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Spatio-temporal Modelling of the Influence of the Number of Business Entities in Selected Urban Centres on Unemployment in the Kujawsko-Pomorskie Voivodeship

Abstract: The paper presents the analysis of the spatial and spatio-temporal tendencies and dependence in matters related to the situation in the labour market of the Kujawsko-Pomorskie Voivodeship across municipalities in the period of 2004–2015. The aim of the investigation is to verify whether, in the presence of the dependence, investing (through a growing number of enterprises) in the development of selected urban centres with a high level of unemployment can significantly reduce the unemployment rate in the whole province. The assessment of the situation in the labour market for each of the municipalities is made with the use of two characteristics, i.e. the share of registered unemployed persons in the number of the working age population and the number of business entities per 10,000 working age population. From the methodological point of view, the values of the variables are treated as realisations of spatial and spatio-temporal stochastic processes. The spatial and spatio-temporal tendencies and dependence were investigated using the concept of spatial and spatio-temporal trends and spatial autocorrelation. Additionally, spatial autoregressive models for individual processes and spatial models of the dependence of unemployment on the number of business entities, for each year of the investigated period, were estimated and verified. The specification of spatio-temporal models of unemployment, including the model which takes into consideration spatial shifts and time lags of the dependence, was carried out. The models were used for simulating the level of unemployment in the province assuming some growth in the number of business entities in selected urban centres.

Keywords: business entities, unemployment, labour market, spatial and spatio-temporal trends, spatial autocorrelation, spatial and spatio-temporal autoregressive models

JEL: C10, E24, R58

1. Introduction

With regard to labour market conditions, as stated by the data from the Central Statistical Office, the Kujawsko-Pomorskie Voivodeship for many years has been located at the bottom of the Polish provinces rankings. The development of individual territorial units within the provinces contributes to jobs creation, and thus, to a reduction of the unemployment rate. According to the 2020 Development Strategy for the Kujawsko-Pomorskie Voivodeship and the 2020+ Modernisation Plan, an improvement in the labour market situation of the province should be the result of investments in the development of the medium-sized urban centres such as Włocławek, Inowrocław and Grudziądz, where the level of unemployment is high. The desire to verify the correctness of this supposition constitutes the main motivation for conducting this research. The mentioned assumption is also the main research hypothesis of the study.

The labour market is a heterogeneous category in different aspects of the analysis (Tokarski, 2005; Perugini, Signorelli, 2007; Gawrycka, Szymczak, 2010; Szulc, 2011; Bal-Domańska, 2014; Nosek, Netrdova, 2014; Pillet et al., 2014; Semerikova, 2015; Vega, Elhorst, 2014; Blinova, Markow, Rusanovskiy, 2016). In particular, in addition to variability in time, the labour market situation is characterised by spatial differentiation occurring at different levels of data aggregation. Simultaneously, certain regularities, tendencies and dependence in this area can be observed.

The primary aim of the paper is to analyse the spatial and spatio-temporal dependence in matters concerning the situation in the labour market of the Kujawsko-Pomorskie Voivodeship across municipalities in the period of 2004–2015. It is the basis to verify that, with the existing dependencies, investments (e.g.: through a growing number of enterprises) in the development of individual medium-sized urban centres of the province can lead to a reduction of unemployment in the whole province. The labour market situation in each of the municipalities has been assessed on the basis of variables which, on the one hand, represent the unexploited labour supply in the form of the number of the unemployed, and, on the other, the demand for labour which is generated by enterprises operating in the market.

The presented considerations refer to our previous work (Szulc, Jankiewicz, 2017). This paper provides some extension and supplement to the investigation.

The structure of the paper is as follows. Section 2 presents the object of the study and its space and time range along with the justification. In Section 3, a preliminary analysis of the data is made pointing to the occurring regularities. Section 4 discusses the methods applied in the study. Section 5 presents empirical results, while the final part contains conclusions and indicates directions for undertaking further research.

2. The subject and scope of the investigation

The study concerns the labour market situation in the Kujawsko-Pomorskie Voivodeship across municipalities in the period of 2004–2015. The following indicators have been analysed: the share of registered unemployed persons in the number of the working age population (Y) and the number of business entities per 10,000 working age population (X). It was considered that these variables were important determinants of the market situation. The data availability at a fixed level of the spatial aggregation (NUTS–5 classification) is also of significance. The adopted level of spatial aggregation and the time range of the research allowed to observe the spatial and temporal tendencies and regularities in the shaping of the analysed variables, which requires a large number of observations.

The spatial and spatio-temporal tendencies and dependence in the behaviour of the analysed variables were in turn investigated using the concept of the spatial and spatio-temporal trend and spatial autocorrelation. Additionally, spatial autoregressive models for individual processes and spatial models of the dependence of unemployment on the number of business entities, for each year of the investigated period, were estimated and verified. The findings concerning the structure of the processes were used for the specification of econometric models for pooled spatial and temporal data, including the model which takes into consideration spatial shifts and time lags of the dependence. The models were used for simulating the level of unemployment in the province assuming some growth in the number of business entities in selected urban centres.

The study verifies the hypothesis of the importance of job creation in certain medium-sized urban centres for the improvement of labour market situation in the whole region. In particular, an attempt was made to determine the extent of spatial influences of stimulation of labour demand in the selected spatial units. The question is whether a growing number of enterprises in selected centres will reduce unemployment in the whole region or in the neighbouring areas only.

3. Data

The data used in the analysis were retrieved from the database of the Central Statistical Office (GUS) (https://bdl.stat.gov.pl/BDL/dane/podgrup/temat). The information regarding variable Y (the share of registered unemployed persons in the working age population) was drawn directly from the database, while the values of variable X (the number of business entities per 10,000 working age population) were obtained through our own calculations. The calculations and figures were made using the R-Cran software (version 3.2.5).

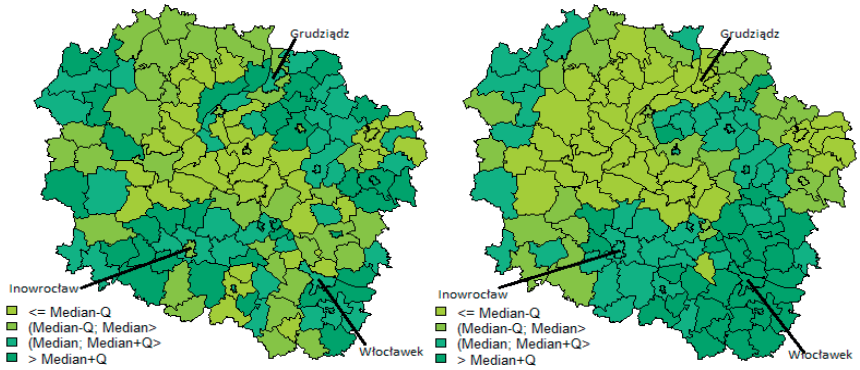

Figure 1. Unemployment across the municipalities in the years 2004 (left) and 2015 (right)

Source: authors' own elaboration

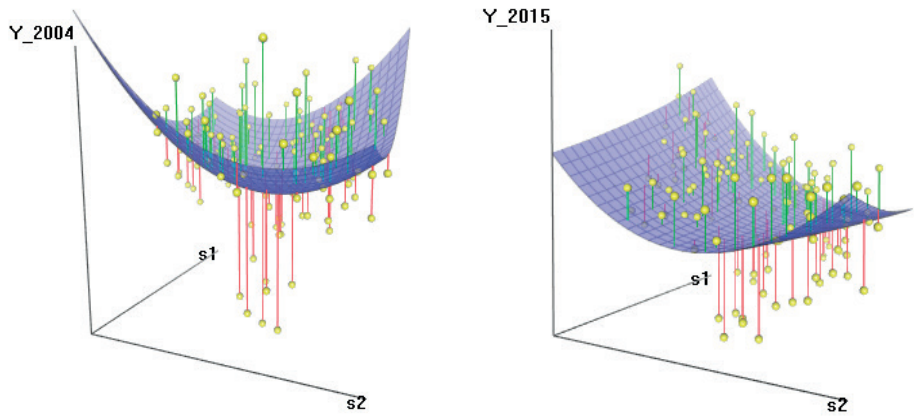

Figure 2. Trend surfaces of unemployment across the municipalities in the years 2004 and 2015

Note: Symbols s_1, s_2 denote the geographic coordinates.

Source: authors' own elaboration

Figure 1 shows the spatial differentiation of variable Y in the two extreme years of analysis, i.e. in the years 2004 and 2015. In Figure 2, the trend surfaces of the considered variable for the respective years are shown. It may be noted that in both years the central part of the province is characterised by a low level of unemployment compared to the remaining part of the region. A higher unemployment rate than that in the centre of the region was noted, among others, in medium-sized urban centres to which particular attention was paid in the investigation, i.e. in Grudziądz, Inowrocław and Włocławek.

In turn, Figure 3 shows the spatial differentiation of variable X. In the case of the number of business entities per 10,000 working age population, the spatial distribution displays an opposite regularity in comparison with the distribution of the variable characterising the level of unemployment. The municipalities located in the cen-

tral part of the region are characterised by a relatively high level of variable X. This concerns both the first and the last year of the investigation. Furthermore, it should be emphasised that a relatively high number of enterprises in the central part of the province was observed also in the urban municipalities which are of particular interest to us, i.e. Grudziądz, Inwrocław and Włocławek. Figure 4 shows the trend surfaces observed in the formation of variable X in the extreme years of our analysis.

Based on Figures 2 and 4, a presumption was formulated regarding the presence of the second-degree spatial trend for both the variables analysed, whereas the spatial directions of the increases and decreases in those variables are opposite.

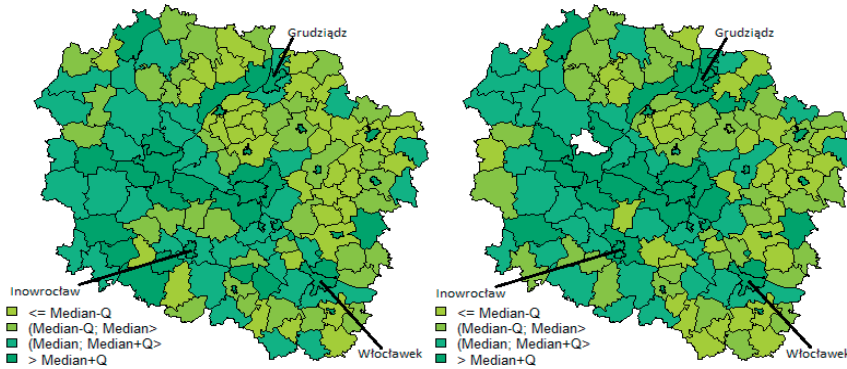

Figure 3. The number of entities per 10,000 working age population across the municipalities in the years 2004 (left) and 2015 (right)

Note: The uncoloured municipality on the right map is the commune of Osielsko which constitutes the so-called outlier with a high value of the variable.

Source: authors' own elaboration

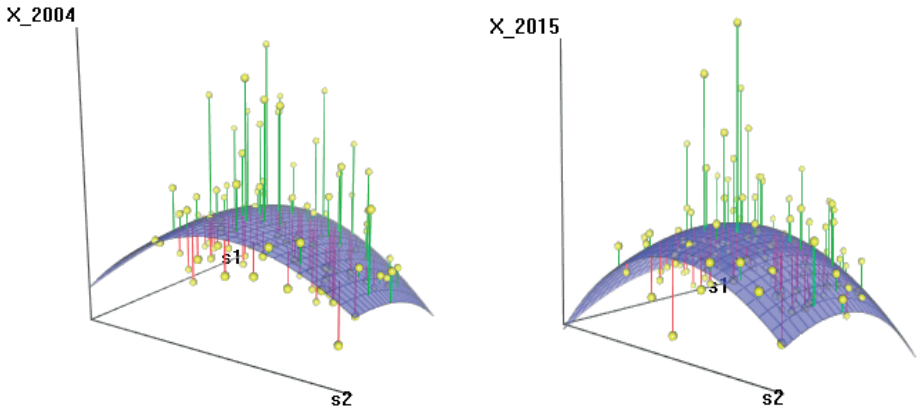

Figure 4. Trend surfaces of the number of entities per 10,000 working age population across the municipalities in the years 2004 and 2015

Source: authors' own elaboration

4. Methodology

The primary tool of analysis conducted in the study is an econometric model specified for the spatio-temporal stochastic process, i.e. a random function $Y(\mathbf{s}, t)$, where non-random arguments are defined as $\mathbf{s} = [s_1, s_2] \in D \subset R^2$, $t \in T \subset R$ (Cressie, 1993: 9; Schabenberger, Gotway, 2005: 432). The two-dimensional argument $\mathbf{s}$ quantifies the locations of spatial units, and t indicates the time. Due to the discreet quantification of space and time in the analyses of economic phenomena, the processes $Y(\mathbf{s}_i, t)$ are considered, where $\mathbf{s}_i = [s_{1i}, s_{2i}]$, $i = 1, 2, \ldots, N$ is the number of the spatial unit, $t = 1, 2, ..., T \subset C$ and C denotes the set of natural numbers.

The specification of the spatio-temporal models was made on the basis of the research of trend-autoregressive structures of the analysed spatial processes $Y(\mathbf{s}_i)$ and $X(\mathbf{s}_i)$, for each year of the period considered. For this purpose, spatial trends were identified and spatial autocorrelation was tested. Moreover, the estimation and verification of the spatial autoregressive models was conducted. The models of spatial trend were used whose general form can be written as follows (Cressie, 1993: 155; Schabenberger, Gotway, 2005: 235):

$$P(\mathbf{s}_i) = \sum_{k=0}^{p} \sum_{m=0}^{p} \theta_{km} s_{1i}^{\ k} s_{2i}^{\ m}, \tag{1}$$

where: $\mathbf{s}_i = [s_{1i}, s_{2i}]$ stands for location coordinates on the plane, $i = 1, 2, ..., N$ (indexes of spatial units), and p means the polynomial trend degree ($k + m \leq p$).

Respectively, the spatio-temporal trend model takes the form (Szulc, 2007: 103):

$$P(\mathbf{s}_i, t) = \sum_{k=0}^{p} \sum_{m=0}^{p} \sum_{l=0}^{p} \theta_{mkl} s_{1i}^{k} s_{2i}^{m} t^{l}, \tag{2}$$

where: $\mathbf{s}_i$, i, p are as above, wherein $k + m + l \leq p$.

The spatial autocorrelation was tested using the test based on the Moran statistic (Moran's I) (first discussed in Moran, 1950 and then, see more in: Cliff, Ord, 1973; 1981) which can be expressed as follows (Schabenberger, Gotway, 2005: 21):

$$I = \frac{N}{\sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij}} \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} w_{ij} \left[z(\mathbf{s}_i) - \overline{z} \right] \left[z(\mathbf{s}_j) - \overline{z} \right]}{\sum_{i=1}^{N} \left[z(\mathbf{s}_i) - \overline{z} \right]^2}, \tag{3}$$

where: $z(\mathbf{s}_i)$ denotes the observation of the phenomenon in the region i, $\overline{z}$ is the average value of the phenomenon, and w_{ij} represents components of the appropriate connectivity matrix. Various types of weights w_{ij} may be pointed out accord-

ing to the established criteria (Haining, 2005: 83–84). In the study, the matrix **W** of connections based on the common border criterion was used.

With the designations adopted above, the spatial autoregressive models (Anselin, 1988; Anselin, Florax, Rey, 2004; Arbia, 2006; LeSage, Pace, 2009) including spatial trends can be written in the following form (Szulc, Müller-Frączek, Pietrzak, 2011: 330):

$$Z\left(\mathbf{s}_i\right)=\sum_{k=0}^{p}\sum_{m=0}^{p}\theta_{km}s_{1i}^{k}s_{2i}^{m}+\rho\mathbf{W}Z\left(\mathbf{s}_i\right)+\varepsilon\left(\mathbf{s}_i\right), \tag{4}$$

where: $\varepsilon(\mathbf{s}_i)$ is the spatial white noise.

In turn, for pooled spatial and temporal data, the models with the spatio-temporal trend take the form (Szulc, 2011):

$$Z\left(\mathbf{s}_i,t\right)=\sum_{k=0}^{p}\sum_{m=0}^{p}\sum_{l=0}^{p}\theta_{mkl}s_{1i}^{k}s_{2i}^{m}t^{l}+\rho\mathbf{W}^{*}Z\left(\mathbf{s}_i,t\right)+\varepsilon\left(\mathbf{s}_i,t\right), \tag{5}$$

where $\mathbf{W}^{*}$ denotes the block matrix of spatio-temporal connections which takes the form:

$$\mathbf{W}^{*}=\begin{bmatrix}\mathbf{W}_1 & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{W}_2 & \cdots & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{W}_T\end{bmatrix},$$

wherein: $\mathbf{W}_1=\mathbf{W}_2=\ldots=\mathbf{W}_T$ represent standard spatial connectivity matrixes, such as in (4), the same for all years, and $\varepsilon(\mathbf{s}_i, t)$ is the spatio-temporal white noise.

In particular, regressive-autoregressive spatial models for the processes $Y(\mathbf{s}_i)$ and $X(\mathbf{s}_i)$ of the form:

$$Y\left(\mathbf{s}_i\right)=\sum_{k=0}^{p}\sum_{m=0}^{p}\theta_{km}s_{1i}^{k}s_{2i}^{m}+\rho\mathbf{W}Y\left(\mathbf{s}_i\right)+\gamma X\left(\mathbf{s}_i\right)+\delta\mathbf{W}X\left(\mathbf{s}_i\right)+\varepsilon\left(\mathbf{s}_i\right) \tag{6}$$

were estimated and verified in the investigation.

Finally, the hypotheses of space-time models for the studied process $Y(\mathbf{s}_i, t)$ were as follows:

$$\begin{aligned}Y\left(\mathbf{s}_i,t\right)=&\sum_{k=0}^{p}\sum_{m=0}^{p}\sum_{l=0}^{p}\theta_{mkl}s_{1i}^{k}s_{2i}^{m}t^{l}+\rho\mathbf{W}^{*}Y\left(\mathbf{s}_i,t\right)\\&+\gamma X\left(\mathbf{s}_i,t\right)+\delta\mathbf{W}^{*}X\left(\mathbf{s}_i,t\right)+\varepsilon\left(\mathbf{s}_i,t\right)\end{aligned}, \tag{7}$$

and

$$Y(\mathbf{s}_i,t)=\sum_{k=0}^{p}\sum_{m=0}^{p}\sum_{l=0}^{p}\theta_{mkl}s_{1i}^{k}s_{2i}^{m}t^{l}+\rho\mathbf{W}^{*}Y(\mathbf{s}_i,t)$$
$$+\gamma X(\mathbf{s}_i,t)+\rho^{\bullet}\mathbf{W}^{*}Y(\mathbf{s}_i,t-1) \tag{8}$$
$$+\delta\mathbf{W}^{*}X(\mathbf{s}_i,t)+\delta^{\bullet}\mathbf{W}^{*}X(\mathbf{s}_i,t-1)+\varepsilon(\mathbf{s}_i,t).$$

The models (7) and (8) were used for simulating the unemployment rate in the province across municipalities assuming some growth in the number of business entities in selected urban centres.

5. Specification of spatial and spatio-temporal econometric models – results of the empirical analysis

5.1. Analysis of spatial trends and spatial autocorrelation

The analysis of the situation in the labour market in the municipalities of the Kujawsko-Pomorskie Voivodeship started with identifying the spatial structure of both variables (processes). For this purpose, the spatial trend models were estimated and verified, and next the spatial autocorrelation using Moran's *I* was tested. Table 1 reports the results of the investigation.

Table 1. Spatial structure of the processes: $Y(\mathbf{s}_i)$ – the share of the registered unemployed persons in the working age population and $X(\mathbf{s}_i)$ – business entities per 10,000 working age population

Year	Degree of spatial trend		R^2		Spatial autocorrelation		Moran's *I*	
	$Y(\mathbf{s}_i)$	$X(\mathbf{s}_i)$	$Y(\mathbf{s}_i)$	$X(\mathbf{s}_i)$	$Y(\mathbf{s}_i)$	$X(\mathbf{s}_i)$	$Y(\mathbf{s}_i)$	$X(\mathbf{s}_i)$
2004	2	2	0.1494	0.0996	+	–	0.3746	0.0295
2005	2	2	0.1612	0.1016	+	–	0.3600	0.0337
2006	2	2	0.1580	0.1004	+	–	0.3807	0.0222
2007	2	2	0.2856	0.1021	+	–	0.4050	0.0339
2008	2	2	0.2736	0.1107	+	–	0.4363	0.0429
2009	2	2	0.2600	0.1343	+	–	0.3951	0.0585
2010	2	2	0.2891	0.1336	+	+	0.4488	0.0842
2011	2	2	0.2839	0.1397	+	+	0.4973	0.0979
2012	3	2	0.3400	0.1386	+	+	0.4722	0.1127

Year	Degree of spatial trend		R^2		Spatial autocorrelation		Moran's I	
	$Y(\mathbf{s}_i)$	$X(\mathbf{s}_i)$	$Y(\mathbf{s}_i)$	$X(\mathbf{s}_i)$	$Y(\mathbf{s}_i)$	$X(\mathbf{s}_i)$	$Y(\mathbf{s}_i)$	$X(\mathbf{s}_i)$
2013	3	2	0.3220	0.1469	+	+	0.4420	0.1320
2014	3	2	0.4309	0.1529	+	+	0.4339	0.1434
2015	2	2	0.4611	0.1504	+	+	0.4404	0.1480

Note: Symbol "+" means that spatial autocorrelation occurs (at the level of significance of at least 0.05), symbol "–" means that spatial autocorrelation does not occur.

Source: authors' own calculations

The presence of spatial tendencies in each year of the investigation for both analysed variables has been identified. More precisely, the spatial trend models of the 2nd degree for the variable X in all the years investigated were fitted and for the variable Y – the models of the 2nd degree in each year of the period 2004–2011 and in 2015, and of the 3rd degree in: 2012, 2013, 2014. In turn, the dependence of the considered variables on their values reported in the neighbouring municipalities was observed in each year for the variable Y and from 2010 for the variable X. Table 2 presents the results of the estimation and verification of the spatial autoregressive models for the analysed variables (processes).

Table 2. The results of estimation and verification of SAR models with the spatial trend for the processes: $Y(\mathbf{s}_i)$ and $X(\mathbf{s}_i)$ in the years 2004–2015

Year	$Y(\mathbf{s}_i)$					$X(\mathbf{s}_i)$				
	Degree of spatial trend	Autoregression		Residuals		Degree of spatial trend	Autoregression		Residuals	
		ρ	p-value	Moran's I	p-value		ρ	p-value	Moran's I	p-value
2004	2	0.6260	0.0000	–0.0110	0.4703	2	–	–	0.0295	0.2489
2005	2	0.6018	0.0000	–0.0161	0.4332	2	–	–	0.0337	0.2249
2006	2	0.6561	0.0000	0.0007	0.4437	2	–	–	0.0222	0.2939
2007	2	0.6724	0.0000	–0.0022	0.4645	2	–	–	0.0339	0.2238
2008	2	0.6751	0.0000	–0.0141	0.4475	2	–	–	0.0429	0.1770
2009	2	0.6424	0.0000	–0.0070	0.5000	2	–	–	0.0585	0.1119
2010	2	0.6763	0.0000	–0.0124	0.4599	2	0.2355	0.0694	–0.0017	0.4605
2011	2	0.7340	0.0000	–0.0127	0.4579	2	0.2613	0.0413	–0.0001	0.4489
2012	3	0.7318	0.0000	–0.0152	0.4398	2	0.2896	0.0222	0.0030	0.4257
2013	3	0.6877	0.0000	–0.0214	0.3954	2	0.3294	0.0084	0.0051	0.4108
2014	3	0.6758	0.0000	–0.0175	0.4229	2	0.3532	0.0045	0.0079	0.3903
2015	2	0.6983	0.0000	–0.0103	0.4754	2	0.3617	0.0036	0.0095	0.3792

Source: authors' own calculations

5.2. The regressive-autoregressive spatial models

Table 3 presents the results of estimation and verification of regressive-autoregressive models, i.e. models of the number of business entities influencing the unemployment rate in the municipalities (parameter γ), including the influence of unemployment in the neighbouring municipalities (parameter ρ), and of the number of business entities observed in the neighbours (parameter δ).

The models contain statistically significant parameters in all the years except 2008 (insignificant parameter γ) and 2010 (insignificant parameter δ). Then, the residuals from all the models do not show spatial autocorrelation.

Table 3. The results of estimation and verification of the regressive-autoregressive models for the process $Y(\mathbf{s}_j)$ in the years 2004–2015

Year	Parameters of regressive-autoregressive models						Autocorrelation of residuals	
	ρ	p-value	γ	p-value	δ	p-value	Moran's I	p-value
2004	0.5581	0.0000	–1.82E–05	0.0076	–5.03E–05	0.0048	–0.0034	0.4737
2005	0.5166	0.0000	–1.81E–05	0.0058	–4.50E–05	0.0027	–0.0100	0.4780
2006	0.5907	0.0000	–1.74E–05	0.0043	–4.50E–05	0.0038	0.0017	0.4362
2007	0.6444	0.0000	–1.30E–05	0.0148	–2.73E–05	0.0424	–0.0021	0.4642
2008	0.6184	0.0000	–8.30E–06	0.0910	–4.36E–05	0.0004	–0.0003	0.4508
2009	0.5804	0.0000	–1.56E–05	0.0088	–3.23E–05	0.0198	–0.0038	0.4766
2010	0.6202	0.0000	–1.89E–05	0.0007	–1.92E–05	0.1342	–0.0175	0.4228
2011	0.6640	0.0000	–2.03E–05	0.0002	–2.49E–05	0.0496	–0.0268	0.3569
2012	0.6426	0.0000	–2.11E–05	0.0000	–2.34E–05	0.0502	–0.0464	0.2330
2013	0.5536	0.0000	–2.27E–05	0.0000	–3.29E–05	0.0119	–0.0481	0.2236
2014	0.5305	0.0000	–2.07E–05	0.0000	–2.61E–05	0.0166	–0.0494	0.2169
2015	0.5738	0.0000	–1.56E–05	0.0005	–2.34E–05	0.0195	–0.0310	0.3288

Source: authors' own calculations

5.3. The specification of the spatio-temporal econometric models

On the basis of the pooled time series and spatial data, the analysis of spatio-temporal trend was conducted for each variable. In the case of unemployment, the presence of second-degree spatio-temporal trend was observed (with regard to both the spatial and temporal component). At the same time, the variable concerning the number of business entities showed a significant tendency of 2nd degree in space and of 1st degree over time.

Next, the SAR models with spatio-temporal trends were estimated and verified. The results are presented in Tables 4 and 5.

Based on the foregoing findings, the estimation and verification of spatio-temporal SAR model was made where the level of unemployment in a given year

in each of the municipalities of the province analysed is explained by the current level of unemployment in the neighbouring municipalities, the number of business entities in the municipality in a given year, and the number of entities in the neighbouring municipalities in the same year.

In accordance with the above-presented findings, the model contained an additional component, i.e. the spatio-temporal trend of degree 2. The model was estimated on the basis of data for the period from 2004 to 2012 in order to enable a simulation of changes in the level of unemployment on assumptive changes regarding the number of enterprises. Table 6 contains the results of the model estimation and verification.

Table 4. The results of estimation and verification of SAR model with the spatio-temporal trend for the process $Y(\mathbf{s}_i)$

Parameter	Estimate of parameter	Standard error	Statistic z	p-value
θ_{000}	1.66E+00	2.71E–01	6.1412	0.0000
θ_{100}	–1.60E–06	4.85E–07	–3.2984	0.0010
θ_{010}	–4.30E–06	6.45E–07	–6.6752	0.0000
θ_{200}	1.24E–12	3.71E–13	3.3507	0.0008
θ_{110}	8.12E–13	4.76E–13	1.7048	0.0882
θ_{020}	3.36E–12	4.47E–13	7.5055	0.0000
θ_{001}	–4.13E–03	7.22E–04	–5.7299	0.0000
θ_{002}	2.29E–04	5.18E–05	4.4213	0.0000
$\rho = 0.7860$ Test LR: 1235.4, p-value: 0.0000				
Moran test: –2.4647, p-value: 0.0069				

Source: authors' own calculations

Table 5. The results of estimation and verification of SAR model with the spatio-temporal trend for the process $X(\mathbf{s}_i)$

Parameter	Estimate of parameter	Standard error	Statistic z	p-value
θ_{000}	–2.94E+04	3.84E+03	–7.6594	0.0000
θ_{100}	3.36E–02	6.93E–03	4.8523	0.0000
θ_{010}	7.97E–02	9.17E–03	8.6881	0.0000
θ_{200}	–2.21E–08	5.28E–09	–4.1886	0.0000
θ_{110}	–2.43E–08	6.80E–09	–3.5713	0,0004
θ_{020}	–6.01E–08	6.32E–09	–9.5003	0.0000
θ_{001}	6.81E+00	2.15E+00	3.1689	0.0015
$\rho = 0.2362$ Test LR: 39.4740, p-value: 0.0000				
Moran test: 0.0727, p-value: 0.4710				

Source: authors' own calculations

In the second variant, the specification of the model was slightly expanded in order to account for the time lags needed to realise the dependence. In this case, parameters δ^*, ρ^* measure the influence, respectively, of the number of entities in the neighbouring municipalities in the previous year and the unemployment in the neighbouring municipalities in the previous year on the unemployment rate in a particular municipality in the current year. Table 7 contains the results of estimation and verification of the model.

Table 6. The results of estimation and verification of the final SAR model with the spatio-temporal trend

Parameter	Estimate of parameter	Standard error	Statistic z	p-value
θ_{000}	1.00E+00	3.29E–01	3.0374	0.0024
θ_{100}	–8.39E–07	5.78E–07	–1.4526	0.1463
θ_{010}	–2.19E–06	7.90E–07	–2.7748	0.0055
θ_{200}	6.12E–13	4.42E–13	1.3829	0.1667
θ_{110}	3.47E–13	5.55E–13	0.6254	0.5317
θ_{020}	1.66E–12	5.52E–13	3.0060	0.0026
θ_{001}	–1.16E–02	1.35E–03	–8.5634	0.0000
θ_{002}	9.56E–04	1.23E–04	7.7413	0.0000
γ	–1.58E–05	1.96E–06	–8.0839	0.0000
δ	–3.27E–05	4.69E–06	–6.9785	0.0000
$\rho = 0.6574$ Test LR: 505.3, p-value: 0.0000 < 2.22E – 16				
Moran test: –1.2234, p-value: 0.1106				

Source: authors' own calculations

Table 7. The results of estimation and verification of the final SAR model with the spatio-temporal trend and time lags of the dependence

Parameter	Estimate of parameter	Standard error	Statistic z	p-value
θ_{000}	0.5523	0.1632	3.3841	0.0007
θ_{100}	–5.45E–08	2.09E–08	–2.6106	0.0090
θ_{010}	–1.46E–06	5.67E–07	–2.5763	0.0099
θ_{020}	1.19E–12	4.92E–13	2.4196	0.0155
θ_{001}	–0.0075	0.0022	–3.4905	0.0005
θ_{002}	0.0007	0.0002	3.5000	0.0000
γ	–1.42E–05	2.05E–06	–6.9206	0.0000
δ	–7.03E–05	2.20E–05	–3.2011	0.0014
δ^*	4.55E–05	2.22E–05	8.4784	0.0000
ρ^*	0.2773	0.0327	8.4801	0.0000
$\rho = 0.5087$ Test LR: 155.3,505.3, p-value: 0.0000 < 2.22E – 16				
Moran test: –3.6435, p-value: 0.0001				

Source: authors' own calculations

5.4. Simulation of the level of unemployment in the municipalities

Using the models described above, a simulation of the level of unemployment in the municipalities of the Kujawsko-Pomorskie Voivodeship was performed on the assumption that the number of enterprises per 10,000 working age population in individual medium-sized urban centres such as Grudziądz, Inowrocław and Włocławek increases (the assumed increase amounted to approximately 20%). The 'predictive' values of Y in the years: 2013, 2014 and 2015 were estimated. Table 8 sets out the municipalities (the same for each year) where a decrease in the level of unemployment was noted.

Table 8. List of the municipalities where a decrease in the unemployment rate was observed

Municipalities: Urban – (1), Rural – (2), Urban-Rural (3)
Inowrocław (1), Pakość (3), Inowrocław (2), Brześć Kujawski (3), Bobrowniki (2), Lubanie (2), Włocławek (2), Dobrzyń nad Wisłą (3), Fabianki (2), Włocławek (1), Grudziądz (2), Dragacz (2), Grudziądz (1), Rogóźno (2)

Source: authors' own elaboration

Table 9. Decrease in the unemployment rate in the municipalities

Municipalities		**2013**		**2014**		**2015**	
		V1	**V2**	**V1**	**V2**	**V1**	**V2**
Urban	Grudziądz (1)	0.39	0.37	0.40	0.38	0.41	0.39
	Inowrocław (1)	0.43	0.41	0.43	0.41	0.44	0.41
	Włocławek (1)	0.49	0.47	0.50	0.47	0.50	0.48
Rural	Bobrowniki (2)	0.14	0.33	0.14	0.12	0.14	0.12
	Dragacz (2)	0.11	0.11	0.11	0.07	0.12	0.08
	Fabianki (2)	0.16	0.16	0.16	0.10	0.16	0.11
	Grudziądz (2)	0.09	0.21	0.09	0.09	0.09	0.09
	Inowrocław (2)	0.09	0.23	0.09	0.08	0.09	0.08
	Lubanie (2)	0.19	0.33	0.20	0.12	0.20	0.12
	Rogóźno (2)	0.19	0.46	0.20	0.17	0.20	0.18
	Włocławek (2)	0.12	0.46	0.12	0.17	0.12	0.17
Urban-Rural	Brześć Kujawski (3)	0.16	0.39	0.16	0.16	0.16	0.16
	Dobrzyń nad Wisłą (3)	0.19	0.29	0.20	0.20	0.20	0.20
	Pakość (3)	0.14	0.34	0.14	0.12	0.14	0.12

Note: Designations V1, V2 denote, respectively, variant 1 and variant 2 of the simulation model.

Source: authors' own calculations

Among the municipalities which showed a decrease in the unemployment as a result of an increase in the number of enterprises per 10,000 working age population are the three centres considered and their neighbouring municipali-

ties. Table 9 shows the values (in p.p.) of the decrease in the unemployment rate wherein two simulation variants were applied: V1 (according to model 7) and V2 (according to model 8). According to the first variant, the decreases in unemployment in urban municipalities are significantly larger than in rural and urban-rural ones in all three years. However, according to the second variant of the simulation, for year 2013 this regularity does not occur (see also maps in Figure 5). Furthermore, in this variant, fewer decreases can be observed in 2014 and 2015 compared to 2013 for rural and urban-rural municipalities. The decreases persist at a similar level only in urban municipalities which have seen an increase in the number of business entities.

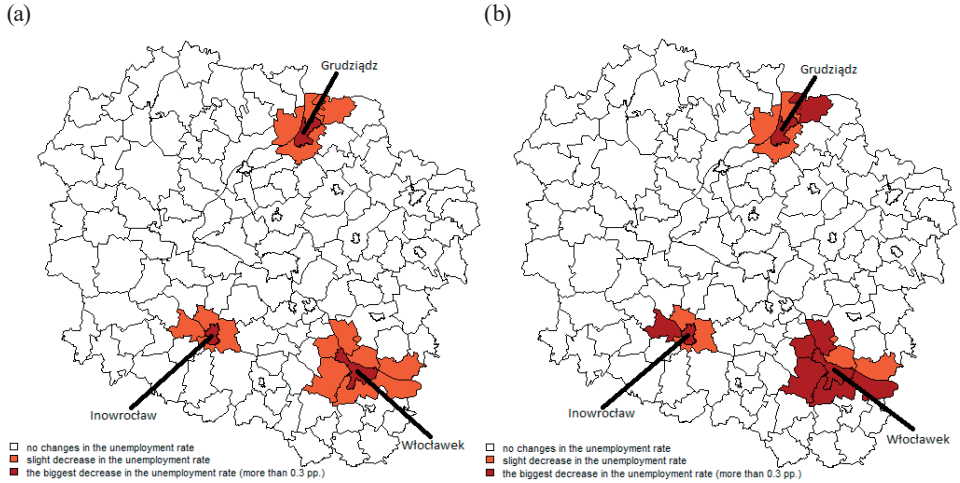

Figure 5. Size and spatial range of decreases in the unemployment rate: (a) according to variant 1 of simulation in all years considered and variant 2 in 2014 and 2015, (b) according to variant 2 of simulation in 2013

Source: authors' own elaboration

6. Conclusions

The analysis shows that, with the existing spatial and spatio-temporal dependencies observable in the labour market in the Kujawsko-Pomorskie Voivodeship across municipalities, an improvement of the situation in this market concerning a decrease in the unemployment rate may be expected as a result of an increase in the number of enterprises in medium-sized urban centres such as Grudziądz, Włocławek and Inowrocław. However, according to the results of the investigation, investments in new business entities in the indicated centres will reduce the level of unemployment only in the said centres and in the neighbouring municipalities.

The obtained results do not provide sufficient grounds to formulate a statement that the indicated actions will significantly improve the labour market situation in the entire province. The research should be continued with the consideration of an extended time span for the analysis, using other spatial connectivity matrices, and, perhaps, focus on investigating the impact of the number of micro, small and medium enterprises on the labour market situation. Furthermore, we intend to continue the work on improving properties of empirical models, for example, with the use of the concept of spatial panel models. Then, in further research, we would like to analyse the influence of the unemployment changes on migration processes.

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Przestrzenno-czasowe modelowanie wpływu liczby podmiotów gospodarczych w wybranych ośrodkach miejskich na bezrobocie w województwie kujawsko-pomorskim

Streszczenie: Artykuł prezentuje analizę przestrzennych i czasowych tendencji i zależności dotyczących sytuacji na rynku pracy w województwie kujawsko-pomorskim w układzie gmin w okresie 2004–2015. Celem badania jest sprawdzenie, czy – przy istniejących zależnościach – wzrost aktywności działalności gospodarczej przez zwiększenie liczby podmiotów gospodarczych w wybranych ośrodkach miejskich województwa, w których poziom bezrobocia jest wysoki, może obniżyć bezrobocie w całym województwie. Sytuacja na rynku pracy w poszczególnych gminach została oceniona na podstawie dwóch zmiennych, tj. udziału bezrobotnych zarejestrowanych w liczbie ludności w wieku produkcyjnym oraz liczby podmiotów gospodarczych na 10 tys. mieszkańców w wieku produkcyjnym. Z metodologicznego punktu widzenia wartości tych zmiennych traktuje się jako realizacje przestrzennych i przestrzenno-czasowych procesów stochastycznych. Przestrzenne i przestrzenno-czasowe tendencje i zależności zostały zbadane przy wykorzystaniu koncepcji trendu przestrzennego i przestrzenno-czasowego oraz autokorelacji przestrzennej. Dodatkowo zostały oszacowane i zweryfikowane przestrzenne modele autoregresyjne poszczególnych procesów, a także przestrzenne modele zależności bezrobocia od liczby podmiotów gospodarczych w poszczególnych latach badanego okresu. Dokonano również specyfikacji przestrzenno-czasowych modeli bezrobocia, w tym modelu uwzględniającego przesunięcia przestrzenne i opóźnienia czasowe rozważanej zależności. Modele te wykorzystano następnie do symulacji poziomu bezrobocia w województwie przy założeniu wzrostu liczby podmiotów gospodarczych w ustalonych ośrodkach miejskich.

Słowa kluczowe: podmioty gospodarcze, bezrobocie, rynek pracy, trend przestrzenny i przestrzenno-czasowy, autokorelacja przestrzenna, przestrzenne i przestrzenno-czasowe modele autoregresyjne

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