INTERACTIONS BETWEEN DISCONTINUITY SURFACES AND THERMO-ELECTRO-MAGNETOELASTIC WAVES

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1. INTRODUCTION

When a wave collides with the interface of two materials having distinct thermo-electro-magnetoelastic properties, various reflected and transmitted (refracted) waves are generated, which have different coupled fields from those of the incident wave. At the same time, discontinuities of the fields across the interface and the surface quantities are induced or influenced by the collision of the wave. General laws are presented which govern such interactions.

2. GENERAL BASIC EQUATIONS FOR THERMO-ELECTRO-MAGNETOELASTIC MATERIALS

In order to investigate intrinsic characters of the above phenomena, we start from a set of general basic equations. Let a singular surface S denote a discontinuity surface of the fields, a boundary surface or an interface. In the reference configuration, the basic equations can be expressed as, respectively, at a point except on S and at a point on S,

$$\dot{\Gamma}^\alpha+\mathrm{DIV}\,\boldsymbol{\Phi}^\alpha+K^\alpha=0\,,\qquad \dot{\Gamma}^\alpha_s+\mathrm{DIV}_s\boldsymbol{\Phi}^\alpha_s+K^\alpha_s+\boldsymbol{M}\cdot[\boldsymbol{\Phi}^\alpha]=0\,, \tag{1}$$

$$\dot{\boldsymbol{Z}}^\Gamma+\mathrm{ROT}\,\boldsymbol{\Psi}^\Gamma+\boldsymbol{L}^\Gamma=\boldsymbol{0}\,,\qquad \dot{\boldsymbol{Z}}^\Gamma_s+\mathrm{ROT}_s\boldsymbol{\Psi}^\Gamma_s+\boldsymbol{L}^\Gamma_s+\boldsymbol{M}\times[\boldsymbol{\Psi}^\Gamma]=\boldsymbol{0}\,, \tag{2}$$

$$\mathrm{DIV}\,\boldsymbol{Z}^\Gamma+M^\Gamma=0\,,\qquad \mathrm{DIV}_s\boldsymbol{Z}^\Gamma_s+M^\Gamma_s+\boldsymbol{M}\cdot[\boldsymbol{Z}^\Gamma]=0\,, \tag{3}$$

$$(\dot{H}+\mathrm{DIV}\,\boldsymbol{\Omega}+N=0\geqslant 0\,,\quad \dot{H}_s+\mathrm{DIV}_s\boldsymbol{\Omega}_s+N_s+\boldsymbol{M}\cdot[\boldsymbol{\Omega}]=0_s\geqslant 0)\,, \tag{4}$$

where Γ^α $(\alpha=1,\ldots,M)$ are scalars or components of vectors, $\boldsymbol{Z}^\Gamma$ $(\Gamma=1,\ldots,N)$ vectors, $\boldsymbol{M}$ a unit normal vector to S, cf. Pao [1] and Kosinski [2]. Eq. (4) corresponds to the second law of thermodynamics. A quantity with subscript s indicates a surface quantity such as the surface charge, the surface current, the surface entropy production. Let $\boldsymbol{a}=(\gamma^\alpha,\zeta^\Gamma)$ be the state variables $(\alpha=1,\ldots,M,$ $\Gamma=1,\ldots,N)$. A homogeneous thermo-electro-magnetoelastic material is defined by the constitutive equations for the quantities in (1)–(4) expressed in terms of the state variables $\boldsymbol{a}$.

3. WEAK DISCONTINUITY WAVE

Let us consider a plane weak discontinuity wave Σ which satisfies the conditions: (i) $\boldsymbol{a}$ has finite jump discontinuities in its first derivatives across Σ, (ii) $\boldsymbol{a}$ is uniformly constant over Σ at each instant, (iii) the surface quantities vanish on Σ. The compatibility conditions for a continuous function $f(\boldsymbol{X}, t)$ across Σ are

$$[\dot{f}] = -U\bar{f}, \quad [f_{,i}] = \bar{f}N_i, \quad \bar{f} = [f_{,i}]\,N_i, \tag{5}$$

where i denotes a derivative with respect to X^i, $[\cdot]$ the jump of a quantity, U the normal speed, $\bar{f}$ the amplitude and N the unit normal vector to Σ in the direction of propagation. Applying (5) to (1)–(3), we have

$$(-UP^{\Gamma'}_{\Delta'} + Q(N)^{\Gamma'}_{\Delta'})\,\bar{a}^{\Delta'} = 0, \quad (\Gamma', \Delta' = 1, \ldots, M+3N) \tag{6}$$

$$R(N)^{\Gamma}_{\Delta'}\,\bar{a}^{\Delta'} = 0, \tag{7}$$

where

$$\boldsymbol{P} = \begin{bmatrix} A^{\alpha}_{\beta} & B^{\alpha}_{\Delta k} \\ E^{\Gamma i}_{\beta} & F^{\Gamma i}_{\Delta k} \end{bmatrix}, \quad \boldsymbol{Q}(N) = \begin{bmatrix} C^{\alpha_i}_{\beta} N_i & D^{\alpha_i}_{\Delta k} N_i \\ \varepsilon^{ijk} G^{\Gamma}_{k\beta} N_j & \varepsilon^{ijl} H^{\Gamma}_{l\Delta k} N_j \end{bmatrix}, \quad \boldsymbol{R}(N) = [E^{\Gamma i}_{\beta} N_i \; F^{\Gamma i}_{\Delta k} N_i] \tag{8}$$

and $A - H$ are derivatives of the constitutive functions. In many cases, the matrix $\boldsymbol{P}$ has an inverse, and the normal speed U and the amplitude $\bar{\boldsymbol{a}}$ are determined by (6). For a wave $U \neq 0$, (7) is fulfilled automatically.

4. REFLECTION AND REFRACTION OF AN OBLIQUE INCIDENT WAVE

We consider the case where a plane wave collides with the interface between two materials. Let $\boldsymbol{M}$ be a unit normal vector to the interface, and $N_{\rm I}$, $U_{\rm I}$ and $\bar{\boldsymbol{a}}_{\rm I}$ the propagation direction, the speed and the amplitude of the incident wave. We assume that the surface quantities except K^{α}_s, L^{Γ}_s, M^{Γ}_s vanish and the distribution of the state variables is two-dimensional associated with the incidence plane spanned by $\boldsymbol{M}$ and $N_{\rm I}$. This imposes that the propagation direction $\boldsymbol{N}$ of any reflected or transmitted wave lies on the incidence plane. Let p be the number of all reflected waves and q the number of all transmitted waves. The speeds of the reflected and the transmitted waves are written as $U^{(1)}_{\rm R}, \ldots, U^{(p)}_{\rm R}, U^{(1)}_{\rm T}, \ldots, U^{(q)}_{\rm T}$. Other quantities are also distinguished by similar super- and subscripts.

Let us pay attention to a neighborhood of the intersection point of the interface and the incident wave in the incidence plane at $t=0$. It follows from a trivial identity that for any quantity f

$$[f]_{\rm I} + [f]_{\rm B} = \sum_{i=1}^{p} [f]^{(i)}_{\rm R} + \sum_{j=1}^{q} [f]^{(j)}_{\rm T} + [f]_{\rm B'}, \tag{9}$$

where a quantity on the interface without or with prime denotes its value at $t=-0$ and $t=+0$, respectively. Eq. (9) means that the total of the jumps of f is conserved just before and just after the collision of the wave. Applying the propagation conditions (6) and (7) to each wave and combining the result with the time derivatives of $(1)_1$–$(3)_1$, we have

$$\boldsymbol{Q}(\boldsymbol{M})^{-}\{U_{\mathrm{I}}\bar{\boldsymbol{a}}_{\mathrm{I}}-\sum_{i=1}^{p}U_{\mathrm{R}}^{(i)}\bar{\boldsymbol{a}}_{\mathrm{R}}^{(i)}\}+\boldsymbol{b}_{s}=\boldsymbol{Q}(\boldsymbol{M})^{+}\{\sum_{j=1}^{q}U_{\mathrm{T}}^{(j)}\bar{\boldsymbol{a}}_{\mathrm{T}}^{(j)}\}+\boldsymbol{b}'_{s}, \tag{10}$$

$$\boldsymbol{M}_{s}=\boldsymbol{M}_{s'}, \qquad \dot{\boldsymbol{M}}_{s}=\dot{\boldsymbol{M}}_{s'}, \tag{11}$$

where superscript − or + denotes a quantity for the first or the second material, respectively, and

$$\boldsymbol{b}=(K^{\alpha}, L^{\Gamma}). \tag{12}$$

In many theories of electromagnetic materials, surface quantities K_s^{α} and L_s^{Γ} are given by smooth functions of M_s^{Γ}. Then (11) implies that

$$\boldsymbol{b}=\boldsymbol{b}'_{s}, \qquad \boldsymbol{b}_{s}=\boldsymbol{b}'_{s}. \tag{13}$$

As in the case of linear waves in isotropic materials, we can derive a Snell law for thermo-electro-magnetoelastic waves

$$\frac{U_{\mathrm{I}}}{\sin\theta_{\mathrm{I}}}=\frac{U_{\mathrm{R}}^{(i)}}{\sin\theta_{\mathrm{R}}^{(i)}}=\frac{U_{\mathrm{T}}^{(j)}}{\sin\theta_{\mathrm{T}}^{(j)}}, \quad (i=1,\dots,p, \quad j=1,\dots,q) \tag{14}$$

where θ is the angle made by $\boldsymbol{N}$ and $\boldsymbol{M}$, cf. Borejko [3] for elastic materials. Since U depends on θ, (14) cannot determine θ. To do it, we write the propagation direction $\boldsymbol{N}$ of a wave as

$$\boldsymbol{N}=\cos\theta_{\mathrm{I}}\boldsymbol{M}+(U\sin\theta_{\mathrm{I}}/U_{\mathrm{I}})\boldsymbol{L}, \tag{15}$$

where $\boldsymbol{L}$ is the unit tangential vector to the interface which makes a sharp angle with $\boldsymbol{N}_{\mathrm{I}}$. Substituting (15) into (6), we have

$$\{-\lambda(\boldsymbol{P}-\sin\theta_{\mathrm{I}}/U_{\mathrm{I}}\boldsymbol{Q}(\boldsymbol{L}))+\boldsymbol{Q}(\boldsymbol{M})\}\bar{\boldsymbol{a}}=\boldsymbol{0}, \tag{16}$$

where $\lambda=U/\cos\theta$. Thus (16) determines λ and the amplitudes of the reflected and the transmitted (refracted) waves. The amplitudes have ambiguity with respect to scalar multiplication. With the aid of (14), the speed of a wave is then given by

$$U^{2}=\lambda^{2}/(1+\lambda^{2}\sin^{2}\theta_{\mathrm{I}}/U_{\mathrm{I}}^{2}) \tag{17}$$

and the propagation direction is obtained by substituting (17) into (15). The magnitude of the amplitude of each wave can be determined by (10)–(13).

As a result, the amplitudes of the reflected waves are influenced by the tensor $\boldsymbol{Q}(\boldsymbol{M})^{+}$ for the second material. Thus we can get information on the properties or the state of the material on the other side by observing the amplitudes of the reflected waves.

5. EXAMPLE OF A THERMO-MAGNETOELASTIC MATERIAL

A linear thermo-magnetoelastic material proposed by Kaliski [4] is governed by the field equations

$$\begin{gathered} \rho\dot{\boldsymbol{v}}=\operatorname{div}\boldsymbol{\sigma}+\boldsymbol{j}\times\boldsymbol{B}_0, \quad \rho T_0\dot{s}=-\operatorname{div}\boldsymbol{q}, \quad \tau\dot{\boldsymbol{q}}+\boldsymbol{q}=-K\operatorname{grad}T+\pi\boldsymbol{j}, \\ \dot{\boldsymbol{D}}=\operatorname{rot}\boldsymbol{H}-\boldsymbol{j}, \quad \dot{\boldsymbol{B}}=-\operatorname{rot}\boldsymbol{E}, \quad \operatorname{div}\boldsymbol{D}=0, \quad \operatorname{div}\boldsymbol{B}=0 \end{gathered} \tag{18}$$

and the constitutive equations

$$\begin{gathered} \boldsymbol{\sigma}=2G\boldsymbol{e}+\lambda(\operatorname{tr}\boldsymbol{e})\mathbf{1}-\alpha_0 T\mathbf{1}, \quad \boldsymbol{B}=\mu\boldsymbol{H}, \quad \boldsymbol{D}=\varepsilon(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}_0), \\ \boldsymbol{j}=(\eta/\varphi)(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}_0)+\kappa/(K\varphi)\boldsymbol{q}, \quad \rho s=\alpha_0(\operatorname{tr}\boldsymbol{e})+\beta_0 T, \end{gathered} \tag{19}$$

where $\boldsymbol{e}$ is the infinitesimal strain, $\varphi=1+\kappa\pi/K$, $\boldsymbol{H}$ is the magnetic field, $\boldsymbol{j}$ the electric current vector, $\boldsymbol{B}$ magnetic induction field, $\boldsymbol{E}$ the electric field, $\boldsymbol{D}$ the electric induction field, $\boldsymbol{u}$ the displacement, $\boldsymbol{\sigma}$ the stress, $\boldsymbol{q}$ the heat flux, T the temperature, s the entropy. We assume that both materials are governed by the above equations but the material constants are different. The state variables are

$$\boldsymbol{a}=(\boldsymbol{v},\boldsymbol{e},\boldsymbol{q},T,\boldsymbol{E},\boldsymbol{H}). \tag{20}$$

We can show that there may exist four kinds of waves in each material, i.e., an electroacoustic wave, a fast and a slow thermoacoustic waves and an electromagnetic wave.

Oblique incident electroacoustic wave

Let us consider an incident electroacoustic wave which satisfies

$$\bar{\boldsymbol{v}}_{\mathrm{I}}=(0,0,\bar{v}_3), \quad \bar{\boldsymbol{E}}_{\mathrm{I}}=B_0\bar{v}_3\sin(\theta_B-\theta_{\mathrm{I}})\boldsymbol{N}_{\mathrm{I}}, \tag{21}$$

where the incidence plane is X_1-X_2 plane and the interface is $X_1=0$. A reflected and a refracted electroacoustic waves are generated.

$$\bar{\boldsymbol{v}}_{\mathrm{R}}^{(\mathrm{EA})}=\frac{G^+/\lambda_{\mathrm{T}}^{(\mathrm{EA})}-G^-/\lambda_{\mathrm{I}}}{G^+/\lambda_{\mathrm{T}}^{(\mathrm{EA})}+G^-\lambda_{\mathrm{I}}}\bar{\boldsymbol{v}}_{\mathrm{I}}, \quad \bar{\boldsymbol{v}}_{\mathrm{T}}^{(\mathrm{EA})}=\frac{2G^-U_{\mathrm{I}}/(\lambda_{\mathrm{I}}U_{\mathrm{T}}^{(\mathrm{EA})})}{G^+/\lambda_{\mathrm{T}}^{(\mathrm{EA})}+G^-/\lambda_{\mathrm{I}}}\bar{\boldsymbol{v}}_{\mathrm{I}}. \tag{22}$$

If the incidence angle is small, a reflected and a refracted electromagnetic waves are also generated. We have

$$\bar{\boldsymbol{H}}_{\mathrm{R}}^{(\mathrm{EM})}=\frac{2\rho^-U_{\mathrm{I}}^2H_0\sin\theta_{\mathrm{B}}(\mu^+\sin\theta_{\mathrm{T}}^{(\mathrm{EA})}-\mu^-\sin\theta_{\mathrm{I}})}{U_{\mathrm{R}}^{(\mathrm{EM})}(\rho^+U_{\mathrm{T}}^{(\mathrm{EA})}+\rho^-U_{\mathrm{I}})(\mu^+U_{\mathrm{T}}^{(\mathrm{EM})}\cos\theta_{\mathrm{T}}^{(\mathrm{EM})}-\mu^-U_{\mathrm{R}}^{(\mathrm{EM})}\cos\theta_{\mathrm{R}}^{(\mathrm{EM})})}\bar{\boldsymbol{v}}_{\mathrm{I}}. \tag{23}$$

The amplitudes of the other fields can be calculated from (22) and (23). The electromagnetic waves are not induced when the magnetic field is perpendicular to the interface or when the incidence angle is not small.

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