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FIXED POINT INDEX FOR A NEW CLASS OF NONLINEAR NONCOMPACT OPERATORS

In the present paper a fixed point index for a large class of mappings in Banach spaces is defined and its properties are examined. This class contains all ultimately compact maps [8] and the identity as well.

1. INTRODUCTION

Most of infinite dimensional generalizations of fixed point index or degree theories has, as their basis, the Leray-Schauder theory [3]. Meanwhile the degree of DC-mappings (see [6], [2], [9]) is closely related to the finite dimensional degree of Brouwer, but it takes values in the large group $\mathcal{G} = \prod_{n=1}^{\infty} / \bigoplus_{n=1}^{\infty}$. Unfortunately, the definitions of DC-mappings and its degree depend on the choice of an increasing sequence of finite dimensional subspaces, the so-called filtration. Next, one of the authors [10] noticed then one can enlarged the class of maps and defined a degree for them, independent of the filtration. In this paper we shall widen again the class of mappings and define their fixed point index. Its properties are standard for the degree; we are not able to prove such important results for indices as the commutativity and the mod p-property. Our approach is similar to that of R. Nussbaum [7] for k-set contractions and B.N. Sadowski [8] for ultimately compact maps.

2. Definition and notations

Let E be a Banach space. By $\{E_t : t \in T\}$ we denote the set of all finite dimensional subspaces of E and by Σ – the set of all sequences $\sigma = (s_n)_{n=1}^{\infty}$ such that $s_n \in T$, $n = 1, 2, \dots$, and $E_{s_n} \subset E_{s_{n+1}}$ for any n. If $\sigma \in \Sigma$, we write

$$E_\sigma = \overline{\bigcup_{n=1}^{\infty} E_{s_n}}. \tag{2.1}$$

This means that $(E_{s_n})_{n=1}^{\infty}$ forms a filtration in E_σ (see [2]).

Let Ω be an open bounded subset of E and $f : \bar{\Omega} \to E$ – a continuous mapping. We put inductively:

$$\Delta^{(1)} = \overline{\mathrm{conv}} f(\bar{\Omega}); \quad \Delta^{(n+1)} = \overline{\mathrm{conv}} f(\Delta^{(n)} \cap \bar{\Omega}),\ n \geq 1;$$

$$\Delta = \bigcap_{n=1}^{\infty} \Delta^{(n)}. \tag{2.2}$$

The standard arguments show ([4], p.96), that $(\Delta^{(n)})_{n=1}^{\infty}$ is a decreasing sequence,

$$f(\Delta \cap \bar{\Omega}) \subset \Delta, \tag{2.3}$$

and that each fixed point of f belongs to Δ. If f is a k-set contraction with $k < 1$ or, more generally, (k, ϕ)–contraction, where ϕ is a measure of noncompactness, then Δ is a compact set (see [7], [8]).

If $\sigma = (s_n) \in \Sigma$, we shall write

$$\Delta_{s_n} = \Delta \cap E_{s_n}, \quad \Delta_\sigma = \Delta \cap E_\sigma. \tag{2.4}$$

Now, we are ready to define a class of mappings for which the fixed point index will be given.

Definition. Let $f : \bar{\Omega} \to E$ be continuous. We shall say that f satisfies condition Λ and write $f \in \Lambda(\Omega)$ if there exists $\sigma = (s_n) \in \Sigma$ such that

$$\overline{\bigcup_{n=1}^{\infty} (\Delta_{s_n} \cap \bar{\Omega})} = \Delta_\sigma \cap \bar{\Omega} \tag{2.5}$$

and

$$\lim_{n\to\infty} \sup_{x \in \Delta_{s_n} \cap \bar{\Omega}} d(f(x), E_{s_n}) = 0 \tag{2.6}$$

($d(y, A)$ is the distance between the point y and the set A).

Remark. If $\Delta \cap \bar{\Omega} = \emptyset$, both conditions are satisfied trivially. When f has fixed points, $\Delta \cap \bar{\Omega} \neq \emptyset$. In this case one can find many $\sigma \in \Sigma$ for which 2.5 holds and the only problem is that 2.6 take place.

3. Examples

(a) Let $\lambda : \bar{\Omega} \to \mathbb{R}$ be continuous and let I stands for the identity mapping on $\bar{\Omega}$. Then $\lambda I \in \Lambda(\Omega)$, since linear subspaces are invariant under λI.

(b) All ultimately compact maps $f : \bar{\Omega} \to E$, i.e. mappings for which Δ is compact, satisfy condition Λ.

In fact, let f be ultimately compact. If $\Delta \cap \bar{\Omega} \neq \emptyset$, take a countable dense subset $\{y_k : k \in \mathbb{N}\}$ in $\Delta \cap \bar{\Omega}$. Then put $E_n = \operatorname{lin}\{y_k : k \leq n\}$, $n = 1, 2, \ldots$, $\sigma = (n) \in \Sigma$ and 2.5 is satisfied. Let $\varepsilon > 0$. Choose a finite ε−net of $f(\Delta \cap \bar{\Omega})$ contained in $\{y_k : k \in \mathbb{N}\}$. This set – say $\{y_{k_1}, \ldots, y_{k_p}\}$ – is included in E_{k_p} and, for any $x \in \Delta \cap \bar{\Omega}$,

$$d(f(x), E_{k_p}) \leq \min_{1 \leq i \leq p} \|f(x) - y_{k_i}\| < \varepsilon.$$

It follows that condition 2.6 holds.

In particular, any k−set contraction or k−ball contraction with $k < 1$ satisfies condition Λ.

(c) Let $(E_n)_{n=1}^{\infty}$ be a filtration in E, i.e. $\sigma = (n) \in \Sigma$ and $E_\sigma = E$. If $f : \bar{\Omega} \to E$ is a DC-mapping, i.e.

$$\lim_{n\to\infty} \sup_{x\in\bar{\Omega}\cap E_n} d(f(x), E_n) = 0,$$

and the limit range Δ satisfies 2.5 with respect to σ, then $f \in \Lambda(\Omega)$.

4. Fixed point index

In our consideration we shall need the following

Lemma 4.1. *Let A be a closed subset of $\bar{\Omega}$ and let $\sigma = (s_n) \in \Sigma$ be such that*

$$\overline{\bigcup_{n=1}^{\infty}(A \cap E_{s_n})} = A. \tag{4.1}$$

If $f : A \to E$ has the property

$$\lim_{n\to\infty} \sup_{x\in E_{s_n}} d(f(x), E_{s_n}) = 0, \tag{4.2}$$

then there exists a mapping $f^* : \bar{\Omega} \cap E_\sigma \to E$ being an extension of f, taking values in $\overline{\mathrm{conv}} f(A)$ and satisfying condition 4.2. Similarly, if $H : \langle 0,1\rangle \times A \to E$ has the property

$$\lim_{n\to\infty} \sup_{t\in\langle 0,1\rangle,\, x\in E_{s_n}} d(H(t,x), E_{s_n}) = 0,$$

then there exists its extension $H^* : \langle 0,1\rangle \times (\bar{\Omega} \cap E_\sigma) \to E$ taking values in $\overline{\mathrm{conv}} H(\langle 0,1\rangle \times A)$ and having the same property.

Proof. Proof is based on the Michael Selection Theorem. It is sufficient to show only the second part. Let n_0 be the smallest positive integer such that $A \cap E_{s_{n_0}} \neq \emptyset$. Let us put

$$\Phi(t,x) = \begin{cases} \{H(t,x)\}, & (t,x) \in \langle 0,1\rangle \\ & \times A, \\ \overline{\mathrm{conv}} H(\langle 0,1\rangle \times (A \cap E_{s_{n_0}})), & (t,x) \in \langle 0,1\rangle \\ & \times (E_{s_{n_0}} \setminus A), \\ \overline{\mathrm{conv}} H(\langle 0,1\rangle \times (A \cap E_{s_n})), & (t,x) \in \langle 0,1\rangle \\ & \times (E_{s_n} \setminus \\ & (A \cup E_{s_{n-1}})), \\ \overline{\bigcup_{n=1}^{\infty} \overline{\mathrm{conv}} H(\langle 0,1\rangle \times (A \cap E_{s_n}))} & \text{other cases.} \end{cases}$$

It is easily seen that, for all $(t,x) \in \langle 0,1\rangle \times (\bar{\Omega} \cap E_\sigma)$, the set $\Phi(t,x)$ is nonempty, closed and convex. Moreover, the multivalued mapping Φ is lower semicontinuous.

In fact, let C be an arbitrary closed subset of E. We have to show that the set $\Phi^{-1}(C) = \{(t,x) : \ \Phi(t,x) \subset C\}$ is closed. There are three possible cases.

(i) $C \cap H(\langle 0,1\rangle \times A) \neq \emptyset$ and $\overline{\mathrm{conv}} H(\langle 0,1\rangle \times (A \cap E_{s_{n_0}})) \cap (E \setminus C) \neq \emptyset$. Then the set $\Phi^{-1}(C) = H^{-1}(C)$ and, from the continuity of H, is closed.

(ii) There exists $n \geq n_0$ such that $\overline{\mathrm{conv}} H(\langle 0,1\rangle \times (A \cap E_{s_n})) \subset C$ and $\overline{conv} H(\langle 0,1\rangle \times (A \cap E_{s_{n+1}})) \cap (E \setminus C) \neq \emptyset$. Then $\Phi^{-1}(C) = (\langle 0,1\rangle \times (\bar{\Omega} \cap E_{s_n})) \cup H^{-1}(C)$.

(iii) For each n, $\overline{\mathrm{conv}} H(\langle 0,1\rangle \times (A \cap E_{s_n})) \subset C$. Then $\Phi^{-1}(C) = \langle 0,1\rangle \times (\bar{\Omega} \cap E_\sigma)$.

We have thus obtained that $\Phi^{-1}(C)$ is always closed. From the Michael Selection Theorem [5] it follows that there exists a continuous mapping $H^* : \langle 0,1\rangle \times (\bar{\Omega} \cap E_\sigma) \to E$ such that $H^*(t,x) \in \Phi(t,x)$ for each (t,x). Obviously, H^* has all needed properties.

Now, we are in a position to define the fixed point index for $f \in \Lambda(\Omega)$. Let $f : \bar{\Omega} \to E$ satisfy condition Λ and let

$$\mathrm{dist}(\Delta, \partial\Omega) > 0$$

where $\operatorname{dist}(B, C)$ stands for the distance between the sets B and C (if $\Delta = \emptyset$, we set $\operatorname{dist}\Delta, \partial\Omega = \infty$). In particular, this means that f has no fixed points on $\partial\Omega$. If $\Delta \cap \bar{\Omega} = \emptyset$, we put

$$\operatorname{Ind}(f, \Omega) = \{\mathbf{0}\} \tag{4.3}$$

where $\mathbf{0}$ is the neutral element of the group

$$\mathcal{G} = \prod_{n=1}^{\infty} \mathbb{Z} / \bigoplus_{n=1}^{\infty} \mathbb{Z}.$$

If $\Delta \cap \bar{\Omega} \neq \emptyset$, fix $\sigma = (s_n) \in \Sigma$ such as in the definition of the class $\Lambda(\Omega)$. By Lemma 4.1, for $f|\Delta_\sigma \cap \bar{\Omega}$, there exists an extension $f^* : \bar{\Omega} \cap E_\sigma \to E$ taking values in Δ and being a DC-mapping with respect to the filtration $(E_{s_n})_{n=1}^{\infty}$. As $\partial_{E_\sigma}(\Omega \cap E_\sigma) \subset \partial\Omega \cap E_\sigma$ where ∂_{E_σ} denotes the boundary in E_σ, we have the inequality for $x \in \partial_{E_\sigma}(\Omega \cap E_\sigma)$:

$$||f^*(x) - x|| \geq \operatorname{dist}(\Delta, \partial\Omega) > 0.$$

Hence the degree of the DC-mapping $I - f^*$ on $\Omega \cap E_\sigma$ at the point 0 is defined and we put

$$\operatorname{ind}_\sigma(f, \Omega) = \operatorname{Deg}(I - f^*, \Omega \cap E_\sigma, 0) \in \mathcal{G}. \tag{4.4}$$

For the definition of degree of DC-mappings, we use [2] as a reference. We mention only that it is based on the possibility of the uniform approximation of DC-mappings by operators which map $\bar{\Omega} \cap E_{s_n}$ into E_{s_n} for sufficiently large n. For these operators, one can define a sequence of Brouwer's degrees on $\Omega \cap E_{s_n}$ and take its equivalence class in the group $\mathcal{G}$ as Deg.

Denote by $J(f)$ the set of all $\sigma \in \Sigma$ satisfying 2.5 and 2.6. We define the fixed point index of f on the set Ω by the formula:

$$\operatorname{Ind}(f, \Omega) = \{\operatorname{ind}_\sigma(f, \Omega) : \sigma \in J(f)\}. \tag{4.5}$$

The index is, in general, multivalued and takes values in $\mathcal{G}$ but, as will be shown later, has most of standard properties.

First, we have to show that the index is independent of the choice of the extension f^*. Let $\Delta \cap \bar{\Omega} \neq \emptyset$ and let $\sigma = (s_n) \in \Sigma$ be such as in the definition of the class $\Lambda(\Omega)$. Take two extensions f_0^* and f_1^* being DC-mappings from $\bar{\Omega} \cap E_\sigma$ into Δ. Due to the convexity of Δ, $H : \langle 0,1\rangle \times (\bar{\Omega} \cap E_\sigma) \to E$ defined by the formula

$$H(t,x) = (1-t)f_0^*(x) + tf_1^*(x)$$

takes values in Δ and, therefore, $(t,x) \mapsto x - H(t,x)$ is a DC-mapping with respect to the filtration $(\langle 0,1\rangle \times E_{s_n})_{n=1}^{\infty}$, such that

$$\inf_{t\in\langle 0,1\rangle, x\in\partial\Omega} \|x - H(t,x)\| > 0.$$

So, by the homotopy invariance of the degree of DC-mappings (see [2]),

$$\mathrm{Deg}(I - f_0^*, \Omega \cap E_\sigma, 0) = \mathrm{Deg}(I - f_1^*, \Omega \cap E_\sigma, 0).$$

By means of the above index, one can define a degree. Let $f : \bar{\Omega} \to E$ belong to $\Lambda(\Omega)$ and let

$$\mathrm{dist}(\Delta + \{y\}, \partial\Omega) > 0$$

where y is a point of E. Then the map $f + \bar{y}$ ($\bar{y}$ – the constant mapping into y) has property Λ and its limit range $\tilde{\Delta} = \Delta + \{y\}$, so $\mathrm{dist}(\tilde{\Delta}, \partial\Omega) > 0$. We define the degree of $I - f$ on Ω at the point y by the formula

$$\mathrm{D}(I - f, \Omega, y) = \mathrm{Ind}(f + \bar{y}, \Omega). \tag{4.6}$$

This degree generalizes the definition given in [10].

5. Properties of the index

We shall need the following

Lemma 5.1. *Let $f : \bar{\Omega} \to E$. Assume that B is a closed convex set containing Δ such that*

$$\mathrm{dist}(B, \partial\Omega > 0, \quad f(B \cap \bar{\Omega}) \subset B$$

and $g = f|B \cap \bar{\Omega}$ satisfies 2.5 and 2.6 for a certain $\sigma \in \Sigma$, where Δ is replaced by B. If f^* and g^* are extensions of $f|\Delta_\sigma \cap \bar{\Omega}$ and $g|E_\sigma$, respectively (as in Lemma 4.1), then

$$\operatorname{Deg}(I - f^*, \Omega \cap E_\sigma, 0) = \operatorname{Deg}(I - g^*, \Omega \cap E_\sigma, 0). \tag{5.1}$$

Proof. As $\Delta \subset B$ and $\operatorname{dist}(B, \partial\Omega) > 0$, both degrees in the assertion are defined. The mapping $h : (t, x) \mapsto x - (1-t)f^*(x) - tg^*(x)$ is a DC-homotopy and $||h(t,x)|| \geq \operatorname{dist}(B, \partial\Omega) > 0$ for $x \in \partial\Omega$, $t \in \langle 0, 1\rangle$ since $(1-t)f^*(x) + tg^*(x) \in B$. Hence 5.1 is a consequence of the homotopy invariance property for DC-mappings.

Now, we prove a form of the homotopy invariance theorem for maps belonging to $\Lambda(\Omega)$. Let $f_0, f_1 : \bar{\Omega} \to E$ be two mappings satisfying condition Λ and $J(f_0) = J(f_1)$. Let $H : \langle 0,1\rangle \times \bar{\Omega} \to E$. constuct, similarly as in Section 2,

$$\begin{aligned} \Gamma^{(1)} &= \overline{\operatorname{conv}} H(\langle 0,1\rangle \times \bar{\Omega}), \\ \Gamma^{(n+1)} &= \overline{\operatorname{conv}} H(\langle 0,1\rangle \times \Gamma^{(n)} \cap \bar{\Omega}), \quad n \geq 1, \\ \Gamma &= \bigcap_{n=1}^{\infty} \Gamma^{(n)}. \end{aligned} \tag{5.2}$$

We shall say that H satisfies condition Λ if there exists $\sigma = (s_n) \in \Sigma$ such that 2.5 holds for Γ instead of Δ, and

$$\lim_{n\to\infty} \sup_{\substack{t \in \langle 0,1\rangle,\\ x \in \Gamma_{s_n} \cap \bar{\Omega}}} d(H(t,x), E_{s_n}) = 0. \tag{5.3}$$

By $J(H)$ we denote the set of such σ's. A mapping H is called a homotopy between f_0 and f_1 if it satisfies condition Λ, $H(0,\cdot) = f_0$, $H(1,\cdot) = f_1$, $\operatorname{dist}(\Gamma, \partial\Omega) > 0$ and $J(H) = J(f_0) = J(f_1)$.

Theorem 5.1. *If there exists a homotopy H between f_0 and f_1, then the fixed point indices for f_0 and f_1 are defined and*

$$\operatorname{Ind}(f_0, \Omega) = \operatorname{Ind}(f_1, \Omega). \tag{5.4}$$

Proof. Let $\sigma = (s_n) \in J(H) = J(f_0) = J(f_1)$. Since H satisfies condition Λ, we can apply the second part of Lemma 4.1 to obtain a DC-mapping $H^* : \langle 0,1\rangle \times \bar{\Omega} \cap E_\sigma \to E$ taking values in Γ. As $\operatorname{dist}(\Gamma, \partial\Omega) > 0$ we get

$$\operatorname{Deg}(I - H^*(0,\cdot), \Omega \cap E_\sigma, 0) = \operatorname{Deg}(I - H^*(1,\cdot), \Omega \cap E_\sigma, 0).$$

Lemma 5.1 applied to $B = \Gamma$ and f_0 (resp. f_1) ends the proof.

Remark. In the definition of homotopy we claim that $J(H) = J(f_0) = J(f_1)$. Without this assumption a filtration good for Γ, need not satisfy even 2.5 for the limit ranges of f_0 and f_1.

The next result is usually called the excision property.

Theorem 5.2. *Let K be a closed subset of $\bar{\Omega}$ and let $f : \bar{\Omega} \to E$. Suupose that* $\operatorname{dist}(\Delta, K \cup \partial\Omega) > 0$ *and both f and $f|\Omega \setminus K$ satisfy condition Λ with that $J(f) = J(f|\Omega \setminus K)$. Then*

$$\operatorname{Ind}(f, \Omega) = \operatorname{Ind}(f, \Omega \setminus K). \tag{5.5}$$

Proof. Assume that $\sigma = (s_n) \in J(f) = J(f|\Omega \setminus K)$ and denote by Δ' the limit range of $f|\Omega \setminus K$. Obviously, $\Delta' \subset \Delta$, so we can apply Lemma 5.1 for $B = \Delta$ and two extensions: f^* with values in Δ' and g^* with values in Δ.

Remark. In our case the additivity property does not give anything more. Indeed, if $\Omega^{(1)}$ and $\Omega^{(2)}$ are disjoint open subsets of Ω, in order to obtain this result, we have to assume that

$$\operatorname{dist}(\Delta, \bar{\Omega} \setminus (\Omega^{(1)} \cup \Omega^{(2)}) > 0$$

and this implie $\Delta \subset \Omega^{(1)}$ or $\Delta \subset \Omega^{(2)}$, so one of the indices vanishes.

Notice that the index of any constant map $\bar{y} : \bar{\Omega} \to E$ equals $\{\mathbf{0}\}$ if $y = \bar{y}(\bar{\Omega}) \notin \bar{\Omega}$ or $\{\mathbf{1}\}$ if $y \in \Omega$, where $\mathbf{1}$ is an element of $\mathcal{G}$ represented by the constant sequence of $1 \in \mathbb{Z}$.

Theorem 5.3. *Let $f : \bar{\Omega} \to E$ have the index* $\operatorname{Ind}(f, \Omega) \neq \{\mathbf{0}\}$. *Then*

$$\inf_{x \in \Omega} \|x - f(x)\| = 0. \tag{5.6}$$

Proof. Let us take $\sigma = (s_n) \in J(f)$ such that, for the extension $f^* : \bar{\Omega} \cap E_\sigma \to \Delta$,

$$\mathrm{Deg}(I - f^*, \Omega \cap E_\sigma, 0) \neq \mathbf{0}. \tag{5.7}$$

Due to Theorem 2.1, [2] already mentioned in Section 4, for an arbitrary $\varepsilon > 0$, there exists $f^*_\varepsilon : \bar{\Omega} \cap E_\sigma \to E$ such that $f^*_\varepsilon(\bar{\Omega} \cap E_{s_n}) \subset \Delta_{s_n}$ (for large n) and

$$||f^*(x) - f^*_\varepsilon(x)|| < \varepsilon \tag{5.8}$$

for any x. Therefore, from the definition of degree of DC-mappings and 5.7 we have a positive integer n_ε such that

$$\deg((I - f^*_\varepsilon)|\bar{\Omega} \cap E_{s_{n_\varepsilon}}, \Omega \cap E_{s_{n_\varepsilon}}, 0) \neq \mathbf{0} \in \mathbb{Z}$$

where deg stands for the Brouwer degree. The standard property of this last degree implies the existence of x_ε such that

$$f^*_\varepsilon(x_\varepsilon) = x_\varepsilon. \tag{5.9}$$

Since $x_\varepsilon \in \Delta_\sigma \cap \bar{\Omega}$ and f^* equals f on this set, from 5.8 and 5.9 we get

$$||x_\varepsilon - f(x_\varepsilon)|| < \varepsilon. ||$$

The last theorem gives only the existence of ε–fixed points. In order to obtain that a fixed point exists, one can additionally assume that $I - f$ is closed or Ω is weakly compact and $I - f$ is demiclosed (see [1], [2]). All properties of Ind can be carried over to the case of degree D.

Now, we shall compare the new fixed point index Ind and the classical one γ for ultimately compact maps. The theory of the index γ can be found in [8].

Theorem 5.4. *Let $f : \bar{\Omega} \to E$ be an ultimately compact map such that $\Delta \cap \partial\Omega = \emptyset$. Then both indices $\mathrm{Ind}(f, \Omega)$ and $\gamma(f, \Omega)$ are defined and*

$$[(m)] \in \mathrm{Ind}(f, \Omega) \tag{5.10}$$

where $[(m)]$ is the equivalence class in $\mathcal{G}$ of the constant sequence of integers $m = \gamma(f, \Omega)$.

Proof. Since Δ is compact, $\mathrm{dist}(\Delta, \partial\Omega) > 0$ and, thereby, f has no fixed points on $\partial\Omega$. Hence both indices exist. Take $\sigma \in \Sigma$ such as in Section 3 (b). Then $f^* : \bar{\Omega} \cap E_\sigma \to \Delta$ is a compact mapping and, by applying Theorem 4.6, [2], we get

$$\begin{aligned} \mathrm{ind}_\sigma(f, \Omega) &= \mathrm{Deg}(I - f^*, \Omega \cap E_\sigma, 0) \\ &= [(\deg_{LS}(I - f^*, \Omega \cap E_\sigma, 0))] \end{aligned}$$

where $\deg_{LS}$ denotes the Leray-Schauder degree.
But $\gamma(f, \Omega) = \deg_{LS}(I - f^*, \Omega \cap E_\sigma, 0)$ by definition.

The theorem can be applied, in particular, to k-set contractions, which gives 5.10 where m is Nussbaum's fixed point index.

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INDEKS PUNKTU STAŁEGO DLA NOWEJ KLASY OPERATORÓW NIELINIOWYCH NIEZWARTYCH

W pracy zdefiniowany jest indeks punktu stałego dla szerokiej klasy odwzorowań zawierającej między innymi odwzorowania granicznie zwarte i DC-odwzorowania. Jest to indeks wielowartościowy w grupie asymptotycznych ciągów liczb całkowitych i ma większość standardowych własności z wyjątkiem komutatywności i mod *p*-własności.

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