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SEQUENCE OF EXPERIMENTAL TRIALS AND FACTORIAL DESIGN OF EXPERIMENT REALIZATION COSTS

Abstract. Design of experiment is one of statistical methods of quality control which allows to appropriate preparation of the production process. Statistical design of experiment allows to set the level and to determine the influence of factors on the results of realized process as well as helps to improve its economic results.

The aim of this article is the issue of the proper order of experimental trials realization of certain design of experiment in order to reduce the cost of its implementation. For this particular problem will be presented an algorithm of design of experiment which uses properly constructed matrix. Proposed method of the experiment will be compared with cost of realization of classical factorial design of experiments.

Key words: Design of experiment, Factorial design, Sequence of experimental trials.

I. INTRODUCTION

The purpose of statistical process control is the improvement of the production process, which leads to maintaining a high level of products quality. Methods used in statistical quality control allow to monitor production process and to determine its capabilities, variability and results. One of the tools of statistical quality control is design of experiments which is an active method that improves the quality of product and economic results of the process.

II. THE THEORY OF DESIGN OF EXPERIMENT

Design of experiments requires the use of suitable rules to realize following experimental trials. The rules preceding production process stage may be presented in the following points (Montgomery, 1997):

– recognition and statement of a problem to determine all the aspects, circumstances and potential objectives of an experiment,

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– choice of factors and description of their levels, ranges over which these factors will be varied as well as determination of a possibility to take them into account in the experiment,

– selection of response variable which provides useful information about the process under study,

– choice of a proper experimental design to determine a number of experiments and possible randomization restrictions,

– performance of the experiment,

– data analysis using statistical methods,

– conclusions and recommendations for a described process following the data analysis.

Experimental trial is a single act of obtaining variable values of a described Y, when each of the factors $X_1, X_2, \ldots, X_m$ is fixed, then the experiment is a sequence of *n* experimental trials. Let the $\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_m$ represent sets of all possible values of factors $X_1, X_2, \ldots, X_m$, then the experimental area is a set of points $\mathrm{x} = (x_1, x_2, \ldots, x_m)$, where $x_i \in \mathrm{X}_i, i = 1, 2, \ldots, m$. A set of pairs

$$P_n = \{\mathrm{x}_j, p_j\}_{j=1}^{n} \tag{1}$$

defines the design of the experiment with *n* experimental trials, where $\mathrm{x}_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ and $p_j = \frac{n_j}{n}$, and n_j is a number of experimental trials in the point x_j of the experimental area, moreover $\sum_{j=1}^{n} n_j = n$ and $\sum_{j=1}^{n} p_j = 1$ for $j = 1, 2, \ldots, n$.

Usually an experimental research involves analyzing the influence of some number of non-random factors $X_1, X_2, \ldots, X_m$ on the result of variable *Y*, but random factors may also have an impact on the starting variable *Y*. This correlation may be presented with the following statistical model (Wawrzynek, 2009)

$$Y(X_1, X_2, \ldots, X_m) = y(X_1, X_2, \ldots, X_m) + \varepsilon \tag{2}$$

where $EY(X_1, X_2, \ldots, X_m) = y(X_1, X_2, \ldots, X_m)$, $E\varepsilon = 0$ and $V\varepsilon = \sigma^2$, where σ^2 is a constant value. The object of the statistical research will be a function $y(x_1, x_2, \ldots, x_m)$ called a response surface. Arguments of the response surface are *m* realization of the non-random factors $X_1, X_2, \ldots, X_m$. The model (2) can be presented as a general linear model (Wawrzynek, 1993), then to estimate the parameters of the response surface function uses the method of least squares (Aczel, 2000).

Generally used in practice of manufacturing company designs of experiments are full and fractional factorial designs of experiments, the realization of which allows to estimate the parameters of response surface function.

III. COSTS OF FACTORIAL DESIGN OF EXPERIMENT REALIZATION

Among statistical tools used in design of experiments there can be distinguished regression analysis, analysis of variance and significance tests. In order to apply all statistical methods the assumptions of sample randomness must be fulfilled. In design of experiment theory the mentioned randomness of sample is a random selection of point of experimental area in which further experimental trials should be carried out (Rasch, Herrendorfer, 1991). Carrying out the experimental trials at random points of experimental area in practice may have a direct influence on elongation time and increased cost of realization of experiment costs, which as a consequence, is connected with time and costs of production process. The problem of dependence of costs of experiment realization and order was first noticed by N. R. Draper and D. M. Stoneman (1968) in their publication and later also by B. L. Joiner and C. Campbell (1976). These authors condition the cost of realization of design of experiment, taking into account the order of experimental trials, on the costs of levels changes of factors included in the experiment.

Based on the expertise of the experimenter or on the historical data, one can determine the costs of level changes of factors taken into account in the design of experiment and combine them in a matrix. So let us consider design of experiment which takes into account *n* point of experimental area in which one should realize further experimental trials. For this particular design of experiment define a matrix **K** of the form

$$\mathrm{K} = \begin{bmatrix} k_{11} & k_{12} & \ldots & k_{1n} \\ k_{21} & k_{22} & \ldots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \ldots & k_{nn} \end{bmatrix}, \qquad (3)$$

where k_{ij} is a cost of the transition from the i-th point of experimental area to the j-th point, for $i \neq j$, or the cost of changing the levels of the various factors included in the experiment, wherein for $i = j$ the element k_{ij} is equal to 0. Such constructed matrix will be called the matrix of the cost of the factorial design of experiment involved n experimental trials.

IV. ALGORITHM OF IMPLEMENTATION FACTORIAL DESIGN OF EXPERIMENT INCLUDING COSTS MATRIX

In this chapter will be presented the algorithm of carrying out the factorial experiment which allows the cost reduction of realization of experiment by using a specific matrix expenses determined in the previous chapter.

Let the construction of factorial design of experiment consist of the following steps:

1. Determination of factorial design of experiment considers n points of experimental area defined as

$$P_n = \{\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_n\}, \tag{4}$$

that is to determine all points of experimental area in which further experimental trials should be achieved.

2. Construct the matrix **K** of cost of factorial design of experiment involving n experimental trials, based on historical data or expert knowledge of the experimenter.

3. Random selection of the starting point x_{m_1}, for $m_1 \in \{1,2,\ldots,n\}$, of constructed design of experiment in which first experimental trial should be realized.

4. Based on m_1-th row of matrix **K**, determination of point of experimental area for which cost of changing levels of the factors is the smallest. If there is more than one point, then the next point of experimental area is randomly selected from the indicated points of the smallest cost of factor level changes.

5. Repeat step 4 of this algorithm to implement a single experimental trial at any point of the experimental area.

Following the above algorithm will result in designation of a scheme of experimental trials realization of full factorial design of experiment presented in the form below

$$P_n = \{\mathrm{x}_{m_1}, \mathrm{x}_{m_2}, \ldots, \mathrm{x}_{m_n}\},\ m_i \in \{1, 2, \ldots, n\}, \tag{5}$$

taking into consideration the cost of implementation of experimental trials, therefore the cost of level changes, at all points of experimental area taken into account in the design.

V. COST OF CLASSICAL AND ALTERNATIVE FACTORIAL DESIGN OF EXPERIMENT REALIZATION – A NUMERICAL EXAMPLE \

Take into consideration the experiment which takes into account factors X_1 and X_2 occurring at four and three levels respectively. Then full factorial design of experiment includes twelve experimental trials as well as the experimental area consists of twelve points. Let the cost matrix be determined as follows

$$K = \begin{bmatrix} 0 & 5 & 10 & 15 & 5 & 7 & 13 & 17 & 10 & 13 & 14 & 23 \\ 5 & 0 & 5 & 10 & 7 & 5 & 10 & 13 & 13 & 10 & 13 & 14 \\ 10 & 5 & 0 & 5 & 13 & 7 & 5 & 7 & 14 & 13 & 10 & 13 \\ 15 & 10 & 5 & 0 & 17 & 13 & 7 & 5 & 23 & 14 & 13 & 10 \\ 5 & 7 & 13 & 17 & 0 & 5 & 10 & 15 & 17 & 7 & 13 & 17 \\ 7 & 5 & 7 & 13 & 5 & 0 & 5 & 10 & 7 & 5 & 7 & 13 \\ 13 & 10 & 5 & 7 & 10 & 5 & 0 & 5 & 13 & 7 & 5 & 7 \\ 17 & 13 & 7 & 5 & 15 & 10 & 5 & 0 & 17 & 13 & 7 & 5 \\ 10 & 13 & 14 & 23 & 17 & 7 & 13 & 17 & 0 & 5 & 10 & 15 \\ 13 & 10 & 13 & 14 & 7 & 5 & 7 & 13 & 5 & 0 & 5 & 10 \\ 14 & 13 & 10 & 13 & 13 & 7 & 5 & 7 & 10 & 5 & 0 & 5 \\ 23 & 14 & 13 & 10 & 17 & 13 & 7 & 5 & 15 & 10 & 5 & 0 \end{bmatrix}.$$

For this specific experiment there has been determined a scheme of realization of experimental trials for factorial design of experiment randomly (classical approach) and in accordance with the presented algorithm (alternative approach). Designated schemes, for the classical and alternative approaches respectively, are as follows:

$$P_{12} = \{x_{10}, x_8, x_2, x_7, x_3, x_6, x_9, x_{12}, x_1, x_5, x_4, x_{11}\}$$

$$P_{12}^* = \{x_{10}, x_{11}, x_7, x_3, x_2, x_6, x_4, x_8, x_{12}, x_9, x_1, x_5\}.$$

It is worth noticing that both, for the classical and alternative design of experiment, the starting point is the point x_{10} and both designs of experiments appear identical sequences of consecutive points of experimental area.

For the designated schemes of experimental trials realization, the assumed matrix cost, the cost of the individual experiments has been calculated. In the case of random order of experimental trials the cost of experiment realization was 128, while in the case of the implementation experimental trials in accordance with the presented algorithm the cost of experiment was 78, therefore, was much lower. The results demonstrate the significant effect of the order of the experimental trials on the cost of experiment, and presented algorithm of design of experiment construction allows to determine design of experiment so that the cost of the experiment is as low as possible.

VI. SUMMARY

Design of experiments as one of the methods of statistical process control can improve the quality of products and have an influence on the economic results of manufacturing process. Carrying out of experiment is the realization of sequence of experimental trials which allows to provide information about capabilities of production process. The realization of experimental trials in classical designs of experiments typically occurs randomly, which significantly affects the cost of implementing the design of experiment. Presented algorithm of determining the order of points of experimental area, using appropriately constructed matrix of costs, helps to reduce the cost of factorial design of experiment realization, which involves the overall cost of the production process.

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KOLEJNOŚĆ DOŚWIADCZEŃ A KOSZTY REALIZACJI PLANU EKSPERYMENTU CZYNNIKOWEGO

Planowanie eksperymentów jest jedną z metod statystycznej kontroli jakości procesu produkcyjnego pozwalającą na jego właściwe przygotowanie. Statystyczne planowanie eksperymentów umożliwia ustalenie poziomu oraz określenie wpływu uwzględnionych czynników na efekty realizowanego procesu, jak również ma wpływ na poprawę jego ekonomicznych rezultatów.

Przedmiotem artykułu jest zagadnienie odpowiedniej kolejności wykonywania doświadczeń pewnego planu eksperymentu w celu zmniejszenia kosztów jego realizacji. Dla tak określonego problemu zaprezentowany został algorytm realizacji planu eksperymentu wykorzystujący odpowiednio skonstruowaną macierz kosztów. Proponowana metoda przeprowadzenia eksperymentu została porównana z kosztami realizacji klasycznych planów eksperymentów czynnikowych.