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A BOTTLENECK PRODUCTION MODEL WITH AND WITHOUT SUBSTITUTION**

In the Polish economy we have an excess of several production factors, their inputs are not in equilibrium. On the other hand, in the production function it is assumed a production factors equilibrium which is related to substitution processes. The output generated from a production function is just an interaction of analyzed production factors.

1. No Substitution Approach

We would like to propose another approach. The output would be related to input of only this production factor which was a bottleneck in a given year. A bottleneck is flexible in a year as well as in an aggregate of the production factor. These are precision constraints of our analysis.

First step - bottleneck search. We look for the bottleneck production factor for each year. To do it we use one factor production functions

$$X = f(Y^i) \tag{1}$$

where: X - production output; Y^i - "i" production factor input.

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The regression curve divides points - observations into two groupe (Figure 1).

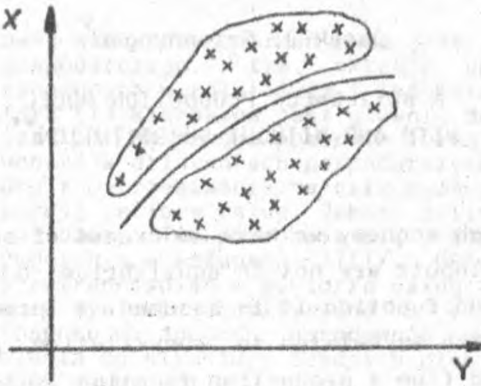


Fig. 1

The points-observations below the regression curve represent years when the production factor was in a relative excess. A relative one because we base on arbitrarily chosen analytical form of the function. We can also meet a case when the production factor is a bottleneck in the whole sample period. But in this case we can expect high goodness of fit.

One by one we drop observations with the biggest residuals (below the curve) because the quantity of the factor in excess is not related to the production output. Then another production factor was a bottleneck and that one determined output. It is not clear when to stop this iteration procedure (one of the proposals is R^2 stabilization). As a result we get a set of one factor production functions estimated on subsamples. Each one of them is based only on the years when the production factor was a bottleneck.

Observations, when the production factor was in excess, not only lower the regression curve, but also often shift it. As a consequence the production function and capacity output curve are shifted.

If in an iteration procedure we drop one of the years for all production factors, it will mean that other factor was a bottleneck in that year.

Any better method, instead of one factor production function can be used to search bottlenecks in the first step.

Second step - the link of bottlenecks (it is not necessarily related to the method used in the first step). Because we are short of observations we link bottlenecks into one model:

$$\frac{X_t - X_{t-k}}{X_{t-k}} = \alpha_0 \cdot k + \alpha_{1i} \frac{Y_t^i - Y_{t-k}^i}{Y_{t-k}^i} + \varepsilon_t \qquad (2)$$

where:

Y_t^i - "i" bottleneck factor in period t,

t - k - previous period when Y^i was a bottleneck,

α_{1i} - production elasticity with regard to factor Y^i,

α_0 - technological progress parameter,

ε_t - disturbance term.

This function is similar to Cobb-Douglas production function, so in the first step we have used one factor Cobb-Douglas production function.

Bottlenecks in the Polish industry. We consider the Polish industry in the period 1961-1981. Bottlenecks after 4-6 iterations are given in the Table 1.

The bottleneck distribution points that in some one factor function autocorrelation appears which can be eliminated by a more convex analytical form. But we can loose the model´s simplicity as well as the connection between the first and second step.

Next, we will concentrate on fixed assets, employment and imports. We shall consider these production factors in 1964. The bottlenecks were then fixed assets, employment and imports from socialist countries. Thus, there are 3 observations for that year. In the first one an annual percentage production increase depends on an annual percentage increase of fixed assets (other variables are equal to zero). In the second observation for 1964 an average 2-year percentage increase of production depends on

T a b l e 1

The Polish industry in the period 1961-1981

Years	K	N	MN	MS	RC	RE	MOI MOP
1961		X		X		X	X
1962	X	X				X	X
1963	X					X	
1964	X	X		X		X	
1965	X	X			X	X	
1966	X			X	X		
1967					X		
1968	X		X	X	X		
1969	X		X		X		
1970			X		X		
1971			X		X		X
1972	X		X		X		X
1973	X				X		
1974	X				X	X	X
1975	X			X	X	X	X
1976				X	X	X	X
1977		X	X	X		X	X
1978		X	X			X	X
1979		X	X				
1980			X				
1981			X				

N o t a t i o n: X - Polish industry production, socialized sector, constant prices, January 1, 1977; K - fixed assets in socialized industry, constant prices January 1, 1977, one year lag; N - employment in socialized industry, one year lag; MN - imports from non-socialist countries, constant 1977 prices; MS - imports from socialist countries, constant 1977 prices; RC - coal for non-market use, million tons; RE - industry electric energy utilization, million kW; MOI - crude oil imports, million tons; MOP - oil product imports, million tons.

an average 2-year percentage increase of employment (other variables are equal to zero). In the third observation an average percentage increase of production for the years 1962, 1963, 1964 depends on an average 3-year percentage increase of imports from socialist countries (other variables are equal to zero).

The following estimation results for the above mentioned function - a time series-cross section sample of 30 observations - were obtained:

$$\frac{X - X_{-k}}{X_{-k}} = \underset{2.3}{1.9} + \underset{4.3}{2.5}\,\frac{N - N_{-k}}{N_{-k}} \qquad \text{6 observations}$$

$$\frac{X - X_{-k}}{X_{-k}} = 1.9 + \underset{6.4}{0.91}\,\frac{K - K_{-k}}{K_{-k}} \qquad \text{10 observations}$$

$$\frac{X - X_{-k}}{X_{-k}} = 1.9 + \underset{4.7}{0.62}\,\frac{MS-MS_{-k}}{MS_{-k}} \qquad \text{6 observations}$$

$$\frac{X - X_{-k}}{X_{-k}} = 1.9 + \underset{8.4}{0.45}\,\frac{MN-MN_{-k}}{MN_{-k}} \qquad \text{8 observations}$$

$R^2 = 0.83$

All the estimated parameters are significant. We get a relatively high goodness of fit in a function estimated on increments. We can observe explicit production elasticity differences from 0.45 with respect to non-socialist imports to 2.5 with respect to employment. Of course, we did not expect that the sum of elasticities would be equal to one but that each elasticity separately would take the value close to one. As we can conclude from the estimated parameters, only fixed assets increased in the growth rate close to the production growth rate. Imports, particularly those from non-socialist countries, increased much faster than production. The estimator value of imports parameter expresses an increasing role of imports in the production process. This has been one of the reasons why production grew faster than employment.

We may also point out that production, fixed assets and imports are measured qualitatively in constant prices except employment. Categories expressed in constant prices are not clean completely from price growth and for that reason they are overestimated. If overestimation of imports, fixed assets and production is in the same range then the estimators will not be biased in one-factor production functions. We shall expect overestimation of the employment parameter.

Disadvantages of the bottleneck production model. Detection of the bottlenecks is related to the analytical form of the function. Because of this we are going to resign from the second step in which we link bottlenecks in the function presented on page 263.

Thus, we have to estimate two parameters for each production factor but assuming several, simple analytical forms. The choice of one of the analytical forms should be done on subsamples constructed from bottlenecks.

The next disadvantage is that it is not clear when to stop the elimination procedure. Another procedure considering year by year may be more useful. Let us analyze three cases (Figure 2).

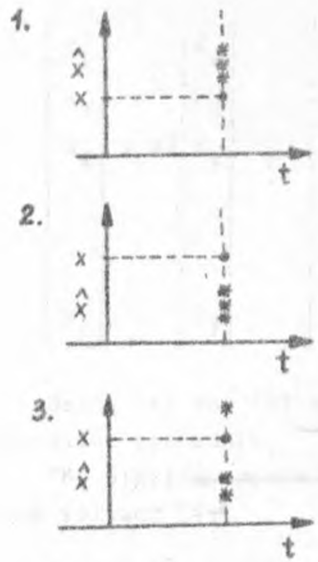
1. The estimated $\hat{X}$'s are above the actual X - all production factors were in excess, another production factor was a bottleneck.

2. The estimated $\hat{X}$'s are below the actual X - all production factors were a bottleneck.

3. The estimated $\hat{X}$'s are considerably above or below the actual X - we can distinguish production factors in excess from the bottleneck production factors.

It is possible to use this model in a forecast. In our example we shall get four forecasts. Their minimum value will point out the bottleneck production factor.

The bottleneck production models seem to be a new approach between the classical production function and the input-output model. From the classical production function we take a variety of analytical forms and high aggregation level. From the input-output model we take the assumption that there is no substitution between production factors.



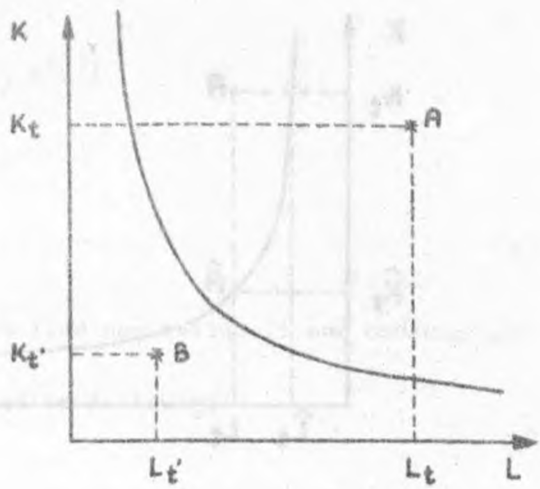


Fig. 2

Fig. 3

2. Approach with Substitution

Let us estimate the production function with substitution:

$$X = f(K, L, \varepsilon) \tag{3}$$

Let us assume that for a given production level X_t we have two observations A and B (Figure 3).

The point A is the observation with relatively large inputs of both production factors. The point B represents year with relatively small factor inputs. It means that now we are going to drop the points above substitution curve.

As in no substitution approach we will look for observations with the largest residuals and drop them. If we drop the observation A it will mean that neither capital nor labour were a bottleneck. Another production factor have to be a bottleneck in this period.

We can consider a case of one production factor excess (for example capital K; see Figure 4).

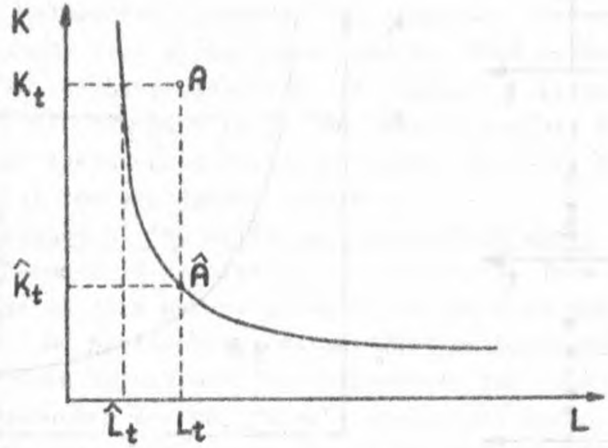


Fig. 4.

In such a case we postulate a model reduction of factor input from K_t to $\hat{K}_t$.

This approach can be formalize by reformulation of the estimated equation (3)

$$\hat{K} = f_K(X, L) \tag{4}$$

$$\hat{L} = f_L(X, K) \tag{4a}$$

$$\left.\begin{aligned} e_t^K &= K_t - \hat{K}_t \\ e_t^L &= L_t - \hat{L}_t \end{aligned}\right\} \quad t = 1, \ldots, T \tag{5}$$

Let us assume that on the basis of residuals (5) we find that in the first period capital was in excess, while in the second period labour was in excess and in the third one both production factors were in excess.

In such a case we have to reestimate equation (3) with following corrections of the sample:

$$X^{(1)} = f[K^{(1)}, L^{(1)}, \varepsilon^{(1)}]$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_4 \\ . \\ . \\ . \\ X_T \end{bmatrix} = f\left[\begin{bmatrix} \hat{K}_1 \\ K_2 \\ K_4 \\ . \\ . \\ . \\ K_T \end{bmatrix}, \begin{bmatrix} L_1 \\ \hat{L}_2 \\ L_4 \\ . \\ . \\ . \\ L_T \end{bmatrix}, \varepsilon^{(1)}\right] \quad (6)$$

Using (4) and (5) we can find new residuals and continue such iterative procedure.

The similar approach will be following:

from (3) and (4)

$$X_L = f(\hat{K}, L, \varepsilon)$$

and from (3) and (4a)

$$X_K = f(K, \hat{L}, \varepsilon)$$

Now we can use one factor approach. In both approaches one sided procedures for detecting outlying observations may be used.

We presume the bottleneck production model to be useful in modelling disequilibrium, forecasting and detecting plan inconsistence.

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MODEL WĄSKICH GARDEŁ PRODUKCJI Z SUBSTYTUCJĄ I BEZ SUBSTYTUCJI

W artykule opisano trzy sposoby modelowania wąskich gardeł procesu produkcji. W pierwszym, najprostszym założono, że nie występuje substytucja czynników produkcji. Oszacowano jednoczynnikowe funkcje produkcji na podpróbkach, w skład których wchodzą te obserwacje, w których czynnik produkcji nie był w nadmiarze. Związek przyczynowy pomiędzy nakładem a wynikiem występuje tylko wtedy, kiedy nakład ten stanowi wąskie gardło. Wąskogardłowe pod-

próby są budowane w procesie kolejnej eliminacji punktów poniżej linii regresji z największymi resztami.

W drugim podejściu zastosowano funkcje produkcji z dwoma lub więcej czynnikami. Nie rozpatrywano typowych reszt względem zmiennej endogenicznej, ale reszty względem każdego czynnika produkcji oddzielnie. Dla dużych reszt nakład czynnika produkcji jest zastępowany przez wartości generowane przez model.

W trzecim podejściu potencjalną produkcją jest to minimum z trzech funkcji produkcji:

$$Y = \min \quad F_1 \quad K,L, \qquad F_2 \; K,M_1, \ldots, M_m \qquad F_3 M_1, \ldots M_m$$

gdzie:

Y - granice możliwości produkcyjnych,

K - środki trwałe,

L - zatrudnienie,

M_i - nakłady i-tego materiału,

F_1 - funkcja skalarna.

F_2, F_3 - funkcje wektorowe.

Następnie przyjęto, że funkcje te są agregatem procesów technologicznych realizowanych bez substytucji nakładów oraz procesów z substytucją typu CES.

Pierwsze podejście zastosowano dla polskiego przemysłu, trzecie dla przemysłu ZSRR.