

# THE EFFECTS OF STRESSES ON THE PROPERTIES OF DOMAIN WALLS IN MAGNETOELASTIC THIN FILMS

SHINYA MOTOGI

Osaka Municipal Technical Research Institute 1-6-50, Morinomiya Joto-ku,  
Osaka 536, Japan

## 1. INTRODUCTION

In magnetoelastic materials, the application of mechanical stresses can have non-negligible effects on some properties of magnetic domain walls. We have already analyzed the effects of magnetostriction and external stresses on the equilibrium distribution of magnetization and the vibration spectrum of Bloch walls in bulk materials [1, 2]. In this paper, we shall examine the case of a Bloch wall in thin films or ribbons. This study aims at basic understanding of domain wall properties when magnetoelastic materials are applied to electronic components such as bubble devices. In Section 2, we summarize the governing equations based on the theory of magnetoelastic interaction by Brown [3] and Tiersten [4]. Section 3 is devoted to the analysis of the influence of stress on the equilibrium distribution of magnetization in a Bloch wall in thin films. We study in Section 4 the change in resonance frequency of wall vibration due to the application of external stresses.

## 2. BASIC EQUATIONS

In ferromagnetic materials with magnetoelastic interactions, the magnetization precession is governed by (Tiersten [4])

$$\frac{\partial \boldsymbol{\mu}}{\partial t} = \gamma \boldsymbol{\mu} \times (\mathbf{H} + \mathbf{H}^{\text{an}} + \mathbf{H}^{\text{ex}}) \quad (1)$$

where  $\boldsymbol{\mu}$  the magnetization per unit mass,  $\gamma$  the gyromagnetic ratio,  $\mathbf{H}$  the Maxwellian magnetic field,  $\mathbf{H}^{\text{an}}$  the anisotropy field,  $\mathbf{H}^{\text{ex}}$  the exchange field. The constitutive quantities  $\mathbf{H}^{\text{an}}$  and  $\mathbf{H}^{\text{ex}}$  can be derived from a potential, the free energy  $\Sigma$ .

$$\mathbf{H}^{\text{an}} = -\frac{1}{M_s} \frac{\partial \Sigma}{\partial \boldsymbol{\alpha}}, \quad \mathbf{H}^{\text{ex}} = \frac{1}{M_s} \nabla \cdot \left( \frac{\partial \Sigma}{\partial \nabla \boldsymbol{\alpha}} \right) \quad (2)$$

where  $M_s = \rho_R \mu_s$  ( $\rho_R$  and  $\mu_s$  are the mass density in the reference state and the saturation magnetization per unit mass, respectively) and  $\alpha$  is the direction cosine of the magnetization which is defined by  $\alpha = \mu/\mu_s$ . The material symmetry is assumed to be uniaxial cubic. The free energy is given by the sum of the exchange, the magnetic anisotropy, the magnetoelastic, the elastic energy, and the interaction energy between some kind of crystal defects and the Bloch wall. We denote in the following equations  $\lambda$  the exchange,  $K$  the anisotropy,  $B$ 's the magnetoelastic,  $C$ 's the elastic and  $\bar{K}$  the interaction constants.

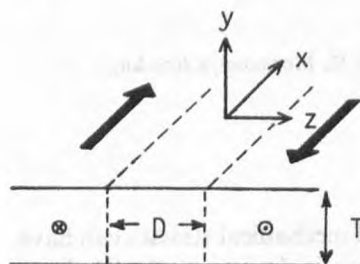


Fig. 1

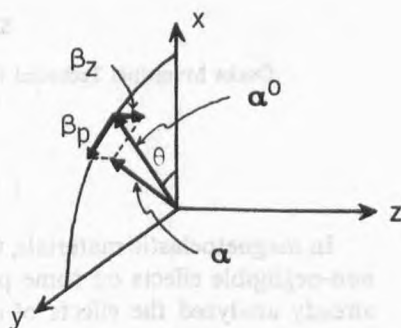


Fig. 2

We consider an isolated Bloch wall in a thin film with thickness  $T$  (Fig. 1). The equilibrium magnetization is denoted by  $\alpha^0$ . We can define the perturbation components  $\beta_p$  and  $\beta_z$  as in Fig. 2. The magnetization equation is reduced to in equilibrium state:

$$\alpha^0 \times H^{\text{eff}} = 0 \quad (3)$$

in perturbed state:

$$\frac{\partial \beta}{\partial t} = \gamma (\beta \times H^{\text{eff}} + \alpha^0 \times h^{\text{eff}}) \quad (4)$$

where  $H^{\text{eff}}$  and  $h^{\text{eff}}$  are the effective fields in equilibrium and dynamic perturbed states, respectively.

It is a fairly complicated problem to obtain analytically the demagnetizing field in a Bloch wall in thin films. We follow here the approximate method by Néel ([5]), which assumes that the cross-section of a Bloch wall is an ellipse, whose major axes are  $T$  and  $D$ , which is uniformly magnetized in the  $y$  direction.  $D$  is the width of the Bloch wall. If concerned with the dynamic component  $\beta_z$ , it gives rise to a demagnetizing field in the  $z$  direction, which can be calculated by Néel's approximation for Néel wall. Therefore the demagnetizing fields are given by

$$H_y = -N_1 M_y = -4\pi \frac{D}{T+D} M_s \alpha_y, \quad h_z = -N_2 M_s \beta_z = -4\pi \frac{T}{T+D} M_s \beta_z. \quad (5)$$

## 3. EQUILIBRIUM DISTRIBUTION OF MAGNETIZATION

Assuming that the angle  $\theta$  between the  $x$  axis and the direction of magnetization depends only on  $z$ , the equation (3) is reduced to

$$\lambda \frac{d^2\theta}{dz^2} - [K + M_s^2 N_1 - 2B_1(e_{xx}^0 - e_{yy}^0)] \sin\theta \cos\theta + 2B_2(\sin^2\theta - \cos^2\theta) e_{xy}^0 = 0 \quad (6)$$

where  $e^0$  is the total initial strain which is split into magnetostriction  $e^{ms}$  and the strain  $e^t$  due to external stresses. Referring magnetostriction to Motogi & Maugin [1], we can rewrite eq. (6):

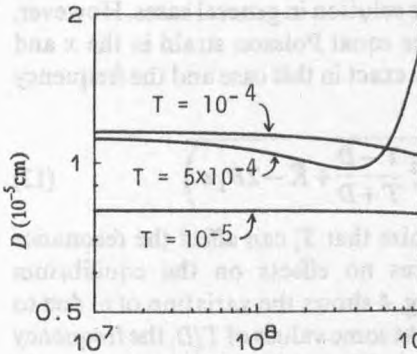
$$\lambda \frac{d^2\theta}{dz^2} - [K + M_s^2 N_1 + \frac{2B_1^2}{C_{11} - C_{12}} - 2B_1(e_{xx}^t - e_{yy}^t)] \sin\theta \cos\theta - 2B_2 \cos 2\theta e_{xy}^t = 0. \quad (7)$$

In view of eq. (7), magnetostriction and the normal components of  $e^t$  can induce uniaxial anisotropy. The existence of the shear component is quite interesting since it induces another type of anisotropy, however, we shall leave the analysis to other occasions. Here we assume that  $e_{xy}^t$  is null and that the strain due to the stresses is uniform, so that eq. (7) gives rise to the Landau-Lifshitz type distribution:

$$\sin\theta = \text{sech}(z/\delta), \quad \cos\theta = -\tanh(z/\delta) \quad (8)$$

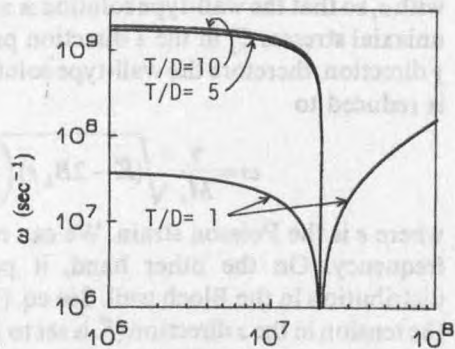
where the width parameter  $\delta$  is defined by

$$\delta = \left\{ \lambda / [K + M_s^2 N_1 + \frac{2B_1^2}{C_{11} - C_{12}} - 2B_1(e_{xx}^t - e_{yy}^t)] \right\}^{1/2}. \quad (9)$$



STRESS  $T_x$  (dyne/cm<sup>2</sup>)

Fig. 3



STRESS  $T_x$  (dyne/cm<sup>2</sup>)

Fig. 4

The demagnetizing factor in eq. (9) contains the wall width  $D$ , hence equating  $\delta$  as  $D/\pi$ , we obtain the relation between the film thickness  $T$  and the wall width  $D$ . Fig. 3 shows the variation of  $D$  with respect to the tension in the  $x$  direction with  $T$  the parameters. The material constants are assumed to be  $\lambda = 10^{-6}$ ,  $K = 10^4$ ,  $M_s = 500$ ,  $B_1 = -5 \times 10^7$ ,  $C_{11} = 10^{12}$ ,  $C_{12} = 0.5 \times 10^{12}$ , all in cgs unit. When the film thickness is small, the change in  $D$  due to stress is also small because the effect of demagnetizing field, which is produced by magnetic surface charge, is dominant. However, when  $T$  is rather large, the demagnetizing effect is negligible, therefore the stress can significantly change the wall width.

#### 4. FREE VIBRATION OF A BLOCH WALL

We consider the free vibration of a Bloch wall. Discarding the dynamic strain, eq. (4) is reduced to

$$\frac{M_s}{\gamma} \frac{\partial \beta_z}{\partial t} = \lambda \frac{\partial^2 \beta_p}{\partial z^2} - K' \cos 2\theta \beta_p - \bar{K} \beta_p + 2 [B_1 (\cos^2 \theta e_{xx}^t + \sin^2 \theta e_{yy}^t) + 2B_2 \sin \theta \cos \theta e_{xy}^t] \beta_p \quad (10)$$

$$\frac{M_s}{\gamma} \frac{\partial \beta_p}{\partial t} = -\lambda \frac{\partial^2 \beta_z}{\partial z^2} + K' \cos 2\theta \beta_z + \bar{K} \beta_z + M_s^2 (N_2 - N_1) \beta_z - 2B_1 [(\cos^2 \theta - \sin^2 \theta) e_{xx}^t + 2 \sin^2 \theta e_{yy}^t] \beta_z \quad (11)$$

where,  $N_2 = 4\pi T/(T+D)$ ,  $K' = K + M_s^2 N_1 + 2B_1^2/(C_{11} - C_{12})$ , and  $\bar{K}$  already includes the effects of magnetostriction. Substituting the wall-type solution which is proportional to  $e^{-i\omega t} \sin \theta$  into  $\beta_p$  and  $\beta_z$ , the resonance frequency is obtained as

$$\omega^2 = \left( \frac{\gamma}{M_s} \right)^2 [\bar{K} - 2B_1 (e_{xx}^t \sin^2 \theta + e_{yy}^t \cos^2 \theta)] [M_s^2 (N_2 - N_1) + \bar{K} - 2B_1 e_{yy}^t] \quad (12)$$

where we have assumed that  $e_{xy}^t$  is zero. The frequency  $\omega$  is not constant but varies with  $z$ , so that the wall-type solution is no true solution in general cases. However, uniaxial stresses  $T_z$  in the  $z$  direction produce equal Poisson strain in the  $x$  and  $y$  direction, therefore the wall-type solution is exact in this case and the frequency is reduced to

$$\omega = \frac{\gamma}{M_s} \sqrt{(\bar{K} - 2B_1 e) \left( 4\pi M_s^2 \frac{T-D}{T+D} + \bar{K} - 2B_1 e \right)} \quad (13)$$

where  $e$  is the Poisson strain. We can recognize that  $T_z$  can affect the resonance frequency. On the other hand, it produces no effects on the equilibrium distribution in the Bloch wall. See eq. (7). Fig. 4 shows the variation of  $\omega$  due to the tension in the  $z$  direction ( $\bar{K}$  is set to  $10^3$ ). At some values of  $T/D$ , the frequency decreases to zero, therefore further application of the tension will produce certain instability of wall vibration. However, when  $T$  and  $D$  are very close, the demagnetizing effect is so small that the frequency rises up again with the tension.

## REFERENCES

- [1] S. Motogi & G.A. Maugin, *phys. stat. solidi (a)* **81**, 519 (1984).
- [2] S. Motogi, *Science and Industry (Osaka)* **58**, 313 (1984) (in Jpn).
- [3] W.F. Brown, Jr., *Magnetoelastic Interactions*, Springer-Verlag, 1966.
- [4] H.F. Tiersten, *J. Math. Phys.* **5**, 1298 (1964) and **6**, 779 (1965).
- [5] R.F. Soohoo, *Magnetic Thin Films*, Harper and Row, New York, 1965.