationships, correlated random components, and latent variables. SEM resolves problems of multicollinearity (a common problem when estimating linear or generalised linear models). Multicollinearity cannot occur because unobserved variables represent different latent constructs. This is important for measuring the SE because traditional techniques only analyse measurable variables. Another noteworthy difference is that the traditional approach specifies the default form of the model, while the structural approach requires a formal specification supported by theory and/or research (the axiomatisation of theory). In this case, speaking about the model means a model that fits the theory, i.e. a model whose structure reconstructs the theory. The specification of SEM (making hypotheses) requires the researcher to have an excellent knowledge of theories and research or excellent intuition and to determine a priori the relationships between the variables. Otherwise, the estimated models, despite the high values of the data-fitting indicators, may have little to do with the described and analysed reality. The advantages of this technique are complemented by a graphical way of presenting a hypothetical structure of connections between variables, which presents a complex structure of a phenomenon in a simple way using a path chart/diagram.

However, the versatility of the method, which is undoubtedly an advantage, may lead to serious difficulties and consequently undermine the validity of the model, hence its mixed reception. There is a well-known debate in the literature between professors Schneider (Dell'Anno, Schneider 2006) and Breusch (2005) considering the substantive correctness of applying the MIMIC method in SE research.

It turns out that the best strategy is to perform numerous statistical tests[5].

5 In most cases, to assess the fit quality of the SEM, measures are used that make it possible to compare the estimated model with the saturated model (i.e. assuming that all variables are correlated with each other) and the independent model (i.e. where there is no correlation between any of the pairs of variables). The following indices are used in SEM-related literature and software: IFI (Incremental Fit Index), TLI (Tucker-Lewis Fit Index), RFI (Relative Fit Index), NFI (Normed Fit Index), CFI (Comparative Fit Index). In addition to these measures, RMSEA (Root Mean Square Error of Approximation) is also determined (Bollen 1989).

Reconstructing the theory in structural terms requires:

- an analysis of the theoretical basis of the phenomenon being studied;
- the formulation of statements concerning the set of variables;
- the a priori establishment of links between the variables, i.e. the specification of the SEM form.

In contrast to the traditional approach, the parameters are not estimated at this stage. The specification covers the construction of a system of two types of equations:

- A structural model (also called an internal model, casual model, or substantival process model) – it reflects interrelationships among constructs and tests proposed casual relationships (reflecting a verifiable theory);
- A measurement model (also called an external model) – it represents the theory that specifies how measured variables come together to represent the unobservable variables (it represents the results of confirmatory factor analysis, making it possible to calculate the loads of individual factors that shape a latent variable).

When trying to apply the SEM, it should be remembered that for a given set of observable variables, it is usually possible to determine many models that present different theoretical consequences but that have an equal fit to the empirical data.

The generally accepted way to present SEM is a path diagram, which is then transformed into a set of equations. The system of equations is solved to test the model fit and estimate the parameters. In the literature (Schneider 2005; Kline 2011; Konarski 2014; Medina and Schneider 2018), the following notation is adopted: observable exogenous (X) and endogenous variables (Y), latent exogenous (ζ) and endogenous variables (η).

It is convenient to consider the SEM (in a general case) in the form of a matrix:

$$\eta = B\eta + \Gamma\zeta + \xi \tag{1}$$

$$X = \Lambda_x\zeta + \delta \tag{2}$$

$$Y = \Lambda_y\eta + \varepsilon \tag{3}$$

where:

$\eta_{m\times 1}$ – vector of hidden endogenous variables, $\zeta_{k\times 1}$ – vector of hidden exogenous variables, $B_{m\times m}$ – matrix of regression coefficients with endogenous variables, $\Gamma_{m\times k}$ – matrix of coefficients with exogenous variables, $\xi_{m\times 1}$ – vector of random components, $Y_{p\times 1}$ – vector of observable endogenous variables, $X_{q\times 1}$ – vector of observable exogenous variables, Λ_x, Λ_y – factorial charge matrices, $\delta_{q\times 1}$, $\varepsilon_{p\times 1}$ – measurement error vectors, $E\left[\zeta\zeta^T\right] = \Phi$, $E\left[\xi\xi^T\right] = \Psi$, $E\left[\delta\delta^T\right] = \Theta_\delta$, $E\left[\varepsilon\varepsilon^T\right] = \Theta_\varepsilon$.

According to the standard approach of the SEM, the following is assumed:

$$E[\xi]=E[\delta]=E[\varepsilon]=0,$$
$$cov\left[\zeta,e^{T}\right]=cov\left[\zeta,\delta^{T}\right]=0,$$
$$cov\left[\eta,\varepsilon^{T}\right]=cov\left[\varepsilon_i,\varepsilon_{j\neq i}\right]=cov\left[\delta_i,\delta_{j\neq i}\right]=0,$$
$$cov\left[\zeta_i,\zeta_{j\neq i}\right]=\varphi_{ji},$$
$$det(I-B)\neq 0.$$

In practice, the model proposed at the specification stage may not have been identified. The lack of traceability of the model parameters occurs when there is no unequivocal solution to the structural and measurement equations that meet the assessment criterion. The conditions for the traceability of SEMs were formulated by Bollen (1989) in the form of several rules. The most general rule is the *t* rule (which is a necessary but not a sufficient condition), which states that the number of unknown parameters of the t model should satisfy the inequality $t\leq\frac{(p+q)(p+q+1)}{2}$, where $p+q$ is the number of all observable variables (endogenous and exogenous).

The equation of observable variables introduced into the SEM is not always used to describe the relationships between the variables measured during the survey but to extract significant information from these values in the context of the survey and eliminate disturbances (Konarski 2014). The purpose of the model is to describe the strength and direction of individual relationships (covariance) between observable variables. In the SEM approach, it is implemented by verifying the hypothesis that the proposed model is correct if the covariance matrix Σ observed in the population is accurately reproduced (implied) by the proposed model of the theoretical process[6]:

$$\Sigma=\Sigma(\theta), \tag{4}$$

where θ is a vector of parameters of the postulated model, and $\Sigma(\theta)$ is a matrix of covariance of observable variables expressed as a function of the parameter vector θ.

Generally, the implied covariance matrix (4) is expressed in the following form:

$$\Sigma(\theta)=\begin{bmatrix}\Sigma_{yy}(\theta) & \Sigma_{yx}(\theta)\\ \Sigma_{yx}(\theta) & \Sigma_{xx}(\theta)\end{bmatrix} \tag{5}$$

6 The essence of estimating parameters depends on finding values for those parameters that result in theoretical correlations that match the possible values of empirical correlations as closely as possible.

where the reduced form of equation (1), that is $\eta = (I-B)^{-1}(\Gamma\zeta + \xi)$, is used to determine the matrix:

$$\Sigma(\theta) = \begin{bmatrix} \Lambda_y (I-B)^{-1}(\Gamma\Phi\Gamma^T + \Psi)((I-B)^{-1})^T \Lambda_y^T + \Theta_\varepsilon & \Lambda_y (I-B)^{-1}\Gamma\Phi\Lambda_x^T \\ \Lambda_x \Phi\Gamma^T ((I-B)^{-1})^T \Lambda_y^T & \Lambda_x \Phi \Lambda_x^T + \Theta_\delta \end{bmatrix}. \quad (6)$$

Parameter estimation methods require the definition of an appropriate function (F) that, when minimised, determines an accurate fitting of the implied covariance matrix to the empirical data covariance matrix S, i.e, $\mathrm{F}\left(\mathrm{S},\ \Sigma\left(\hat{\theta}\right)\right)$.

Expressing the model in the implied form (4) and verifying the condition of model identification are key elements of its specification. In practice, estimation is conducted by solving a system of equations using one of the iterative methods:

- the maximum likelihood method (assuming that the distribution is a multidimensional normal distribution);
- the generalised least squares method (which requires a large sample, i.e., more than 2500 observations);
- ADF (an augmented Dickey–Fuller test) methods (insensitive to distribution, cf. Konarski 2014).

At the end of the twentieth century, Jöreskog (1973) and Jöreskog and Sörbom (1993) developed an original program to estimate and verify the linear form of the SEMLIS-REL model (LInear Structural RELations). Since then, new opportunities brought about by the development of the computer market and the software industry have increased the number of programmes, making way for a variety of SEM applications. The most frequently used software include the following: Mplus, SPSS (Amos), STATISTICA (SE-PATH), R (SEM, Lavaan), SAS/STAT (CALIS), and Stata.

If the estimated theoretical model is correctly verified[7], then the identified and estimated relationships and the strength of their influence will provide accurate information about the causal process underlying these variables (i.e. the substantial process). SEM calculations are not difficult to perform thanks to the specialised software mentioned above. The problem is using it accurately and to correctly interpret the results.

7 It will be confirmed in the analysed set of observations.

Proposal for an SEM to measure the shadow economy

The idea behind the proposed approach depends on three facts being considered: 1) The estimated size of the SE, as it is not directly observed, is represented by a rich set of observable variables ($X_1, \ldots X_9$) and (Y_1, Y_2, Y_3). 2) Its existence determines the functioning of various economic spheres (processes) (measurable employing specific macroeconomic indicators). 3) The complexity of the economy – a strong interdependence and the interrelationship of various economic spheres.

A list of variables has been proposed and their interdependencies determined in light of the above (Figure 1). This was done bearing in mind the unique characteristics of Poland, based on both theoretical considerations and empirical research carried out by other researchers in Poland and other countries (in particular: Mróz 2002; Schneider 2005; Cichocki 2006; Grzegorzewska-Mischka and Wyrzykowski 2015; Buehn and Schneider 2016; Dymarski 2016; Dybka et al. 2017; Błasiak 2018; Medina and Schneider 2018; Misztal 2018; Fundowicz et al. 2019; Malczewska 2019) and the experience of the authors derived from estimating the SE in Statistics Poland.

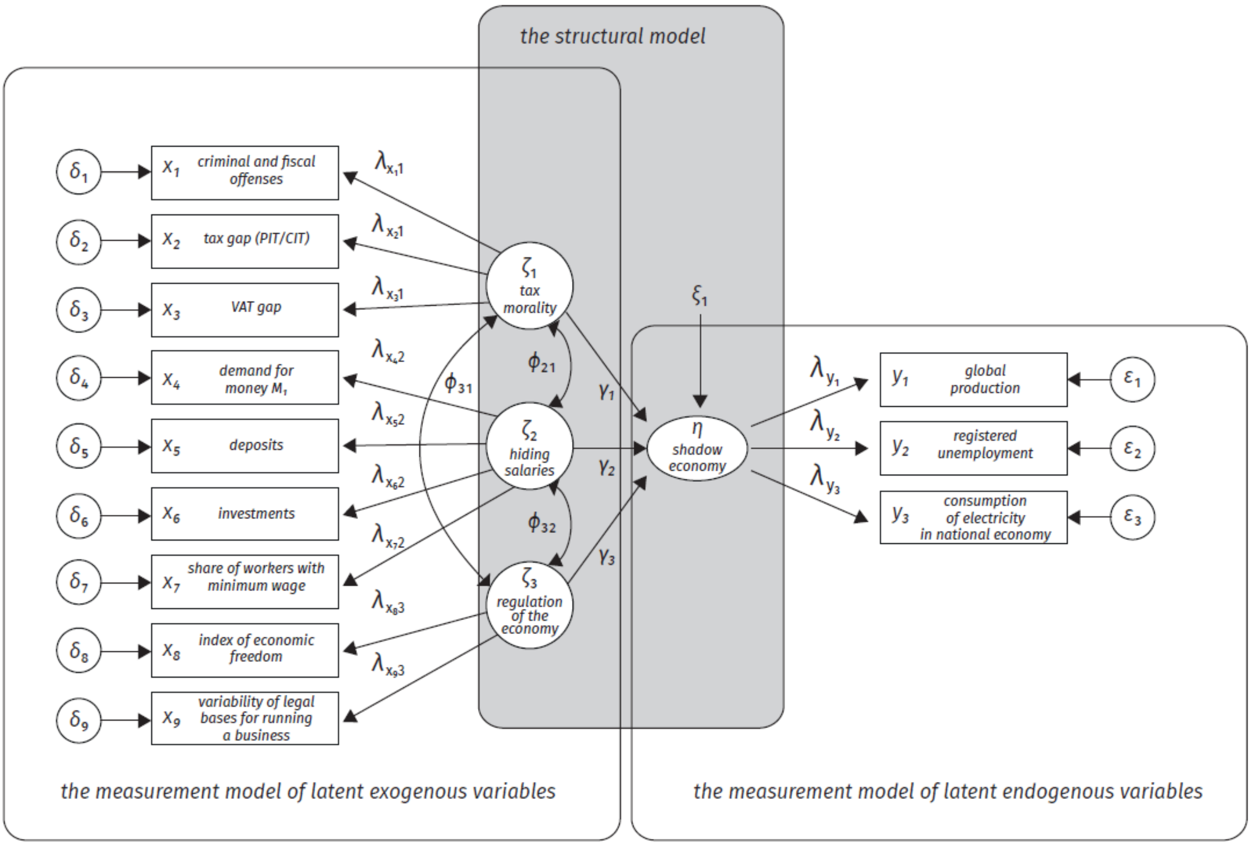

Figure 1. Original SEM describing the internal and external model of the model hypothesis concerning the determinants of the level of the SE in Poland

Source: own elaboration.

In order to overcome the limitations of fragmentation caused by the one-dimensionality of the model (i.e. estimating the SE by e.g. measuring the consumption of electricity (Lackó 2000) or by a monetary approach based on the analysis of the amount of money in circulation[8]), three dimensions have been taken into account in the above model (Y_1, Y_2, Y_3), which reflect the size of the SE.

The innovation of the proposed approach (Figure 1) involves replacing the previously considered MIMIC type models (which contain only endogenous latent[9] variables) with the SEM type. The SEM type combines the structural model with the model (the sub-model) that measures exogenous latent variables (i.e. the factor analysis model) (see Figure 2).

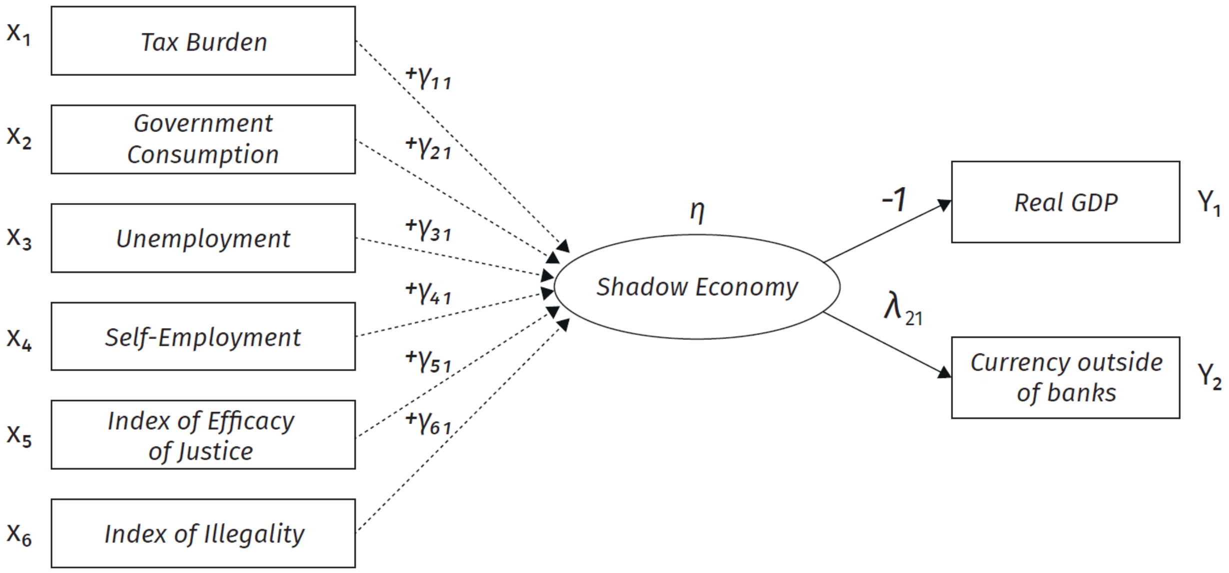

Figure 2. Schneider's MIMIC model of the shadow economy – an international perspective

Source: Dell'Anno and Schneider (2003).

The proposed concept of modelling SE considers that there are unobserved exogenous variables (ζ_1, ζ_2, ζ_3) (theoretical constructs, latent factors) which reflect the causes of the SE. In the proposed model (Figure 1), the reflective approach of defining a hidden variable has been used[10]. Each of the ζ_i constructs has its empirical argumentation in measurable indicators (for ζ_1 they are observational indicators X_1, X_2, X_3, for ζ_2 – observational indicators X_4, X_5, X_6, X_7 and for ζ_3 – indicators X_8, X_9). Furthermore, the model assumes

8 Dymarski (2016) reviewed such models in his doctoral dissertation.

9 Frey and Weck-Hanneman (1984), Dell'Anno and Schneider (2006) and Medina and Schneider (2018) estimated the SE using the MIMIC methodology.

10 The use of a reflective approach implies the assumption that a hidden variable exists and is strongly correlated with observable variables that are the result of the interaction of the hidden variable(s) (Gatnar 2003). The assumed direction of causal flow results from the fact that, e.g., an increase in tax morality means a simultaneous change in the adopted indicators, and it is not expected that direct manipulation of a specific indicator will have a causal impact on a hidden variable.

the existence of a correlation between constructs ζ_1, ζ_2 and ζ_3, which in practice means assuming that the sources of covariance $\phi_{21}, \phi_{31}, \phi_{32}$ are outside the scope of the model.

Such a solution is a chance to eliminate one of the basic objections concerning the assumption made in the MIMIC method, i.e., that variables that act as indicators (and/or determinants) in the model interact only through the endogenous latent variable, i.e., the SE (Dymarski 2016).

In the approach proposed by Schneider (2005) (Figure 2), the causality direction leads from the exogenous observable variables to a hidden variable. This is the formative way of determining a hidden variable (the formative indicators model), the essence of which is the assumption that observable variables (indicators) are hypothetical reasons for the existence of a hidden variable (Bollen 1989). In this approach, the hidden variable does not have to actually exist (Kaplan 2000; Gatnar 2003). Of course, considering more observable variables minimises estimation errors, allowing us to obtain a more reliable and complete picture of the analysed unobservable variable.

The model assumes that the path to shaping the size of the SE is reflected by changes in the level of global production, the rate of registered unemployment, and the size of electricity consumption in the national economy. The size of production depends on the situation in the labour market, and changes in energy consumption are strongly correlated with changes in the size of production (this interdependence results from, among other things, the indirect relationship between the variables, precisely since the SE exists).

Three factors that are not directly observable are proposed as determinants of activity being undertaken in the shadow sphere, i.e., tax morality, concealing salaries, and regulation of the economy. Their task is to reflect information about various (multifaceted) reasons that play an important role in making decisions on how business owners function in the shadow economy.

Finally, the process of data collection, which will provide the theoretical model with the proposed variables is tedious and time-consuming. It requires a broad exploration of both statistical and non-statistical information resources (e.g. administrative data sources, the results of the work of research centres) while maintaining methodological reliability in terms of their comparability in longer time series.

Conclusions

The issues discussed in the article have been discussed for many years. The SE (its causes, consequences, measurement, and evaluation) is still of interest to experts in many fields of academia: sociology, psychology, banking, finance, economics, and statistics.

Despite the passage of time, it still causes difficulties not only in terms of reliable measurement but also in terms of interpreting results. The complexity of the SE means that it is still a problem to find a reliable way to measure and analyse it.

The review of national practices in selected EU countries showed different approaches to estimating the SE and publishing results. This article presents the idea of a developed methodology based on the SEM technique with latent variables. SEM is a combination of techniques that allow multivariate data analysis enriched by the aspect of causality. This makes it possible to capture and explain empirical phenomena more precisely and effectively than with classical methods of statistics and econometrics.

In the literature on the subject, there are studies being conducted based on a special case of the SEM, i.e. the MIMIC rule. The originality of our approach proposed is based on the introduction of exogenous latent variables, which give a better chance to accurately estimate the SE. As a result, the research approach presented in the article will provide a new, objective quality in estimating the size of the SE, and it will deepen our knowledge about its complex structure. The identification of important reasons that determine the existence of the SE will make it possible to better estimate indicators provided by public statistics while indicating the potential directions for limiting its functioning and development.

The presented model is a starting point for further research. There is a high probability that many SEMs will need to be tested and modified to achieve the final result.

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Koncepcja modeli równań strukturalnych w pomiarze szarej strefy – perspektywa międzynarodowa i polska

W artykule podjęto próbę zaprezentowania niewykorzystanego potencjału eksplikacyjnego modelu równań strukturalnych oraz jego specyfikacji w kontekście pomiaru szarej strefy. Szara strefa jako zjawisko interdyscyplinarne budzi wiele pytań i kontrowersji wśród badaczy oraz urzędów statystycznych w wielu krajach. W artykule zaprezentowano różne podejścia krajów w tym zakresie, ze szczególnym uwzględnieniem Włoch i Polski. Jedną z najważniejszych trudności stanowi brak możliwości przeprowadzania bezpośredniego pomiaru wielkości tego zjawiska. Rozwiązaniem tego problemu w opinii autorów jest zastosowanie modeli ze zmiennymi nieobserwowalnymi. Złożoność szarej strefy wymaga uwzględnienia nie tylko prostych relacji między zmiennymi zależnymi i niezależnymi (a właściwie endogenicznymi i egzogenicznymi), równie istotne dla poprawności analizy jest zbadanie związków między samymi zmiennymi o charakterze zależnym lub niezależnym. Powyższe fakty skłaniają do zastosowania niestandardowych technik umożliwiających modelowanie złożonych relacji między zmiennymi oraz uwzględnienie, a następnie szacowanie zmiennych nieobserwowalnych. Artykuł ma charakter przeglądowo-konceptualny i jest przyczynkiem w międzynarodowej dyskusji dotyczącej doskonalenia technik szacowania szarej strefy.

Słowa kluczowe: szara strefa, krajowe urzędy statystyczne, modele równań strukturalnych, zmienne nieobserwowane, gospodarka międzynarodowa