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Hidden Lorentz symmetry of the Hořava–Lifshitz gravity

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ABSTRACT

In this Letter it is shown that the Hořava–Lifshitz gravity theory admits Lorentz symmetry preserving preferred global time foliation of the spacetime.

The possibility that gravity may exhibit a preferred foliation at its most fundamental level has attracted a lot of attention recently, mainly due to the Hořava's papers [1–3] devoted to gravity models characterized by certain specific anisotropic scaling between space and time. The leading idea of the Hořava approach to the quantization of gravity is to achieve power-counting renormalizability by modifying the graviton propagator. This is obtained by adding to the action terms containing higher order spatial derivatives of the metric which, in turn, naturally leads to the preferred co-dimension one foliation $\mathcal{F}$ of spacetime manifold $\mathcal{M}$ topologically equivalent [1–3] to $R^1 \times \Sigma$. The resulting theory, known as the Hořava–Lifshitz (HL) gravity, is then invariant under a group of diffeomorphisms $\mathcal{D}iff(\mathcal{F},\mathcal{M})$ preserving this foliation

$$\tilde{t} = \tilde{t}(t), \qquad \tilde{x}^i = \tilde{x}^i(t, \boldsymbol{x}) \tag{1}$$

where $i = 1, 2, \ldots, D$. The above mentioned anisotropic scaling characterizing HL gravity is of the form

$$t \longrightarrow b^z t, \qquad \boldsymbol{x} \longrightarrow b\boldsymbol{x}. \tag{2}$$

Thus the (momentum) dimension $[t] = -z$, $[x^i] = -1$, so the light velocity c has the dimension $[c] = z - 1$. When z equals the number of spatial dimensions D the theory becomes power-counting renormalizable provided all terms allowed are compatible with the gauge symmetries in the action.

The HL theory is naturally described by the ADM decomposition [4] of the relativistic metric, namely by the lapse function N ($[N] = 0$), the shift vector N^i ($[N^i] = [N_i] = z - 1$) and the metrics γ_{ij} ($[\gamma_{ij}] = 0$) on the spacial slices Σ. In the HL gravity the lapse $N = N(t)$ is only a function of time t which is constant along Σ whereas the shift vector N^i depends on the spacetime point $(t, \boldsymbol{x})$. In terms of the ADM variables the metrics can be written as

$$\begin{aligned} ds^2 &= g_{\mu\nu}\, dx^\mu\, dx^\nu \\ &= -c^2 N^2\, dt^2 + \gamma_{ij}\big(dx^i + N^i\, dt\big)\big(dx^j + N^j\, dt\big). \end{aligned} \tag{3}$$

The HL action, respecting the symmetries $\mathcal{D}iff(\mathcal{F},\mathcal{M})$ is [1–3]

$$S = \frac{2}{\kappa^2}\int dt\, d^D\boldsymbol{x}\,\sqrt{\gamma}N\big[\big(K_{ij}K^{ij} - \lambda K^2\big) - V\big], \tag{4}$$

where $K = K^i{}_i$, λ is a dimensionless coupling constant and

$$K_{ij} = \frac{1}{2N}(\partial_t\gamma_{ij} - \nabla_i N_j - \nabla_j N_i) \tag{5}$$

is the extrinsic curvature of the leaves hypersurface Σ. A scalar potential function V is built out of the spatial metrics, the spatial Riemann tensor and its covariant spatial derivatives but is independent of the time derivatives of fields. For a review and extensions of the Hořava's approach see [5–9]. In the following we restrict ourselves to the physically important $z = D = 3$ case.

One of the problems of the Hořava–Lifshitz gravity is that this theory does not exhibit Lorentz symmetry. A proposed way out of this situation is an appropriate preparation of the potential to restore dynamically local Lorentz invariance in the low-energy limit [1–3]. However, for each finite energy scale the Lorentz symmetry is in fact broken. In this Letter we suggest a way how to overcome the difficulty with the Lorentz symmetry in the Hořava–Lifshitz gravity in a physically acceptable way. To do this let us consider

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a coordinate independent solution to the model defined by the action (4) where the potential V is chosen as in Ref. [1] with the cosmological constant equal to zero. Namely, let us choose the shift vector $\boldsymbol{N}$ as

$$\boldsymbol{N} = \frac{-c\boldsymbol{\epsilon}}{1-\boldsymbol{\epsilon}^2} \tag{6}$$

with $0 \leqslant \boldsymbol{\epsilon}^2 < 1$, while the lapse N is given by

$$N = \frac{1}{\sqrt{1-\boldsymbol{\epsilon}^2}}. \tag{7}$$

Furthermore, the space metrics is chosen as

$$\gamma = \left(I - \boldsymbol{\epsilon} \otimes \boldsymbol{\epsilon}^{\mathrm{T}}\right), \tag{8}$$

where T denotes transposition of the coordinate independent dimensionless column vector $\boldsymbol{\epsilon} = (\epsilon^a)$, $a = 1, 2, 3$. With help of the classical equations of motion [10] it can be verified that Eqs. (6)–(8) define the flat solution to the HL theory determined by (4). The spacetime metrics (3) takes the form

$$\begin{aligned} \mathrm{d}s^2 &= \zeta_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta \\ &= -c^2\,\mathrm{d}t^2 - 2c\boldsymbol{\epsilon}\cdot\mathrm{d}\boldsymbol{x}\,\mathrm{d}t + \mathrm{d}\boldsymbol{x}^{\mathrm{T}}\left(I - \boldsymbol{\epsilon}\otimes\boldsymbol{\epsilon}^{\mathrm{T}}\right)\mathrm{d}\boldsymbol{x} \end{aligned} \tag{9}$$

with the metric tensor

$$\zeta_{\alpha\beta} = \left(\begin{array}{c|c} -1 & -\boldsymbol{\epsilon}^{\mathrm{T}} \\ \hline -\boldsymbol{\epsilon} & I - \boldsymbol{\epsilon}\otimes\boldsymbol{\epsilon}^{\mathrm{T}} \end{array}\right). \tag{10}$$

Here $\alpha, \beta = 0, 1, 2, 3$. It is easy to see that the metrics form (9) is related to the Minkowski spacetime as well as the space geometry is Euclidean. Now, let us consider the rotations

$$t' = t, \qquad \boldsymbol{x}' = R\boldsymbol{x}, \qquad \boldsymbol{\epsilon}' = R\boldsymbol{\epsilon}, \tag{11}$$

where R belongs to the group of orthogonal matrices, and the transformations defined by

$$t' = \frac{t}{a + \boldsymbol{a}\cdot\boldsymbol{\epsilon}}, \tag{12}$$

$$\boldsymbol{x}' = \left(I + \boldsymbol{a}\otimes\boldsymbol{\epsilon}^{\mathrm{T}} + \frac{\boldsymbol{a}\otimes\boldsymbol{a}^{\mathrm{T}}}{1+a}\right)\boldsymbol{x} + \boldsymbol{a}ct, \tag{13}$$

$$\boldsymbol{\epsilon}' = \frac{1}{a + \boldsymbol{a}\cdot\boldsymbol{\epsilon}}\left[\boldsymbol{\epsilon} + \boldsymbol{a}\left(1 + \frac{\boldsymbol{a}\cdot\boldsymbol{\epsilon}}{1+a}\right)\right], \tag{14}$$

where $\boldsymbol{a}$ parametrizes the standard Lorentz boost $L(\boldsymbol{a})$

$$L(\boldsymbol{a}) = \left(\begin{array}{c|c} a & \boldsymbol{a}^{\mathrm{T}} \\ \hline \boldsymbol{a} & I + \frac{\boldsymbol{a}\otimes\boldsymbol{a}^{\mathrm{T}}}{1+a} \end{array}\right), \tag{15}$$

with $a = \sqrt{1+\boldsymbol{a}^2}$. It can be shown that the transformations (11)–(14) taken together form the realization of the Lorentz group and it is obvious that they do not destroy the foliation $\mathcal{F}$. Moreover, the metrics (9) is invariant under the transformations (11)–(14). We point out that in view of (11)–(14) the above transformations form a nonlinear realization of the Lorentz group [11,12]. Nonlinearity affects the coordinate independent vector $\boldsymbol{\epsilon}$ only, whereas $\boldsymbol{x}$ and t transform linearly. The nonlinear realization (11)–(14) was firstly introduced in a different context and form in [13] and was applied to localization problem in Lorentz-covariant quantum mechanics [14,15] and in statistical physics [16]. There is a simple relationship between the standard Lorentz transformations and those given by (11)–(14). Indeed, introducing the new time coordinate by the affine transformation (not belonging to the $\mathcal{D}iff(\mathcal{F}, \mathcal{M})$)

$$t_E = t + \frac{\boldsymbol{\epsilon}\cdot\boldsymbol{x}}{c} \tag{16}$$

we arrive at the standard Minkowski form of the metrics (9). Moreover, we can easily recover for $\boldsymbol{x}$ and t_E the standard Lorentz transformations in the pseudoorthogonal frame. Thus the time redefinition (16) should be interpreted as the change of distant clock synchronization [17–21]. Consequently, the vector $\boldsymbol{\epsilon}$ plays the role of the Reichenbach synchronization coefficient [17,22]. Notice, that in the Einstein general relativity the corresponding group of diffeomorphisms contains both realizations of the Lorentz group: the standard one and the realization defined by (11)–(14). It is not surprising because of the physical equivalence of different synchronization schemes on the classical level (see, e.g., Refs. [20] and [21]). However, only the latter survives as the result of the reduction of the diffeomorphisms group arising in the HL theory.

Now, it is not difficult to apply the above Lorentz covariant flat solution as the local reference frame in a general case. This can be done by introducing the tetrad fields $\omega^\alpha = \lambda^\alpha{}_\mu\,\mathrm{d}x^\mu$ satisfying

$$\zeta_{\alpha\beta}\omega^\alpha\omega^\beta = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu, \tag{17}$$

with $\zeta_{\alpha\beta}$ and $g_{\mu\nu}$ given by (9) and (3) respectively. The solution has the form

$$\omega^0 = \left(cN - \epsilon^a e^a_i N^i\right)\mathrm{d}t - \epsilon^a e^a_i\,\mathrm{d}x^i, \tag{18}$$

$$\omega^a = e^a_i\left(\mathrm{d}x^i + N^i\,\mathrm{d}t\right), \tag{19}$$

where the triads e^a_i determine the space metrics $e^a_i e^a_j = \gamma_{ij}$. The tetrads ω^α transform with respect to the index α according to the law (11)–(14) treated as the frame transformations. Notice, that in general the synchronization vector $\boldsymbol{\epsilon}$ is frame dependent because it transforms from frame to frame according to the formula (14). In particular, we can specify the boost parameter $\boldsymbol{a}$ to obtain the synchronization vector $\boldsymbol{\epsilon}$ equal to zero in a distinguished frame. In this peculiar frame the Einstein synchronization convention applies. Finally, let us stress that the synchronization change (16) does not affect the physical content of theory on the classical level because of the conventionality of the synchronization procedure [17–22]. However, it breaks the quantization procedure essential to the Hořava approach. This can indicate that result of quantization depends on the adapted synchronization scheme. Indeed, an analysis of the quantum-mechanical models discussed in Refs. [14–16], shows that there is not unitary equivalence between quantum theories incorporating different schemes of synchronizations. This means that on the field theory ground one can expect noninvariance of the vacuum state with respect to the transformations implementing a change of the synchronization scheme.

Concluding, the Hořava–Lifshitz gravity admits Lorentz symmetry preserving preferred global time foliation of the spacetime. This symmetry can be related to the standard Lorentz transformations by the frame dependent change of synchronization (16) to the Einstein one. However, (16) breaks the preferred foliation of the HL gravity. Thus the HL theory forces Lorentz symmetry realized in the synchronization scheme related to the transformation laws (11)–(14). Our observation can be also applied to the causal dynamical triangulation theory [23], where the global time foliation is assumed too (however see [24]).

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