ACTA UNIVERSITATIS LODZIENSIS
FOLIA MATHEMATICA 4, 1991

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ON A K.M. GARG'S PROBLEM IN RESPECT TO DARBOUX FUNCTIONS

There is considered the problem 3.11 from [3] by K. M. Garg in the class of Darboux functions.

In his paper [2] K. M. Garg has proved that every Darboux function $f: \mathbb{R} \to \mathbb{R}$ is continuous with more abstract domains In [3] the following question has been put: under which assumptions with respect to X and f a connected function $f: X \to \mathbb{R}$ is monotone, or weakly monotone, relatively to the set $\overline{S_c(f)}$ in general. Some partial answers can be found in [4], [5], [6] and [7].

As an immediate consequence of the mentioned papers we obtain:

THEOREM. Every connected function $f: I^2 \to \mathbb{R}$ is continuous and weakly monotone on $\overline{S_c(f)}$.

In the face of the Theorem 2 of [2] it is natural to set the question: will the above theorem remain valid for the wider class of Darboux functions i.e. the functions mapping arcs onto connected sets (see [8])? Since the answer is negative (Theorem 1) it is natural to look for additional assumption under which, the function $f|_{\overline{S_c(f)}}$ could be continuous and weakly monotone. As a result we obtain Theorems 2 and 3.

Within the whole paper we use well-known traditional symbols and notation, the same like in [1], [2] or [8]. In particular $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{N}$ and I denote the set of: real numbers, rational numbers, natural numbers and the closed interval $[0, 1]$, respectively. If $a, b \in \mathbb{R}$ the symbol (a, b) denotes an open interval with end points a and b ($a > b$ is also possible).

Let A, B be subsets of a topological space X such that $A \subset B$. Then the closure of A in the subspace B will be denoted by $\mathrm{cl}_B(A)$ and also by $\overline{A}$ if $B = X$. For every net $\{x_\sigma\}_{\sigma\in\Sigma}$ in X the symbols $A - \lim\limits_{\sigma\in\Sigma} x_\sigma$ and $A - \operatorname{acp}\limits_{\sigma\in\Sigma} x_\sigma$ will mean the sets of limit points and accumulation points belonging to A, respectively; if $A = X$, we write shortly: $\lim\limits_{\sigma\in\Sigma} x_\sigma$ and $\operatorname{acp}\limits_{\sigma\in\Sigma} x_\sigma$. Let f be a mapping of a topological space X into a topological space Y, A be a subset of X, $\{x_\sigma\}_{\sigma\in\Sigma}$ be a net in X. We say that the above net is f-agree with A, if $\overline{A} - \operatorname{acp}\limits_{\sigma\in\Sigma} x_\sigma \neq \emptyset$ or $\overline{f(A)} - \operatorname{acp}\limits_{\sigma\in\Sigma} f(x_\sigma) \neq \emptyset$. The combination of mappings f_1 and f_2 is denoted by $f_1 \nabla f_2$ (see [1]). We say that X is σ-coherent (see [3]), if the intersection of every decreasing sequence of closed connected sets in X is connected.

Let $f: X \to Y$ be a mapping where X, Y are Hausdorff spaces. We say that $y_o \in Y$ is a limit element or cluster element of the function f at a point $x_o \in X$, if there exists a net $\{x_\sigma\}_{\sigma\in\Sigma} \subset X \setminus \{x_o\}$ such that $x_o = \lim\limits_{\sigma\in\Sigma} x_\sigma$ and $y_o = \lim\limits_{\sigma\in\Sigma} f(x_\sigma)$. We denote by $L(f, x_o)$ the set of all cluster elements of f at x_o.

We say that $y_o \in Y$ is a (*) - limit element or (*) - cluster element of a function f at x_o, if $y_o \in L(f, x_o)$ and, moreover, if $\{x_\sigma\}_{\sigma\in\Sigma}$ is a net such that $x_o = \lim\limits_{\sigma\in\Sigma} x_\sigma$ and $y_o = \lim\limits_{\sigma\in\Sigma} f(x_\sigma)$, then there exists $\sigma_o \in \Sigma$ such that $f(x_\sigma) = y_o$ for $\sigma \geq \sigma_o$. We denote by $L^*(f, x_o)$ the set of all (*) - limit elements of f at x_o.

We say that f is a closed function at the point $x_o \in X$, if $L(f, x_o) \setminus \{f(x_o)\} \subset L^*(f, x_o)$ (see [9]).

According to the notation in [3] for any function $f: X \to \mathbb{R}$ we write:

$Y_c(f) = \{\alpha \in f(X): f^{-1}(\alpha)$ is a connected set$\}$,

$S_c(f) = f^{-1}(Y_c(f))$.

We say that a function $f: X \to \mathbb{R}$ is weakly monotone if $S_c(f) = X$. It is weakly monotone on $A \subset X$ if $f_{|A}$ is a weakly monotone function.

The paper is based on the continuum hypothesis.

THEOREM 1. There exists a Darboux function $f: I^2 \to \mathbb{R}$ such that $f_{|\overline{S_c(f)}}$ is not a continuous function and it is not weakly monotone.

P r o o f. Let $A_o = \{(x, y) \in I^2: x = 0 \text{ or } y = \frac{1}{4}\sin\frac{1}{x} + \frac{1}{2}\}$. Then $A_o = \overline{A}_o$. Denoting by $A_\alpha = \{\beta \in I^2: \rho(\beta, A_o) = \alpha\}$ (for a positive number α) we easily obtain $I^2 = \bigcup_{\alpha \geq 0} A_\alpha$. Let us define the function $f_o: A_o \to \mathbb{R}$ by setting

$$f_o((x, y)) = \begin{cases} y & \text{when } x = 0 \text{ and } y \in [0, \frac{1}{2}] \\ 1-y & \text{when } x = 0 \text{ and } y \in [\frac{1}{2}, 1] \\ 1 & \text{when } x \neq 0 \text{ and } y = \frac{1}{4}\sin\frac{1}{x} + \frac{1}{2} \end{cases}$$

In the family of sets $\{A_\alpha: \alpha > 0\}$ we can define an equivalence relation in the following way:

$A_\alpha * A_\beta \iff \alpha - \beta \in \mathbb{Q}$.

Let $\mathcal{P}$ be the family of all classes of abstraction for that relation $*$ and let φ be a one-to-one map of $\mathcal{P}$ onto $(-\infty, 1)$.

Write $f_1(x) = \varphi([A_{\alpha_x}])$, where A_{α_x} denote the set of the family $\{A_\alpha: \alpha > 0\}$, to which x belongs; $[A_{\alpha_x}]$ - the class of abstraction determined by A_{α_x}, and $f_1: I^2 \setminus A_o \to \mathbb{R}$. An important fact is that $f = f_o \triangledown f_1: I^2 \to \mathbb{R}$ is a Darboux function. Indeed, let $Ł \subset I^2$ be an arbitrary arc. There are three possible cases:

1) $Ł \subset A_o$, then it is obvious that $f(Ł)$ is connected;

2) $Ł \subset A_\alpha$ for $\alpha > 0$, then $f(Ł)$ is a one-element set;

3) there exist $\alpha_1, \alpha_2 \in [0, 1]$ such that $\alpha_1 \neq \alpha_2$ and $Ł \cap A_{\alpha_1} \neq \emptyset \neq Ł \cap A_{\alpha_2}$, then the definition of the function f implies that:

$$f(Ł) = \begin{cases} (-\infty, 1) & \text{when } Ł \cap [A_o \setminus \{(x, y): x = 0\}] = \emptyset \\ (-\infty, 1] & \text{when } Ł \cap [A_o \setminus \{(x, y): x = 0\}] \neq \emptyset. \end{cases}$$

It is easily seen, that $S_c(f) = \{(x, y) \in I^2: x > 0$ and $y = \frac{1}{4}\sin\frac{1}{x} + \frac{1}{2}\}$, whence $A_o \setminus \{(x, y) \in I^2: x = 0$ and $y \in (0, \frac{1}{4}) \cup (\frac{3}{4}, 1)\} \subset \overline{S_c(f)}$. Thus, really $f|_{\overline{S_c(f)}}$ is not a continuous function (at the point $(0, \frac{1}{2})$) and it is not a weakly monotone function (for example $f^{-1}|_{\overline{S_c(f)}}(\frac{1}{4}) = \{(0, \frac{1}{4}), (0, \frac{3}{4})\}$).

We shall introduce now some new definitions which are needed to answer the question put in the introduction.

DEFINITION 1. A function $f: X \to Y$ is said to satisfy a condition (D_1) in respect to a set $A \subset X$ (we denote it by: $f \in D_1^A$) if, for any net $\{x_\sigma\}_{\sigma\in\Sigma}$ and for any $x_o \in A - \lim_{\sigma\in\Sigma} x_\sigma$, $f|_{\{x_\sigma : \sigma\in\Sigma\} \cup \{x_o\}}$ is a closed function at x_o.

DEFINITION 2. A function $f: X \to Y$ is said to satisfy a condition (D_2) in respect to a set $A \subset X$ (denote: $f \in D_2^A$), if for any net $\{x_\sigma\}_{\sigma\in\Sigma} \subset A$ such that $A - \operatorname{acp}_{\sigma\in\Sigma} x_\sigma = \emptyset$ and for any arbitrary $B = \overline{B} \subset \overline{\{x_\sigma: \sigma \in \Sigma\}}$ the set $f(A \cap B)$ is closed in $f(A)$ (as a subspace of the space Y).

DEFINITION 3. A function $f: X \to Y$ is said to satisfy a condition (D_3) in respect to a set $A \subset X$ (denote: $f \in D_3^A$) if for any f-agree with A net $\{x_\sigma\}_{\sigma\in\Sigma}$ such that $A - \operatorname{acp} x_\sigma = \emptyset$ and for any closed in $\{x_\sigma: \sigma \in \Sigma\}$ set B (as a subspace of the space X), the set $f(B)$ is closed in $f(A \cup \{x_\sigma: \sigma \in \Sigma\})$ (as a subspace of the space Y).

For a function $f: X \to Y$, we also write $f \in D_i$ if $f \in D_i^X$ ($i = 1, 2, 3$).

It is supposed throughout this paper that X is a arcwise connected and locally arcwise connected T_5-space [1] and $f: X \to \mathbb{R}$ is a Darboux function. To simplify the notation we shall write $f \in D_iD_j$, whenever $f \in D_i$ and $f \in D_j$.

THEOREM 2. Let X be σ-coherent space. If $f \in D_1D_2$, then f is continuous and weakly monotone on $\overline{S_c(f)}$.

THEOREM 3. Let $f \in D_3$. A function f is continuous and weakly monotone if and only if $f \in D_1$.

At first we shall prove the following lemma:

LEMMA. A function $f \in D_1$ if and only if f is a continuous function on $\overline{S_c(f)}$.

P r o o f. Necessity. We have to prove that f is a continuous function on $\overline{S_c(f)}$. Suppose that it is not so. Then there is a point $x_o \in S_c(f)$ such that f is discontinuous at x_o. It means that there exists $\varepsilon > 0$ such that

$f(x_o) \in f(V) \not\subset (f(x_o) - \varepsilon,\ f(x_o) + \varepsilon)$ for any neighbourhood V of the point x_o (1)

Now let us denote by $B(x_o)$ a base of the space X at the point x_o. For each $U \in B(x_o)$ there exist an open set V_U such that $x_o \in V_U \subset U$ and

for any $y \in V_U$ there exists an arc $Ł = L(x_o, y)$ such that $Ł \subset U$ (2)

In face of (1) and (2) we come to conclusion that for any $U \in B(x_o)$ there exists an arc $Ł \subset U$ such that $f(x_o) \in f(Ł) \not\subset$ $\not\subset (f(x_o) - \varepsilon, f(x_o) + \varepsilon)$. Since f is a Darboux function, for every $U \in B(x_o)$ we have

$(f(x_o) - \varepsilon, f(x_o)] \subset f(U)$ or $[f(x_o), f(x_o) + \varepsilon] \subset f(U)$ (3)

Let $\Sigma = \{(U, n): U \in B(x_o)$ and $n \in \mathbb{N}\}$. Now we define a relation $\leqq$, which direct the set Σ as follow

$(U, n) \leqq (U_1, n_1) \iff U \supset U_1$ and $n \leqq n_1$.

The symbol $\leqq$ between two natural numbers means a common relation (less or equal). Selecting for each $\sigma = (U, n) \in \Sigma$ an element x_σ belonging to $U \cap (f^{-1}((f(x_o) - \varepsilon, f(x_o) - \frac{n}{n+1}\varepsilon)) \cup f^{-1}((f(x_o) + \frac{n}{n+1}\varepsilon,\ f(x_o) + \varepsilon)))$ we define a net $\{x_\sigma\}_{\sigma\in\Sigma}$ such that $x_o = \lim\limits_{\sigma\in\Sigma} x_\sigma$ (it is possible due to (3)). Write:

$$N_1 = \{n \in \mathbb{N}: x_\sigma \in f^{-1}((f(x_o) - \varepsilon,\ f(x_o) - \tfrac{n}{n+1}\varepsilon))\},$$

$$N_2 = \{n \in \mathbb{N} : x_\sigma \in f^{-1}((f(x_o) + \frac{n}{n+1}\varepsilon,\ f(x_o) + \varepsilon))\}.$$

Then at least one of these sets is infinite (it may always be assumed that card $N_1 \geq \aleph_o$).

Let $\Sigma_1 = \{(U, n): U \in B(x_o)$ and $n \in N_1\}$. Then Σ_1 is cofinal in Σ, therefore $\{x_\sigma\}_{\sigma\in\Sigma_1}$ is a subnet of $\{x_\sigma\}_{\sigma\in\Sigma}$, hence $x_o = \lim_{\sigma\in\Sigma_1} x_\sigma$, moreover $f(x_o) - \varepsilon = \lim_{\sigma\in\Sigma_1} f(x_\sigma)$ and $f(x_o) - \varepsilon \notin L^*(f_{|\{x_\sigma:\sigma\in\Sigma_1\}\cup\{x_o\}}, x_o)$, but it means that $f \notin D_1$.

Sufficiency. Let $\{x_\sigma\}_{\sigma\in\Sigma}$ be an arbitrary net such that $x_o \in \overline{S_c(f)} - \lim_{\sigma\in\Sigma} x_\sigma$. Than $f_{|\{x_\sigma:\sigma\in\Sigma\}\cup\{x_o\}}$ is continuous at the point x_o, hence

$$L(f_{|\{x_\sigma:\sigma\in\Sigma\}\cup\{x_o\}}, x_o) \setminus \{f(x_o)\} = \emptyset \subset L^*(f_{|\{x_\sigma:\sigma\in\Sigma\}\cup\{x_o\}}, x_o).$$

P r o o f. of Theorems 2 and 3. In face of the above lemma it is sufficient to show that $f_{|\overline{S_c(f)}}$ is weakly monotone (if only X is σ-coherent and $f \in D_1D_2$ or $f \in D_1D_3$). First we shall prove that:

(*) if α is a two-sided point of accumulation of the set $Y_c(f)$, then $f^{-1}(\alpha) \cap \overline{S_c(f)}$ is a connected set.

Indeed, let $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty} \subset Y_c(f)$ be arbitrary sequences satisfying the conditions $\alpha_n \nearrow \alpha \swarrow \beta_n$. Then by the lemma $f^{-1}(\alpha_n)$, $f^{-1}(\beta_n)$ are closed sets in $S_c(f)$ for $n = 1, 2, \ldots$, whence they are also closed in X.

Now we shall prove that:

(**) $f^{-1}((\alpha_n, +\infty))$, $f^{-1}((-\infty, \alpha_n))$ are open sets in X for $n = 1, 2, \ldots$.

Let $x_o \in f^{-1}((\alpha_n, +\infty))$. Since $f^{-1}(\alpha_n)$ is a closed set, there exists a neighbourhood V_{x_o} of the point x_o such that: given any $y \in V_{x_o}$ there exists an arc $Ł = L(x, y)$ such that $Ł \cap f^{-1}(\alpha_n) \neq \emptyset$. Since f is a Darboux function we have $V_{x_o} \subset f^{-1}((\alpha_n, +\infty))$, which leads to the conclusion that $f^{-1}((\alpha_n, +\infty))$ is an open set.

Arguing similarly as above we can state that $f^{-1}((-\infty, \alpha_n))$ is also an open set. In face of (**) we may conclude that

$f^{-1}((-\infty, \alpha_n])$, $f^{-1}([\alpha_n, +\infty))$ are closed sets.

Moreover $f^{-1}((-\infty, \alpha_n]) \cup f^{-1}([\alpha_n, +\infty)) = X$ is closed, arcwise connected set $(n = 1, 2, \ldots)$ and $f^{-1}((-\infty, \alpha_n]) \cap f^{-1}([\alpha_n, +\infty)) = f^{-1}(\alpha_n)$ is a closed connected set $(n = 1, 2, \ldots)$. Similar argumentation may be applied for sets $f^{-1}((-\infty, \beta_n)$ and $f^{-1}([\beta, +\infty))$. Using a method of the proof given by K. M. G a r g in the paper [3] (Lemma 32, p. 23) it is possible to show that $f^{-1}([\alpha_n, \beta_n])$ is a connected set (for $n = 1, 2, \ldots$).

Suppose that the assumptions of Theorem 2 are fulfilled (X is σ-coherent) then $f^{-1}(\alpha) = \bigcap_{h=1}^{\infty} f^{-1}([\alpha_n, \beta_n])$ is a connected set in X, thus $\alpha \in Y_c(f)$ which evidently yields to (*).

Therefore let us suppose that the assumptions of Theorem 3 are satisfied and assume that there exists $\alpha \in f(\overline{S_c(f)})$ such that $f^{-1}_{|\overline{S_c(f)}}(\alpha)$ is a nonempty, disconnected set. Then $f^{-1}(\alpha)$ is also a disconnected set, so $f^{-1}(\alpha) = P_1 \cup P_2$, where P_1, P_2 are nonempty separated sets in X. Since X is a T_5-space (see [1] Theorem 2.17, p. 97), there exist disjoint open sets U and V such that $P_1 \subset U$ and $P_2 \subset V$. For $n = 1, 2, \ldots$ let us choose $z_n \in f^{-1}([\alpha_n, \beta_n]) \setminus (U \cup V)$. The element z_n exists (for $n = 1, 2, \ldots$) because $f^{-1}([\alpha_n, \beta_n])$ is a connected set such that $U \cap f^{-1}([\alpha_n, \beta_n]) \neq \emptyset \neq V \cap f^{-1}([\alpha_n, \beta_n])$.

Then $f(z_n) \to \alpha$, thus $\{z_n\}_{n=1}^{\infty}$ is f-agree with $\overline{S_c(f)}$ and obviously $z_n \notin f^{-1}(\alpha)$ $(n = 1, 2, \ldots)$. Now there are two possible cases:

1. There exists an element x_* belonging to $\overline{S_c(f)}$ - $\operatorname{acp}_{n=1,2\ldots} z_n$. Then $x_* \notin f^{-1}(\alpha)$. Let $\{s_t\}_{t\in T}$ be a sequence finer than $\{z_n\}_{n=1}^{\infty}$ and $\{s_t\}_{t\in T}$ converge to x_*. It follows that $\lim_{t\in T} f(s_t) = \alpha$ and $\alpha \neq f(s_t)$ for $t \in T$ and $f(x_*) \neq \alpha$. Therefore:

$\alpha \in L(f_{|\{s_t:\ t\in T\}\cup\{x_*\}}, x_*) \setminus (\{f(x_*)\} \cup$

$\cup\ L^*(f_{|\{s_t:\ t\in T\}\cup\{x_*\}}, x_*))$.

But it is in contradiction with the fact that $f \in D_1$.

2. $\overline{S_c(f)} - \operatorname*{acp}_{n=1,2...} z_n = \emptyset$. Let us set $B = \{z_n:\ n = 1, 2,...\}$. Then $\alpha \in \operatorname{cl}_{f(\overline{S_c(f)}\cup\{z_n:\ n=1,2,...\})}(f(B)) \setminus f(B)$, which follows from the fact that $f \in D_3$.

The obtained contradictions have proved finally, that α is a two-sided accumulation point of a set $Y_c(f)$, then $f^{-1}_{|\overline{S_c(f)}}(\alpha)$ is a connected set.

Let us suppose now that α is not a two-sided accumulation point of the set $Y_c(f)$, but $f^{-1}_{|\overline{S_c(f)}}(\alpha)$ is nonempty disconnected set.

Then because of the continuity of $f_{|\overline{S_c(f)}}$, we have $f^{-1}_{|\overline{S_c(f)}}(\alpha) = \hat{P}_1 \cup \hat{P}_2$ where $\hat{P}_1$, $\hat{P}_2$ are closed, disjoint sets. Let U_1, U_2 be open, disjoint sets such that $\hat{P}_1 \subset U_1$ and $\hat{P}_2 \subset U_2$ and let $p \in \hat{P}_1$, $q \in \hat{P}_2$. We can notice that α is one-sided accumulation point of $Y_c(f)$. It results from facts: $p \in \overline{S_c(f)} \setminus S_c(f)$, α is a not two-sided accumulation point of $Y_c(f)$, the function $f_{|\overline{S_c(f)}}$ is continuous.

Suppose at the moment that there exist a sequence $\{\alpha_n\}_{n=1}^{\infty} \subset Y_c(f)$ such that $\alpha_n \nearrow \alpha$ and there is $\varepsilon > 0$ such that $[\alpha, \alpha + \varepsilon) \cap Y_c(f) = \emptyset$.

Let $\beta(p)$, $\beta(q)$ denote basis of a space X at the points p and q consisting of sets contained in U_1 and U_2, respectively. Then

$$(***)\ Y_c(f) \cap f(U_o) \cap f(V_o) \cap (\alpha - \tfrac{1}{n}, \alpha\,] \neq \emptyset \quad \text{for every } n \in \mathbb{N}$$

and arbitrary $U_o \in \beta(p)$, $V_o \in \beta(q)$.

Write $\Delta = \{(U, V, n):\ U \in \beta(p)$ and $V \in \beta(q)$ and $n \in \mathbb{N}\}$.

Let $\dashv$ be a relation directing the set Δ in the following way:

$(U, V, n) \dashv (U_1, V_1, n_1) \Leftrightarrow U \supset U_1$ and $V \supset V_1$ and $n \leq n_1$.

By condition (***), for each $\delta = (U, V, n) \in \Delta$ there exists $\gamma_\delta \in Y_c(f) \cap f(U) \cap f(V) \cap (\alpha - \frac{1}{n}, \alpha]$. Then $f^{-1}(\gamma_\delta)$ is a connected set such that $f^{-1}(\gamma_\delta) \cap U_1 \neq \emptyset \neq f^{-1}(\gamma_\delta) \cap U_2$.

Thus for arbitrary $\delta \in \Delta$ there exists $\omega_\delta \in f^{-1}(\gamma_\delta)$ such that $\omega_\delta \notin U_1 \cup U_2$. Now let us notice now that $\overline{S_c(f)} - \operatorname{acp}_{\delta\in\Delta} \omega_\delta = \emptyset$. Indeed, if it is not true, there is $\omega_o \in \overline{S_c(f)} - \operatorname{acp}_{\delta\in\Delta} \omega_\delta$. Then, because of the continuity $f_{|\overline{S_c(f)}}$ we have $\omega_o \in f^{-1}(\alpha)$. But it leads to a false conclusion that $(U_1 \cup U_1) \cap \{\omega_\delta: \delta \in \Delta\} \neq \emptyset$. Moreover, we may deduce that $\{\omega_\delta\}_{\delta\in\Delta}$ is f-agree with $\overline{S_c(f)}$.

Write $B_1 = \{\omega_\delta: \delta \in \Delta\}$ and $B_2 = \{\omega_\delta: \delta \in \Delta\}$. Let us notice that $\alpha \in cl_{f(\overline{S_c(f)})}(f(B_2)) \setminus f(B_2)$ and $\alpha \in cl_{f(\overline{S_c(f)})}(f(B_1)) \setminus f(B_1)$. This evidently shows that $f \notin D_2$ and $f \notin D_3$ is contrary to the assumptions of Theorems 2 and 3.

The contradiction obtained have ended a proof of the Theorems 2 and 3.

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O PROBLEMIE GARGA W ODNIESIENIU DO FUNKCJI DARBOUX

W prezentowanym artykule rozważany jest problem 3.11 z pracy [3], postawiony przez K. M. Garga, przy czym założenie, że rozpatrywane funkcje są spójne zastąpione zostało przypuszczeniem, że posiadają one własność Darboux.