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INFLUENCE OF INACCURACY MEASUREMENT ON MIDDLE-OPERATING CONTROL COSTS

Abstract. Control charts are commonly used in plants to monitor the correctness of manufacturing process, due to their simplicity and explicitness of diagnosis. Qualification mistakes make that the signals, appear either with lag, or in case of regulated process. False signals, as well as lack of appropriately early signal make financial loss. The purpose of this article is to pay attention to shaping of middle – operating control costs, especially in case of appearance of qualification mistakes. The accepted hypothesis states that qualification mistakes cause considerable increase in the middle – operating control costs.
Key words: Middle – operating control costs, qualification mistake.

I. INTRODUCTION

Control chart being a simple and effective tool of statistical quality control is commonly used in manufacturing to monitor the production process. Explicitness of diagnosis of the production process, is often disturbed by incorrect estimation of quality of elements. As a result of these mistakes, the control chart may give false signals, in case of regulated process, and not indicate or indicate with a lag, that something is wrong. Situations like these are source of redundant costs borne by the company.

II. CONTROL CHART

Control chart is a tool, which enables statistical monitoring of the production process. It gives information about running of the process, and it contributes to the rise in production quality. Its characteristic features are simplicity and effectiveness. A huge advantage of the control chart is its possibility to indicate the moment, in which it is necessary to search for causes of process fluctuation.

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Points configuration in relation to levels of control and warning lines are fundamental to conclude about correctness of the process run (Kończak, 2007).

Control charts are described by operating characteristic curves, which give a probability that production process is regulated. It is a measure of the sensitivity of the control chart on upsetting the process (Montgomery, 1997). The second characteristic of the control chart is ARL, which is the most universally used criterion to compare the control charts. In case of assuming only crossing upper or lower control line, it is expressed by the formula:

$$ARL = \frac{1}{p_s} \tag{1}$$

where p_s is a probability of appearance a signal for single observation (sample).

III. COST OF QUALITY CONTROL

The concept of quality control costs means all enterprise's costs connected with carrying out quality control. Among costs of statistical quality control, there were distinguished costs of prevention, costs of examination and estimation, loss of internal lack and loss because of external lack (Iwasiewicz, 1999).

The total middle – operating control cost consists of costs of examination and estimation, loss because of internal and external lack. The costs of examination and estimation depend on costs of control single element (k_k) and on amount of controlled pieces.

Loss because of internal lack, in case of middle – operating control, includes the cost of exchange (or repair) of defective elements detected during control and the cost of regulation process, which is made when the signal appears on the control chart. The cost of regulation involves the cost of analysis which causes defects, the costs connected with a halt, a downtime and a restart of the production process, and the costs of not meeting the deadline of a delivery. The cost of internal lack, is a random variability whose value depends on the number of defective elements detected in controlled samples and on the number of regulations of the production process. The number of defects detected in a single sample is a random variable (X_D), which depends on defectiveness of the production process *(w)* and number of sample *(n)*. Its expected value is the following:

$$E(X_D) = w \cdot n \tag{2}$$

The number of regulations of the production process is also a random variable (X_R), whose value depends on probability of appearance of a signal (p_s) and on the number of measurements registered on the control chart (*N*). Its expected toward value is the following:

$$E(X_R) = p_s \cdot N \tag{3}$$

Let us denote the single cost of repair (exchange) of defects as k_n, the single cost of the production process regulation as k_r, the number of defects detected in i – sample as d_i, and the amount of the production process regulation as r, the cost of internal lack in case of middle – operating control may by presented by the formula:

$$K_{bw} = N \cdot k_n \cdot \sum_{i=1}^{N} d_i + k_r \cdot r \tag{4}$$

And its random variability may be presented by the formula:

$$E(K_{bw}) = w \cdot n \cdot N \cdot k_n + p_s \cdot N \cdot k_r \tag{5}$$

Loss because of external lack in case of run in company the acceptance control is becoming an accepted cost of internal lack, and in case of lack acceptance procedure it involves the exchange of defective elements, guarantee repairs, loss as a result of forfeiture of customers and reputation of company. It depends on the number of defective elements, which penetrate through the control and on value of single external lack for middle – operating control (k_{bz}).

Presented structure of the middle – operating control cost is correct under the assumption of faultless estimation of quality of the products. Unfortunately, in practice there are no statistical quality control methods, which are perfect. Consequently, neither the regulation of the incorrect process, nor a lack of regulation of the correct one is a certain event (Iwasiewicz, 2005).

In case of running attributes control chart, an element, which is controlled may be judged as defective, if it is really defective or in case of being correct and being falsely categorized. Similarly, an element, which is controlled may by judged as correct, if it is really correct, or in case of being a defect and being falsely categorized. According to this, the probability of categorized elements as defected is as follows:

$$p^{*} = p(1-\varepsilon) + q\eta \tag{6}$$

where p is a real defectiveness of elements, ε is a probability of commitment qualification mistake relied on categorize defective element as correct element, η is a probability of commitment qualification mistake relied on categorize correct element as defective element.

The probability of categorized element as correct is the following:

$$q^{*} = 1 - p^{*} \tag{7}$$

The middle – operating control costs in case of appearance of qualification mistakes should supplement also with cost of redundant repair of elements, which are in fact correct and which were categorized as defects (k_{zn}). This cost embraces the exchange or repair of correct elements and cost of renewed quality control of those elements after conducted repair.

IV. SIMULATION ANALYSIS

Getting precise result of the total middle – operating control costs is very difficult, it is often impossible in practice. For the purpose of estimation of those costs, computer simulations were carried out. The middle – operating control costs connected with p control chart were set in case of correct control as well as in case of appearance of the qualification errors. In analyses assumed, that the conducted control is a full control, in which as an out – of – control signal, a crossing upper or lower control line is recognized. In simulations it was assumed that the cost of regulations is equal $k_r = 300$ units, the single cost of repair is $k_n = 1$ unit, the cost of external lack is $k_{bz} = 30$ units, and the cost of redundant repair of flawless elements is $k_n = 1$ unit. Calculations were conducted for branch of different number in case of the in-control process, as well as the out-of-control one. The cost of examination and estimation was not analyzed, because of equal joint amount of controlled elements in all cases. This means, that the cost of examination and estimation was also on the same level.

During generating qualification mistakes, assumed, that the probability of committing qualification mistake relied on categorizing correct element as defected is equal $\eta = 0{,}001$, and the probability of committing qualification mistake relied on categorizing defect as correct element is equal $\varepsilon = 0{,}005$. Moreover, we assumed, that the defectiveness of the monitored production process is equal $w = 2\%$ (Iwasiewicz, 2005).

The simulation procedure is as follows. A production process, whose defectiveness is equal w, is given. Chosen characteristic of the product is estimated in attributed way, making the p control chart. For N controlled elements there should be $N \cdot w$ defects. Qualification mistakes which occur during control are generated in the procedure with a given probability η and ε. In the analysis, it was counted how many times correct element was categorized as defected, this sum was multiplied by its single cost of repair getting the cost of redundant repair of flawless elements. It was also counted how many times defective element was categorized as correct, what in case of full control is a source of loss of external lack. In analysis it was also calculated how many times the committed mistakes caused incorrect estimation of production process. Information like this allowed for estimation cost of regulation in case of appearance of mistakes, as well as in case of faultless control. The whole procedure was repeated 1,000 times and the results were averaged.

Table 1 presents value of probability of appearance a signal on the p control chart in dependence of number of samples and in dependence of level of shift in process defectiveness in case of faultless control, as well as in case of appearance of qualification mistakes.

According to Table 1 the probability of appearance a signal decreases with the increase of the number of controlled samples, independently of the correctness of the conducted control. It should be noticed, that this probability in case of appearance the qualification mistakes, independently of the size of sample and independently of the level of shift in defectiveness, is bigger than in case of faultless control. It results from the fact, that the mistake relied on categorizing correct element as defective one, in spite of the probability of oversight defect is bigger. This dependence is, of course, reflected in shaping middle – operating control cost (Table 2).

Table 1. The probability of appearance a signal on the p control chart in dependence of number of sample, shift in process defectiveness and in dependence of correctness of control process

The shift in the process defectiveness	Control	Number of sample		
		n=50	n=200	n=500
Lack of shift	Regulated	0,0180	0,0077	0,0032
	Inregulated	0,0207	0,0101	0,0050
Shift to a level of 2,1%	Regulated	0,0209	0,0104	0,0053
	Inregulated	0,0238	0,0134	0,0081
Shift to a level of 2,2%	Regulated	0,0242	0,0138	0,0085
	Inregulated	0,0273	0,0175	0,0129
Shift to a level of 2,3%	Regulated	0,0278	0,0181	0,0133
	Inregulated	0,0312	0,0226	0,0189

Source: Own elaboration.

On the basis of Table 2 it may be stated that together with an increase in the number of controlled samples, the middle – operating costs decrease. From the table above it also follows that the analyzed costs are higher in case of the control with appearing mistakes than in case of the faultless control. This is caused by the fact, that categorizing a correct element as defective one, produces the cost of its redundant repair and it may cause receiving a false signal on the control chart, what obviously results in the cost of redundant process regulation. Moreover, oversight of defect may result in giving a customer the incompatible batch of products, what is a source of the loss because of external lack.

Table 2. The middle – operating control cost in dependence of number of sample, shift in process defectiveness and in dependence of correctness of control process

The shift in the process defectiveness	Control	Number of sample			
		n=50	n=100	n=200	n=500
Lack of shift	Regulated	6327	1669	1635	1085
	Inregulated	7431	2031	1970	1339
Shift to a level of 2,1%	Regulated	7257	1797	1635	1214
	Inregulated	8390	2158	1970	1488
Shift to a level of 2,2%	Regulated	8726	2041	2206	1354
	Inregulated	9985	2448	2644	1656
Shift to a level of 2,3%	Regulated	9717	2452	2480	1522
	Inregulated	11046	2935	3011	1914

Source: Own elaboration.

Detailed analysis of a structure of the control costs confirms this dependence. Table 3 presents the structure of the middle – operating costs by assumption, that on the control chart there were registered 2500 results coming from analysis of 200-element samples, appropriately in case of correct and incorrect run of control process. Their analysis leads to the conclusion that independently of correctness of the production process, as well as correctness of the control process, the significant part of analysis middle – operating costs constitutes the cost of repair of defects. This cost points inheritance a tendency in relation to the total middle – operating control cost with increase of defectiveness of the production process. Its value is always lower in case of defective efficiency of diagnosis than in case of the correct control.

Table 3. The structure of middle – operating control cost

Process	Control	Cost of repair of defective elements	Cost of redundant repair of flawless elements	Cost of regulation process	Loss because of external lack
In-control	Regulated	991,96	0	93	0
	Inregulated	986,83	48,54	150	153,9
Out-of- control	Regulated	1150,45	0	372	0
	Inregulated	1144,74	49,12	549	171,3

Source: Own elaboration.

The cost of regulation is shaped quite differently. Dependence between its share in the total middle – operating control cost and the level of shift in defectiveness of the production process is positive. Moreover this cost achieves higher values in case of incorrect control process, than in case of correct estimation of products quality. In case of the control during which the qualification mistakes occurred in the structure of the middle – operating costs, next to the cost of repair of defects and the cost of regulation, there are also two additional components – the cost of redundant repair of flawless elements and the cost of external lack, which in case of full control are directly connected with committed qualification errors.

Summing up the middle – operating control cost born in case of incomplete diagnostic efficiency is higher than the cost which might be born if the qualification mistakes were not committed. It is necessary to pay attention to the fact that in practice independently on awareness of incomplete diagnostic efficiency, in case of occurring qualification errors, the cost of repair of flawless elements is not observed. Other components of the middle – operating control cost are observed, but the cost of repair or exchange defects is really the cost of repair or exchange of elements categorized as defects during control.

V. CONCLUSION

On the basis of the carried out simulation analysis, the accepted hypothesis, which stated that the qualification mistakes cause substantial increase of the middle – operating control costs, should be acknowledged as true. The errors committed are not only the source of additional costs like the cost of repair of flawless elements, or the cost of external lack, but also disturb a value of internal lacks (the cost of repair or exchange of defects and cost of process regulations).

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O WPŁYWIE NIEDOKŁADNOŚCI POMIARÓW NA KOSZTY KONTROLI MIĘDZYOPERACYJNEJ

Karta kontrolna jest powszechnie stosowana do monitorowania prawidłowości przebiegu procesów produkcyjnych, ze względu na prostotę prowadzenia i jednoznaczność diagnozy. Popełniane błędy obniżają jednak jej skuteczność. Błędne sygnały, jak i brak odpowiednio wczesnego sygnału narażają przedsiębiorstwo na straty. Celem niniejszego artykułu jest zwrócenie uwagi na kształtowanie się kosztów kontroli międzyoperacyjnej, szczególnie w przypadku występowania błędów kwalifikacji. Postawiona została hipoteza głosząca, iż błędy kwalifikacji przyczyniają się do znacznego wzrostu kosztu kontroli międzyoperacyjnej.